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FORMATION OF DARK-STATE POLARITONS AND TWO-POLARITON BOUND STATES IN ARRAYS OF ATOMS AND OPTICAL CAVITIES

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Анђело Мађити

ФОРМИРАЊЕ ТАМНИХ ПОЛАРИТОНА И ДВО-ПОЛАРИТОНСКИХ ВЕЗАНИХ СТАЊА У НИЗОВИМА АТОМА И ОПТИЧКИХ МИКРОРЕЗОНАТОРА

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Day of the defense:

Dedicated to my family and friends...

Захвалница

На пруженој подршци током истраживања, а и ван сарадње на изради доктората, дубоко бих се захвалио ментору др Милану Радоњићу и коментору др Бранislаву Јеленковићу који су бројним дискусијама и саветима допринели стварању ове тезе и проширењу мог знања превашодно из области квантне оптике, али и из физике уопште.

Даље бих се захвалио колегама са Института за физику и Физичког факултета у Београду који су ми пружили подршку и пријатељство током целог доктората. Опуштена дружења и разговори који нису били непосредно везани за физику допринели су свеукупној позитивној атмосфери.

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Својим дивним родитељима, мајци Гордани и оцу Алфреду желим најискреније да се захвалим на пруженој љубави и подршци, не само током доктората него и током целог свог живота.

Београд, 25. децембар 2015.

FORMATION OF DARK-STATE POLARITONS AND TWO-POLARITON BOUND STATES IN ARRAYS OF ATOMS AND OPTICAL CAVITIES

Abstract

This thesis covers a theoretical analysis of non-interacting and interacting quasi particles, called polaritons. Polaritons are composites, based on photonic and atomic excitations in a tunable and controlled manner. Dark-state polaritons, as a subclass of polaritons, are very curious objects, as they can act as a quantum memory of photons within an ensemble of alkali-metal atoms in Λ-type configuration which admit electromagnetically induced transparency (EIT). This thesis focus on the formation of dark-state polaritons in degenerate two-level systems, where two light fields, a quantum probe and a classical driving field, couple the same transition between the ground state and excited state manifold. An algorithm is going to be derived in order to determine the dispersion relation and inherent composition of the dark-state polaritons in the degenerate two-level system. The algorithm is based on a microscopic equation of motion technique and provides an extension of the non-degenerate case. Depending on the polarization of the light fields, it will be shown that either one or two dark-state polaritons can exist. Further, the calculated dark-state polaritons can be used in order to perform a frequency conversion and even a conversion of linear light polarization.

In the second part of the thesis, interacting polaritons in arrays of coupled QED cavities will be discussed. In first place, the standard Jaynes-Cummings model is discussed in order to provide a better insight to polaritons in QED cavities. Then, a modified Jaynes-Cummings model is introduced and derived in order to study dark-polaritons and their interaction. It will be shown that dark-polaritons directly depend on the common single photon detuning of the two coupling fields, quantum probe and classical driving field. The interaction between dark-polaritons will be discussed where the emergence of dark-polariton bound pairs will be shown. By tuning the Stark-shift due to the classical control field, the number and the composition of dark-polariton bound pairs can be controlled. It will be demonstrated that there exists a dark-polariton bound pair which differs from the aforementioned ones, because it represents a ground state of the system. Using that kind of a dark-polariton bound pair, storage and retrieval of a single photon can be performed, even though that the photons are in a two-photon bound state.

In the last part of the thesis, disorder between QED cavities will be introduced in a controlled manner through staggered inter-cavity photon hopping strengths J_1 and J_2. Further, the appearance of a dark-polariton bound pair of a completely

different type will be presented.

Keywords: EIT, dark-state polaritons, cavity QED arrays, interacting dark-polaritons, dark-polariton bound pairs, quantum memory of light and disorder

Scientific field: Physics

Research area: Quantum Optics

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ФОРМИРАЊЕ ТАМНИХ ПОЛАРИТОНА И ДВО-ПОЛАРИТОНСКИХ ВЕЗАНИХ СТАЊА У НИЗОВИМА АТОМА И ОПТИКИХ МИКРОРЕЗОНАТОРА

Резиме

Ова теза представља теоријску анализу неинтерагујућих и интерагујућих квази-честица, тзв. поларитона. Поларитони су комбинације фотонских и атомских ексцитација у контролисано променљивом односу. Врста поларитона, тамни поларитони, су веома необични објекти будући да могу да служе као квантна меморија за фотоне унутар ансамбла атома алкалних метала у Λ-конфигурацији, а у вези са ефектом електромагнетно индуковане транспаренције (ЕИТ). Теза се фокусира на проучавање формирања тамних поларитона у системима са два нивоа, основним и побуђеним, који поседују многострукости дегенерисаних поднивоа и спрегнути су квантним пробним и класичним контролним ласерским пољем. Биће изведен алгоритам за добијање дисперзионе релације и одговарајућег састава тамних поларитона у системима са два-нивоа и дегенерацијом. Алгоритам је заснован на техници микроскопских једначина кретања и представља проширење случаја без дегенерације. Биће показано да у зависности од поларизације поља могу постојати један или два тамна поларитона. Тако добијени тамни поларитони могу бити коришћени за конверзију фреквенције и линеарне поларизације светлости.

У другом делу тезе разматраћемо интерагујуће поларитоне у низовима спрегнутих квантно-електродинамичких оптичких микрорезонатора. Као прво, приказан је стандардни Џејнс-Камингсов модел да би се обезбедио бољи увид у поларитоне унутар оптичких микрорезонатора. Затим је уведен модификовани Џејнс-Камингсов модел ради проучавања тамних поларитона и интеракције међу њима. Биће показано да тамни поларитони директно зависе од заједничке једнофотонске раздешености (детјунинга) коришћених поља, квантног пробног и класичног контролног. Интеракција између тамних поларитона ће бити продискутована, при чему ће бити показана појава везаног пара тамних поларитона. Подешавањем Штарковог помераја услед контролног поља може се утицати на број и особине тих везаних парова. Биће показано да постоји везани пар тамних поларитона који се разликује од поменутих пошто представља основно стање система. Такав везани пар тамних поларитона омогућава заробљавање и ишчитавање једног од два везана фотона. У последњем делу тезе биће уведено контролисано неуређење у облику наизменичних параметара спрезања међу микрорезонаторима, J_1 и J_2, и биће показана појава потпуно новог типа везаног пара тамних поларитона.

Кључне речи: електромагнетно индукована транспаренција (ЕИТ), тамни поларитони, низ квантно-електродинамичких микрорезонатора, интерагујући тамни поларитони, везани парови тамних поларитона, квантна меморија за фотоне и неуређење

Научна област: Физика

Ужа научна област: Квантна оптика

УДК број: 539:535.14(043.3)

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Introduction and Thesis Outline

1.1 Introduction

An important aspect in the research of physics is the emergence of cross-connections between two promising research fields. One such prosperous connection can bee seen in cavity-QED (cavity quantum electrodynamics). It brings together quantum optics with many-body physics (condensed matter physics). Quantum optics itself has its roots in studying fundamental quantum features of matter and their interaction with light which has led to new developments such as different kinds of lasers and their numerous applications. Light is composed of particles, called photons. They carry information of the light polarization, which is determined through the components of the electric and magnetic field, and their spin. As bosonic particles, they have an integer spin. One of the most extensively studied aspects of quantum optics is the interaction of laser light with atomic vapours. Especially laser excited alkali-metal-atom vapours are in the focus of interest because of many ongoing emergent electromagnetically induced coherent effects. Those provide the opportunity for numerous applications. Moreover, a deep inquiry of coherent effects, transfer of coherence and population precedes to a better understanding of various phenomena in quantum optics and laser-matter interaction in general.

Characteristic examples of coherent effects are those that originate from coupling a single atomic excited state with two long-lived ground atomic states by the use of two laser fields. The laser fields are called probe and pump. These two laser fields and the atomic three-level system form a so called Λ-configuration. This system enables the realization of interference between two transition pathways, which are generated by the laser fields, and the creation of so called dark-states. Dark-states

represent coherent superpositions between the two long-lived ground atomic states, uncoupled from the excited state. Very remarkable features are connected to the dark-states, among them the increase of transparency if the pair of laser fields in two-photon Raman resonance prepare the absorbing medium into the dark-state. This is the common physical picture of coherent population trapping (CPT) [1, 2] and electromagnetically induced transparency (EIT) [3, 4, 5]. Within the spectral bandwidth of the EIT there is a strong dispersive behaviour of the index of refraction which results in the non linearity of EIT media and slow, stopped and stationary light [6, 7, 8]. Slow, stopped and stationary light are also directly related to the formation of so called dark-state polaritons (DSPs). DSPs are in particular low energy, single probe photon driven, collective excitations that do not have a contribution of the excited atomic states. They were firstly introduced by Lukin and Fleischhauer as an extension to the well-known concept of adiabatic Raman polaritons in Λ-systems that admit EIT [9, 10]. Their formation relies on the dark-states and assumes an adiabatic change of the pump field. Further, DSPs possess unique properties and features which have made them attractive for theoretical and experimental studies. Lukin and co-workers have shown the usage of DSPs as a storage medium for photons[11]. Quantum states of photons are transferred onto collective Raman excitations in a loss-free and reversible manner, thereby the state of photons e.g. spin or polarization is stored in the atomic spin-coherences between the two ground state levels. Additionally, it has been shown by Chong *et al.* that DSPs in double Λ-system enable the down-conversion of frequencies in alkali-metal atoms [12]. Furthermore, DSPs have shown to mediate coherent interactions between atoms in atomic vapours [13]. An additional property of DSPs, which was shown by Unayan and co-workers, is the spinor like behaviour and fulfilment of a Dirac like equation of motion in case two pairs of counter-propagating laser fields interact with a tripod linkage in atomic vapours [14]. However, CPT and EIT can also be observed in multilevel systems as those involving two atomic degenerate-level manifolds with multiple Zeeman substates. The observation of CPT and EIT in such systems is also a direct consequence of the existence of a dark-state within the ground atomic level if $F_g \geq F_e$ (F_g and F_e being the angular momenta quantum numbers of the ground and excited state respectively). Theoretical investigations of these kind of systems haven't been that intense, compared to non-degenerate case. Especially degenerate two-level system have been under minor consideration. A better understanding of them could lead to new applications, especially with possible new building blocks in

quantum information processing. In these systems, multiple dark-states and hence multiple DSPs can be observed. The multiplicity of them depends on the considered polarization of the coupling fields as it has been shown in [15].

Quantum optics and many-body physics share two common features which are the type of considered particles and the non linearity of interactions between them. One aspect of interest in many-body physics are strong correlations between bosonic systems, but clearly between fermionic systems as well. A well-known model that incorporates strong correlated bosonic systems is the so called Bose-Hubbard-model (BHM). It is defined for strongly interacting bosons on an optical lattice and accounts for large number of bosonic particles [16, 17, 18]. BHM with its inherent nonlinearity is mainly given by the onsite potential U_{BHM} which appears along with the combination of number operators of the form $\hat{n}(\hat{n}-1)$. Depending on the strength of the onsite potential U_{BHM} relatively to other parameters of the system, one can observe the appearance of quantum phase transitions (QPT). BHM in particular supports quantum phase transitions of Mott-insulator to super-fluid [16]. Mott-insulators are in general described by a gap in the eigenspectrum of the BHM, zero compressibility and localized bosonic excitations. Mott-insulator to the super-fluid transition is linked through a Bose glass phase [16], whose characteristic is a finite compressibility without the presence of a gap. Mean-field theory has shown to be an accurate method in order to study ground state QPTs of this kind [16, 19].

Strong correlations between photons, atoms and photons and between atoms gained a remarkable interest around 2004 and were substantially inspired by the study of strong correlations in many-body physics. To obtain strong correlations of photons, photon-photon interactions are required. One proposal of introducing photon-photon interactions within alkali-metal atomic vapours was based on the mechanism of photon exchange that is very weak [20]. Further, atom-photon interactions in EIT media are not that weak due to the provided nonlinearity but are not strong enough to reach strong correlations as the probe field is considered and defined as weak [10, 11, 15]. As a consequence, QED cavities have been proposed to enhance these interactions and correlations. QED cavities because on one hand, to quantize the light-matter interaction and on the other hand to reach the strong coupling regime as the absorption cross section for photons is reduced. Consequently, the cooperativity parameter η increases which is a key feature of the strong coupling regime. The description of quantized light-matter interactions is based on the standard Jaynes-Cummings model (JC) which was firstly introduced in [21]. They

focused on a single mode cavity that interacts nearly resonantly with a single two-level atom and showed a nonlinear behaviour of the spectrum. This nonlinearity in the spectrum, known as Kerr-nonlinearity, induces an effective photon-photon repulsion. It is directly related to the photon-blockade effect which can be tuned by controlling the level spacing as well as cavity mode frequency [22, 23]. A measure for the photon-photon repulsion is the effective photon number dependent onsite repulsion strength $U(n)$. The eigenstates of the Jaynes-Cummings model are called polaritons. To be more precise, in optical cavities these eigenstates are also known as bright-polaritons because they have a contribution of the excited atomic level.

Exploration of strongly interacting bosons in quantum optics as in condensed-matter physics is doable by increasing the number of coupled QED cavities and forming one- and two-dimensional lattices. This increase in the number of coupled QED cavities leads to an extension of the standard Jaynes-Cummings model to a standard Jaynes-Cummings-Hubbard model (JCH). It provides an opportunity to simulate the Bose-Hubbard model and its properties as well as features, but with fewer number of bosonic particles. Thus, the complicated many particle Bose-Hubbard Hamiltonian can be effectively reduced in complexity by considering JCH of coupled QED cavities. An important advantage that has attracted a lot of attention in the scientific community of quantum optics as well as condensed-matter physics [24]. Similarly to the BH model, the JCH model supports QPT of Mott- to superfluid, but for photons, as Greentree and co-workers have shown [25].

Apart from strong photon-photon and atom-atom interactions, strongly interacting polaritons in coupled QED cavities have encouraged profound theoretical investigations. Hartmann and co-workers have demonstrated that interacting DSPs in QED cavities with atoms in N-configuration can be exactly mapped to the BH model and show the Mott-insulator to superfluid QPT [26]. Mott-insulator to superfluid transition for interacting polaritons in Jaynes-Cummings and Jaynes-Cummings-Hubbard lattices has been under extensive study in the past few years [27, 28, 29]. Remarkably, even for interacting polaritons in one-dimensional arrays of coupled QED cavities a glassy phase of polaritons called polariton glass was discovered [30]. Analogues of extrinsic semiconductors, which are made of interacting polaritons in one-dimensional arrays of coupled QED cavities, have been demonstrated. It was shown that doping can be performed by changing the single photon detuning in a staggered way which in turn affects the Mott-insulator to superfluid QPT [31]. As polaritons in one-dimensional arrays of QED cavities are strongly interacting, one

question arises. Can bound states of polaritons be formed out of the interaction? Very little is known about polaritonic bound states. Wong and Law have demonstrated the existence of two-polariton bound states in the standard one-dimensional JCH model where the involved polaritons are bright-polaritons [32]. Recently, it has been shown that an equivalent spin-orbit interaction in two-polariton bound states exists [33]. A profound investigation of two-polariton bound states and dark-polariton bound pairs (DPBPs), which haven't been considered so far, can lead to rich physics and new applications. Possible applications are creation of lattices based on two-polariton bound states or DPBPs, quantum networks or optical topological insulators.

1.2 Thesis outline

This thesis deals with a theoretical investigation of coherent effects which are related to DSPs in degenerate two-level system admitting EIT. In a further step, interacting dark-polaritons are considered in one-dimensional arrays of coupled QED cavities. In order to provide existence of dark-polaritons, a new model is derived. This new model represents a modified Jaynes-Cummings- Hubbard model which supports the formation of DPBPs in the two-excitation subspace. It will be shown that for uniform inter-cavity photon hopping there exists a ground state DPBP which functions as a quantum memory of a single photon, even though the two photons are tightly bound to the atomic bound state which protects one of the photons from the storage and retrieval process. Moreover, we consider a disordered array of coupled QED cavities. The disorder is introduced through a staggered, non-uniform inter-cavity photon hopping. Under this setting and in dependence of (non)-compensating control field Stark shift, quantum state transfer of a ground state DPBP between the two sub lattices is reported. The thesis is organized as follows:

- **chapter 2**: Here we provide the theoretical basics in order to follow the results. Especially, we provide the background of the non-degenerate microscopic equation of motion for DSP-field operator. In depth, we not only derive and discuss the standard and modified Jaynes-Cummings model, but also compare them.

- **chapter 3+4**: In this chapter a detail presentation of the results, obtained

by investigating the degenerate two-level system in an ultra cold atomic gas of alkali-metal atoms as well as the investigation of interacting dark-polaritons in the two-excitation subspace of an array of coupled QED cavities with uniform inter-cavity photon hopping configuration is given.

- **chapter 5**: In this chapter a detail presentation of the results, obtained by investigating dark-polaritons in the two-excitation subspace of an array of coupled QED cavities with staggered inter-cavity photon hopping configuration is provided.

- **chapter 6+7**: In this last chapter conclusions are drawn. Further, future investigations, which are based on the so far obtained results, are discussed.

2

Dark-state polaritons in free space and polaritons in cavity-QED

2.1 Theory of dark-state polaritons in free space

2.1.1 Electromagnetically induced transparency and dark-state polaritons

Three-level Λ system (shown in Fig. 2.1) represents the simplest system presenting two physically closely related coherent phenomena - coherent population trapping (CPT) and electromagnetically induced transparency (EIT). Basic physical picture of these phenomena is based on the existence of *dark states* that are uncoupled to the laser fields. The atoms trapped into the dark state cannot be further excited by the laser fields and cannot fluorescence -they are dark.

Let us consider the case when the two fields are in Raman resonance with a

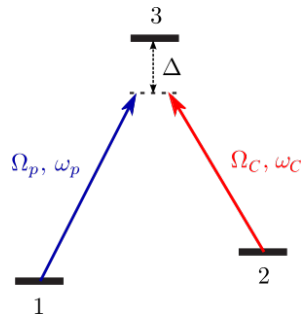

Figure 2.1: Three-level Λ system. Pump field (label p) couples the transition $1 \rightarrow 3$, while control field (label C) couples the transition $2 \rightarrow 3$. Ω_p and Ω_C are Rabi frequencies of the fields, ω_p and ω_C are carrier frequencies, while Δ is the transition detuning (common single-photon detuning).

common single-photon detuning Δ. The Λ system Hamiltonian in a basis composed from the states $|\psi_1\rangle$, $|\psi_2\rangle$ and $|\psi_3\rangle$, respectively,

$$\mathsf{H}(t) = \hbar \begin{bmatrix} 0 & 0 & \frac{1}{2}\Omega_p(t) \\ 0 & 0 & \frac{1}{2}\Omega_C(t) \\ \frac{1}{2}\Omega_p^*(t) & \frac{1}{2}\Omega_C^*(t) & \Delta \end{bmatrix} \tag{2.1}$$

has the following eigenenergy $\varepsilon_0(t) = 0$ and eigenstate $\Psi_0(t) = \frac{\Omega_C^*(t)}{\Omega(t)}|\psi_1\rangle - \frac{\Omega_p^*(t)}{\Omega(t)}|\psi_2\rangle$, with $\Omega(t) = \sqrt{|\Omega_p(t)|^2 + |\Omega_C(t)|^2}$. It is important to note that the state $\Psi_0(t)$ is composed entirely from ground states 1 and 2 and has no contribution of the excited state 3. Moreover, the state $\Psi_0(t)$ is a dark state that is effectively decoupled from the excited state 3, since $\mathsf{H}(t)\Psi_0(t) = 0$. This decoupling is a consequence of destructive interference of the probability amplitude for the transition $1 \to 3$ with the probability amplitude for the transition $2 \to 3$. If the medium is prepared in this state, there is no possibility of excitation by means of the coupling laser fields. This leads to an enhanced transparency of the medium when the laser fields are close to Raman resonance. Increased transparency for near resonant coupling fields is common to CPT and EIT. Preparation into the dark state via optical pumping (via spontaneous decay from the excited state 3 is one way to trap population into that state. Note once again that necessary conditions for the CPT and EIT appearance are the existence of dark states and two-photon Raman resonance of the coupling laser fields.

Formation of dark-state polaritons in optically thick media is related to the Raman adiabatic passage (naturally provided in EIT) and can either happen via optical pumping, i.e., by an incoherent process, or via a coherent preparation scheme. Optical pumping requires that the atomic ensemble is initially prepared in a mixed state. An undesirable property of optical pumping is the unrecoverable loss of photons. However, in coherent preparation all atoms are initially in a pure state, e.g. $|1\rangle$ if $|1\rangle$ is a non degenerate ground state with a sufficient energy gap to the state $|2\rangle$. This is achievable by the so called stimulated Raman adiabatic passage (STIRAP). STIRAP assures that the atoms return to their initial state after the interaction with the weak coupling field. Further, it conserves the number of photons in the weak field. A key point within STIRAP is that the two coupling fields $\Omega_p(t)$ and $\Omega_c(t)$ change adiabatically, thus a complete transfer of the initial population from $|1\rangle$ to $|2\rangle$ is reversible and therefore conserved.

2.1.2 Dark-state polariton-field operator equation

Suppose we have a two level system which is prepared in the ground state $|\psi_0\rangle$ with the energy E_0. Further, a time independent Hamiltonian $\hat{H}$ governs the system's evolution so that $\hat{H}|\psi_0\rangle = E_0|\psi_0\rangle$. We are looking for a field operator $\hat{A}^\dagger$ with the property $\hat{H}(\hat{A}^\dagger|\psi_0\rangle) = (E_0+\hbar\omega)\hat{A}^\dagger|\psi_0\rangle$, where $\hbar\omega$ corresponds to the energy gap that divides the ground and excited state (this is basically the amount of energy needed to excite the system). Time-independent Schrödinger equation for the excited state $\hat{A}^\dagger|\psi_0\rangle$ yields the following requirement for the field operator

$$\begin{aligned} \hat{H}(\hat{A}^\dagger|\psi_0\rangle) &= (E_0 + \hbar\omega)\hat{A}^\dagger|\psi_0\rangle \\ \hat{A}^\dagger\hat{H}|\psi_0\rangle &= E_0\hat{A}^\dagger|\psi_0\rangle \\ (\hat{H}\hat{A}^\dagger - \hat{A}^\dagger\hat{H})|\psi_0\rangle &= \hbar\omega\hat{A}^\dagger|\psi_0\rangle \\ [\hat{H}, \hat{A}^\dagger] &= \hbar\omega\hat{A}^\dagger, \end{aligned} \tag{2.2}$$

where the action on the state $|\psi_0\rangle$ is implicitly assumed.

If we extend the single two level to a degenerate two level system with the two coupling fields, weak quantum probe field and classical driving field, the field operator $\hat{A}^\dagger$ is going to be expressed by a superposition of operators as it will be presented in chapter 3 Dark-state polaritons in a degenerate two-level system.

2.2 Bright-polaritons in cavity-QED: Jaynes-Cummings Model

In 1963, the two American physicists Edwin Jaynes and Fred Cummings proposed a theoretical model in order to investigate the relation between quantum theory and the semi-classical theory of radiation. In fact, they were motivated by describing the process of spontaneous emission [21]. Within their approach they analysed the interaction of a monochromatic electromagnetic field mode with a quantum mechanical two-level system. Although this model is obviously an approximation, it turned out to be of crucial importance in order to understand the fundamental interaction between light and matter. The fact that Jaynes and Cummings found a quantum mechanical description of light-matter interaction that showed a very good quantitative agreement with experiments, strongly enhanced the progress in the field of Quantum Optics and has been a corner stone especially in this field

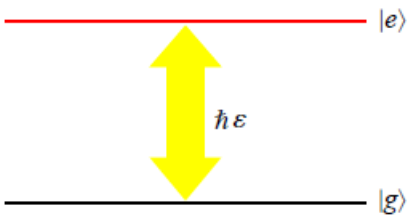

Figure 2.2: Energy structure of the two-level system. The two-level system consists of a ground state $|g\rangle$ with energy $E_0 = 0$ and an excited state $|e\rangle$ with energy $E_e = \hbar\epsilon$.

of physics ever since. In the following years and decades the further developments, based on their model, led to multiple applications and improvements such as masers, lasers and optical trapping and cooling techniques [34, 35, 36]. A derivation of this model can be found in almost every physics book concerning quantum optics, see for instance the Refs. [37, 38, 39, 40, 41]. In this thesis I derive the Jaynes-Cummings Model where I focus on the derivation of the two-level system Hamiltonian as well as the interaction Hamiltonian. I mainly point out the two-level system Hamiltonian and the interaction Hamiltonian because in case of the modified Jaynes-Cummings Model, which is going to be derived in the sequel section, they change significantly.

2.2.1 Two-level system Hamiltonian

I now consider a two-level system that is placed in an optical micro-cavity and interacts with the intra-cavity field mode. Further, I assume that the two-level system has an energy structure as depicted in Figure (2.2). It consits of a ground state $|g\rangle$ with energy $E_0 = 0$ and an excited state $|e\rangle$ with energy $E_e = \hbar\epsilon$.

By taking this point of view, I make the most general ansatz to deal with this kind of system which is applicable to all experimental setups. Since the energy eigenstates and their respective eigenvalues are known by definition, one can immediately write down the Hamiltonian of this system in the energy representation:

$$\hat{H}_{atom} = \hbar\epsilon|e\rangle\langle e| + 0|g\rangle\langle g|. \tag{2.3}$$

By choosing a specific representation for the abstract eigenstates $|g\rangle$, $|e\rangle$, one can transform this Hamiltonian in a more convenient form. Obviously, there are quite a lot of possible representations. I use the simplest representation and associate the

kets with the two dimensional Cartesian unit vectors as follows

$$|g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{2.4}$$

As we use this representation for the state vectors, it leads to a matrix representation for the projection operators in the Hamiltonian (2.3). Inserting (2.4) into (2.3) yields

$$\hat{H}_{atom} = \hbar\epsilon \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + 0 \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \hbar\epsilon & 0 \\ 0 & 0 \end{pmatrix}. \tag{2.5}$$

Furthermore, remembering the definition of the Pauli matrices:

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2.6}$$

and their combinations

$$\hat{\sigma}^+ = \frac{1}{2}(\hat{\sigma}_x + i\hat{\sigma}_y) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{2.7a}$$

$$\hat{\sigma}^- = \frac{1}{2}(\hat{\sigma}_x + i\hat{\sigma}_y) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \tag{2.7b}$$

one finds that, suitable combinations of the state vectors can be expressed in terms of these matrices. In fact, it can be shown that, in this choice of representation, the following relations hold

$$|e\rangle\langle g| = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \hat{\sigma}^+ \tag{2.8a}$$

$$|g\rangle\langle e| = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \hat{\sigma}^-. \tag{2.8b}$$

From the above relations follows, that the operators $\hat{\sigma}^+$, $\hat{\sigma}^-$ are the creation and annihilation operators of the two-level system, i.e. $\hat{\sigma}^+$ creates an atomic excitation, whereas $\hat{\sigma}^-$ destroys it. With the help of these relations, one finds for the remaining projection operators the following form

$$|e\rangle\langle e| = \hat{\sigma}^+\hat{\sigma}^-, \quad |g\rangle\langle g| = \hat{\sigma}^-\hat{\sigma}^+. \tag{2.9}$$

Hence, one can finally express the projection operators, occurring in the two-level Hamiltonian, in terms of combinations of Pauli matrices. If we plug in the expressions (2.9) into (2.3), the desired simplified form of the atomic Hamiltonian reads

$$\hat{H}_{atom} = \hbar\epsilon\hat{\sigma}^{+}\hat{\sigma}^{-} = \hbar\epsilon\hat{n}_a, \tag{2.10}$$

where I introduced the occupation number operator $\hat{n}_a = \hat{\sigma}^{+}\hat{\sigma}^{-}$ for the two-level system. Formally, this Hamiltonian shows quite similarities with the Hamiltonian of the monochromatic free electromagnetic field

$$\hat{H}_{field} = \hbar\omega(\hat{n} + \frac{1}{2}). \tag{2.11}$$

Namely, the Hamiltonian takes on the form of a product of the occupation number of the excited level and the energy of this level. Note that, in literature one often finds other notations for this Hamiltonian corresponding to another choice of energies. Furthermore, one can see that the Hamiltonian (2.10) commutes with the occupation-number operator $\hat{n}_a$ and, therefore, the conserved quantities for this Hamiltonian are the excitations of the two-level system. After having derived the quantum mechanical description of the energy contributions of the intra-cavity photon field and the two-level system, I now investigate the energy contribution arising from the interaction between those two, in the following section.

2.2.2 Interaction Hamiltonian

In the previous section, I derived the Hamiltonian for a two-level system without specifying the actual experimental setup. However, in order to derive the interaction Hamiltonian in the following paragraph, I need to be more precise, since there is a variety of possible realizations of this system, leading to very different interactions. For example, one can implement the two-level system using the spin of an electron, which could be manipulated by a magnetic field. Hence, the interaction Hamiltonian for this case would be proportional to the magnetic field. On the other hand, one can use the electronic transitions in an atom, to realize the system. In this case the electron would couple to the electric field vector. Within this thesis I focus on the latter case. Hence, the energy states $|g\rangle$, $|e\rangle$ correspond to electronic states. The transition between these states is characterized by the electronic-transition dipole moment. In this case the interaction is mediated via the coupling of the electri-

cal field to the transition dipole moment. Assuming that the wavelength of the monochromatic electric field is large compared to the dimension of the atom, one can work in the so called *dipole approximation*. In this approximation, one considers only the field strength at the centre-of-mass position of the atom. Thus, using this approach, the classical interaction Hamilton function takes on the form

$$H_{int} = -q\mathbf{r} \cdot \mathbf{E}(\mathbf{R}, t), \tag{2.12}$$

where the vector $\mathbf{R}$ labels the centre-of-mass position of the atom and $q\mathbf{r}$ is the classical electronic dipole moment. Since I assume that the intra-cavity photon field is monochromatic (specified by the mode index ρ) and has a fixed polarization σ, I drop the index ρ and σ in the electric field $\mathbf{E}(\mathbf{R}, t)$ and, thus, the explicit form of the quantized form of the electric field is given by

$$\hat{\mathbf{E}}(\mathbf{R}, t) = i\xi[\hat{a}(t)\mathbf{u}(\mathbf{R}) - \hat{a}^{\dagger}(t)\mathbf{u}^{*}(\mathbf{R})], \tag{2.13}$$

where I introduced the abbreviation

$$\xi = (\frac{\hbar\omega}{2\epsilon})^{\frac{1}{2}}. \tag{2.14}$$

Note that, the photonic ladder operators $\hat{a}(t)$, $\hat{a}^{\dagger}(t)$ in the above equation are now formally time dependent. The reason for this is, that I absorbed the time dependence of the mode function $\mathbf{u}(\mathbf{R})$, which is characterized as a plane wave, and defined

$$\hat{a}(t) = \hat{a}e^{-i\omega t}. \tag{2.15}$$

To the end of this section, I drop the explicit time dependence for the sake of clarity. In order to write down the fully quantized version of (2.12), one still needs to find the quantum mechanical description of the dipole moment. Trying to use the same procedure as in (2.3), I expand the dipole operator in the energy eigenbasis of the two-level system. This approach leads to

$$q\mathbf{r} = \sum_{i,j} |i\rangle\langle i|q\mathbf{r}|j\rangle\langle j|, \tag{2.16}$$

where the indices i and j label the respective energy eigenstates $|g\rangle$, $|e\rangle$. Since transition dipole moments can just arise from electronic transition from $|g\rangle$ to $|e\rangle$

or vice versa, the terms corresponding to the even transitions $\langle e|q\mathbf{r}|e\rangle$ and $\langle g|q\mathbf{r}|g\rangle$, have to be zero. Hence, I define

$$\mathbf{p} := q\langle e|\mathbf{r}|g\rangle, \tag{2.17a}$$

$$\mathbf{p}^* := q\langle g|\mathbf{r}|e\rangle. \tag{2.17b}$$

These results yield the following expression for the dipole operator expansion (2.16):

$$q\mathbf{r} = \mathbf{p}\hat{\sigma}^+ + \mathbf{p}^*\hat{\sigma}^-. \tag{2.18}$$

For the last equivalence in (2.18), I used the relation of the state vectors to the Pauli matrices, introduced in (2.9). Subsequently, one can combine expressions (2.13) and (2.18) to give the quantized version of the interaction Hamiltonian (2.12), which reads

$$\hat{H}_{int} = -i\xi[\hat{a}\mathbf{u}(\mathbf{R}) - \hat{a}^\dagger\mathbf{u}^*(\mathbf{R})](\mathbf{p}\hat{\sigma}^+ + \mathbf{p}^*\hat{\sigma}^-). \tag{2.19}$$

Expanding this expression yields

$$\hat{H}_{int} = -i\xi[\hat{a}\mathbf{u}(\mathbf{R})\cdot\mathbf{p}\hat{\sigma}^+ + \hat{a}\mathbf{u}(\mathbf{R})\cdot\mathbf{p}^*\hat{\sigma}^- - \hat{a}^\dagger\mathbf{u}^*(\mathbf{R})\cdot\mathbf{p}\hat{\sigma}^+ - \hat{a}^\dagger\mathbf{u}^*(\mathbf{R})\cdot\mathbf{p}^*\hat{\sigma}^-]. \tag{2.20}$$

Having a closer look at the occurring terms, one can see that, two of them describe rather unphysical processes, that violate conservation laws. In fact, the term proportional to $\hat{a}\hat{\sigma}^-$ describes the decay of the excited atomic level together with the annihilation of an intra-cavity photon field, whereas the term proportional to $\hat{a}^\dagger\hat{\sigma}^+$ describes the excitation of the atom together with the creation of a photon. Both processes obviously violate the conservation of energy and particle number in the system and, hence, I neglect them in the further calculations. This approach is known in the literature as the *rotating wave approximation* (RWA)[38, 40, 41] and leads to a Hamiltonian of the form

$$\hat{H}_{int} = -i\hbar(g\hat{a}\hat{\sigma}^+ - g^*\hat{a}^\dagger\hat{\sigma}^-), \tag{2.21}$$

where I introduced the complex coupling strength g defined by $\hbar g = \xi\mathbf{u}(\mathbf{R})\cdot\mathbf{p}$ and $\hbar g^* = \xi\mathbf{u}^*(\mathbf{R})\cdot\mathbf{p}^*$. Note that the implicit time dependence of the Hamiltonian (2.21) due to relation (2.15) vanishes in a rotating frame and, thus, can be neglected in the following considerations. At first appearance it seems that, one has arrived at the simplest form for the interaction Hamiltonian. However, I show within the next

section that there exists a property of the Hamiltonian (2.21), which leads to an even more compact form.

2.2.3 Symmetry of the interaction Hamiltonian

Within this section I am going to examine the invariance of the Hamiltonian (2.21) under global U(1) phase transformations. This symmetry offers the possibility to restrict the complex coupling strength g to real values and, thus, leads to the final form of the interaction part of the Jaynes-Cummings Hamiltonian. The standard approach to analyse this property of the Hamiltonian is to apply a global phase transformation to either the state vectors or the appearing operators. I use the first method and perform the following transformation for the two-level eigenstates

$$|e\rangle' \rightarrow e^{i\alpha}|e\rangle, \quad |g\rangle' \rightarrow e^{i\beta}|g\rangle. \tag{2.22}$$

The introduced parameters α and β are global constants independent of space and time. Using the relations (2.8) I see that this transformation immediately yields new expressions for the annihilation and creation operators of the two-level system, namely

$$\hat{\sigma}^{+\prime} \rightarrow e^{i(\alpha-\beta)}\hat{\sigma}^{+}, \quad \hat{\sigma}^{-\prime} \rightarrow e^{-i(\alpha-\beta)}\hat{\sigma}^{-}. \tag{2.23}$$

By inserting these expressions into (2.10), it is easy to see that the performed phase transformation leaves the Hamiltonian invariant.

$$\hat{H}'_{atom} = \hbar\epsilon\hat{\sigma}^{+\prime}\hat{\sigma}^{-\prime} = \hbar\epsilon e^{i(\alpha-\beta)}\hat{\sigma}^{+}e^{-i(\alpha-\beta)}\hat{\sigma}^{-} = \hbar\epsilon\hat{\sigma}^{+}\hat{\sigma}^{-} = \hat{H}_{atom}. \tag{2.24}$$

Following the same procedure, I analyse the symmetry of the Hamiltonian for the free electromagnetic field (2.11), by performing a global phase transformation on the photonic annihilation and creation operators as follows

$$\hat{a}' \rightarrow e^{i\phi}\hat{a}, \quad \hat{a}^{\dagger\prime} \rightarrow e^{-i\phi}\hat{a}^{\dagger}, \tag{2.25}$$

with the global constant ϕ. An investigation of the effect of this transformation on equation (2.11) immediately shows that the Hamiltonian of the intra-cavity photon

field stays invariant:

$$\hat{H'}_{field} = \hbar\omega\left(\hat{a}^{\dagger'}\hat{a}' + \frac{1}{2}\right) = \hbar\omega\left(e^{-i\phi}\hat{a}^{\dagger}e^{i\phi}\hat{a} + \frac{1}{2}\right) = \hbar\omega\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right) = \hat{H}_{field}. \tag{2.26}$$

Therefore, both, the intra-cavity field Hamiltonian as well as the two-level system Hamiltonian remain invariant under some global phase transformation. The interesting question is: what happens to the interaction part? This can be easily investigated by simultaneously performing both transformations (2.23) and (2.25) on equation (2.21), which results in the modified interaction Hamiltonian:

$$\hat{H}_{int} = -i\hbar\left[g\hat{a}\hat{\sigma}^{+}e^{i(\phi+\alpha-\beta)} - g^{*}\hat{a}^{\dagger}\hat{\sigma}^{-}e^{-i(\phi+\alpha-\beta)}\right]. \tag{2.27}$$

Furthermore, using the fact that complex numbers can be separated into a real modulus and a complex phase, equation (2.27) can be rewritten as

$$\hat{H}_{int} = -i\hbar|g|\left[\hat{a}\hat{\sigma}^{+}e^{i(\theta+\phi+\alpha-\beta)} - \hat{a}^{\dagger}\hat{\sigma}^{-}e^{-i(\theta+\phi+\alpha-\beta)}\right], \tag{2.28}$$

where I defined $g = |g|e^{i\theta}$. The arbitrariness of the introduced parameters α, β and ϕ allows to choose their values in such a way as to compensate the phases of the coupling constant. For this reason, I demand that the following relation has to hold

$$\theta + \phi + \alpha - \beta \stackrel{!}{=} \frac{\pi}{2}. \tag{2.29}$$

Here I choose $\frac{\pi}{2}$ because I want to use the over determination to get rid of the prefactor i in (2.28). With the phase parameters obeying equation (2.29), one finally arrives at the most compact formulation of the interaction Hamiltonian, which reads

$$\hat{H}_{int} = \hbar g\left(\hat{a}\hat{\sigma}^{+} + \hat{a}^{\dagger}\hat{\sigma}^{-}\right), \tag{2.30}$$

where the coupling constant g is now a real quantity and is defined as

$$\hbar g = \xi|u(\mathbf{R})\cdot\mathbf{p}|. \tag{2.31}$$

2.2.4 Jaynes-Cummings model Hamiltonian

The Jaynes-Cummings (JC) system is depicted schematically in Figure (2.3). In this picture I have indicated the main processes one has to deal with in further

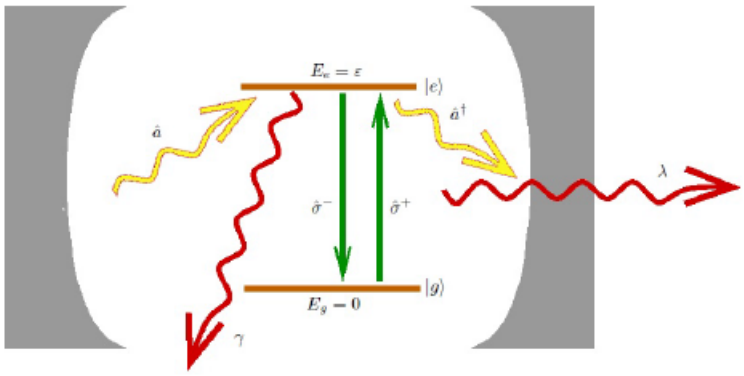

Figure 2.3: Two-level system in a micro-cavity and the most important processes in this system. Yellow arrows indicate the creation annihilation of cavity photons, green arrows indicate the excitation and relaxation of the two-level system and the red arrows indicate the loss processes ($\hbar = 1$).

calculations. Specifically, there are: the creation and annihilation of intra-cavity photons via the photonic operators $\hat{a}$,$\hat{a}^\dagger$, the excitation and relaxation of the two-level system via the electronic operators $\hat{\sigma}^+, \hat{\sigma}^-$. Furthermore, there are additional processes, which haven't been discussed so far. These new processes, which are indicated by the red waving arrows, correspond to loss processes in the cavity. In a real experiment, there will be two main sources for energy dissipation out of the system. The first one is simply due to the fact that, in general there is a non-vanishing probability for spontaneous emission of a photon from the excited level of the atom. In my formulation this probability is proportional to γ. The second process amounts for the fact, that the cavity itself is not perfectly closed and, therefore, gives rise to the possibility of a photon to leak out of the cavity at a rate λ. However, within the further calculations, I explicitly neglect these loss processes, assuming that the coupling g is much bigger than the dissipation, i.e.

$$\eta = \frac{g^2}{\gamma\lambda} \gg 1. \tag{2.32}$$

This approach is known in the literature as working in the *strong coupling regime* with η the cooperativity parameter.This regime has already been shown to be experimentally feasible in many different setups [42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54]. From equation (2.31), I deduce that, this regime can be established by maximizing the electronic-transition dipole moment and choosing a small cavity volume. For completeness, I mention that there also exists some calculations

[53, 55, 56, 57, 58] and experiments [59, 60], that explicitly describe and test the dissipative regime by taking loss processes into account. Hitherto, I have thoroughly derived and justified the basic constituents of the form of the Jaynes-Cummings Hamiltonian. Therefore, I can now write down the full Hamiltonian, which reads

$$\hat{H}^{JC} = \hat{H}_{field} + \hat{H}_{atom} + \hat{H}_{int}. \tag{2.33}$$

Shifting the energy of the system by $\frac{\hbar\omega}{2}$ in order to get rid of the zero-point energy contribution, which arises from the electromagnetic field Hamiltonian, one finds the following form for the Jaynes-Cummings Hamiltonian in the rotating wave approximation:

$$\hat{H}^{JC} = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\epsilon\hat{\sigma}^+\hat{\sigma}^- + \hbar g\left(\hat{a}\hat{\sigma}^+ + \hat{a}^\dagger\hat{\sigma}^-\right). \tag{2.34}$$

One can further transform this expression to a more convenient form, by introducing the composed occupation number operator

$$\hat{n} = \hat{a}^\dagger\hat{a} + \hat{\sigma}^+\hat{\sigma}^-, \tag{2.35}$$

and the detuning parameter

$$\Delta = \epsilon - \omega, \tag{2.36}$$

which is a measure for the detuning between the monochromatic photon field frequency and the two-level transition frequency. The resulting Hamiltonian reads

$$\hat{H}^{JC} = \hbar\omega\hat{n} + \hbar\Delta\hat{\sigma}^+\hat{\sigma}^- + \hbar g\left(\hat{a}\hat{\sigma}^+ + \hat{a}^\dagger\hat{\sigma}^-\right), \tag{2.37}$$

which I will use within the further calculations. First, I notice some general properties of this Hamiltonian. One thing I observe is that in the case of resonant pumping, i.e. $\Delta = 0$ the second term vanishes, leaving just the contribution proportional to $\hbar\omega$ and the interaction term, which is proportional to g. Considering the latter, I place emphasis on the fact that, this term describes the conversion of atomic excitations to photonic excitations and vice versa. The next step to analyse the Jaynes-Cummings model is to determine the eigenstates and eigenvalues of the Hamiltonian (2.37). It turns out that to perform these calculations, the rotating wave approximation introduced in Section (2.2.2) is absolutely crucial. Within this approximation it is possible to analytically diagonalize the Hamiltonian. However, as proposed by Feranchuk et. al.[61] and others [37, 62, 63, 64, 65] it is also possible

to analytically solve the Jaynes-Cummings model without the RWA, but within this thesis, I explicitly make use of this simplification. In order to diagonalize the Hamiltonian (2.37), one has to investigate its commutator with the occupation number operator $\hat{n}$, which leads to

$$\left[\hat{n}, \hat{H}^{JC}\right] = \hbar\omega \left[\hat{n}, \hat{n}\right] + \hbar\Delta \left[\hat{n}, \hat{\sigma}^{+}\hat{\sigma}^{-}\right] + \hbar g \left(\left[\hat{n}, \hat{a}\hat{\sigma}^{+}\right] + \left[\hat{n}, \hat{a}^{\dagger}\hat{\sigma}^{-}\right]\right). \tag{2.38}$$

Remembering that each operator commutes with itself and noticing that the following relations have to hold

$$\left[\hat{a}, \hat{\sigma}^{+}\right] = \left[\hat{a}, \hat{\sigma}^{-}\right] = \left[\hat{a}^{\dagger}, \hat{\sigma}^{+}\right] = \left[\hat{a}^{\dagger}, \hat{\sigma}^{-}\right] = 0, \tag{2.39}$$

since the appearing operators $\hat{a}^{\dagger}, \hat{a}$ and $\hat{\sigma}^{+}, \hat{\sigma}^{-}$ operate on independent subspaces, one can immediately conclude that

$$\left[\hat{n}, \hat{a}^{\dagger}\hat{a}\right] = \left[\hat{n}, \hat{\sigma}^{+}\hat{\sigma}^{-}\right] = 0. \tag{2.40}$$

Hence, the commutator (2.38) simplifies to

$$\left[\hat{n}, \hat{H}^{JC}\right] = \hbar g \left(\left[\hat{n}, \hat{a}\hat{\sigma}^{+}\right] + \left[\hat{n}, \hat{a}^{\dagger}\hat{\sigma}^{-}\right]\right). \tag{2.41}$$

Using the fundamental commutator relations for the photonic ladder operators as bosonic operators and the commutator relation of the Pauli matrices:

$$\left[\hat{\sigma}^{+}, \hat{\sigma}^{-}\right] = \hat{\sigma}_{z}, \tag{2.42}$$

one can easily calculate the remaining commutators, which results in the following relations

$$\begin{aligned} \left[\hat{\sigma}^{+}\hat{\sigma}^{-}, \hat{a}\hat{\sigma}^{+}\right] &= \hat{a}\hat{\sigma}^{+}, \quad \left[\hat{\sigma}^{+}\hat{\sigma}^{-}, \hat{a}^{\dagger}\hat{\sigma}^{-}\right] = -\hat{a}^{\dagger}\hat{\sigma}^{-}, \\ \left[\hat{a}^{\dagger}\hat{a}, \hat{a}\hat{\sigma}^{+}\right] &= -\hat{a}\hat{\sigma}^{+}, \quad \left[\hat{a}^{\dagger}\hat{a}, \hat{a}^{\dagger}\hat{\sigma}^{-}\right] = \hat{a}^{\dagger}\hat{\sigma}^{-}, \end{aligned} \tag{2.43}$$

which yields for the commutators in equation (2.41)

$$\left[\hat{n}, \hat{a}\hat{\sigma}^{+}\right] = \left[\hat{n}, \hat{a}^{\dagger}\hat{\sigma}^{-}\right] = 0. \tag{2.44}$$

Thus, I found the very important property, that the Jaynes-Cummings Hamiltonian $\hat{H}^{JC}$ commutes with the bosonic occupation number operator $\hat{n}$:

$$\left[\hat{n}, \hat{H}^{JC}\right] = 0. \tag{2.45}$$

This result has two essential implications. The first one is that $\hat{n}$ obviously describes a conserved quantity in the Jaynes-Cummings model. This quantity is the number of so called *polaritons* in the system. In the present model, a polariton is basically a coupled excitation of the atomic and the photonic system. The second important implication is that $\hat{H}^{JC}$ and $\hat{n}$ share a common set of eigenstates, in which both operators are diagonal. In order to find these states, it is advisable to have a closer look at the occupation number operator. As introduced in (2.35), this operator is the sum of the occupation number operators for the intra-cavity photon field $\hat{n}_p = \hat{a}^\dagger \hat{a}$ and the occupation number operator $\hat{n}_a = \hat{\sigma}^+ \hat{\sigma}^-$ of the two-level system, respectively. Because these operators commute as well, they also share a set of common eigenstates. Nevertheless, since they operate in different subspaces and, therefore, have distinct sets of eigenvalues, the only possible candidate for a common set of eigenstates are the product states of the photonic Fock states and the atomic two-level states. For this reason, I consider the ansatz

$$|n_p, s\rangle = |n_p\rangle \otimes |s\rangle, \quad s \in \{e, g\}\,, \tag{2.46}$$

where these new states have to satisfy the eigenvalue equations

$$\begin{aligned} \hat{n}_p |n_p, s\rangle &= n_p |n_p, s\rangle, \\ \hat{n}_a |n_p, s\rangle &= n_a |n_p, s\rangle. \end{aligned} \tag{2.47}$$

Furthermore, the new set of product states inherits the completeness and orthogonality relations from the subspaces of its components yielding

$$\sum_{n=0}^{\infty} \sum_{s=e,g} |n_p, s\rangle\langle n_p, s| = 1, \tag{2.48}$$

$$\langle n_p', s'| \cdot |n_p, s\rangle = \delta_{n_p, n_p'} \delta_{s,s'}. \tag{2.49}$$

Throughout this thesis, I refer to these product states as the *bare basis set* of the Jaynes-Cummings model. The presented set of states leads, according to equations

(2.46) and (2.47), to the following eigenvalue equation of the polariton occupation number operator

$$\hat{n}|n_p, s\rangle = (\hat{n}_p + \hat{n}_a)\,|n_p, s\rangle = (n_p + n_a)\,|n_p, s\rangle = n|n_p, s\rangle. \tag{2.50}$$

Remembering from Section (2.2.1) that n_a can only take on the values 1 and 0, corresponding to the excitation states $|e\rangle$ and $|g\rangle$, one can immediately deduce from (2.50) the important fact that for a fixed number n of polaritons there exist *two possible micro states*. Having found a set of eigenstates of the polariton occupation number operator,now, one can make use of this result and write down the representation of the Jaynes-Cummings Hamilton operator with respect to the bare basis by using the completeness relation (2.48):

$$\hat{H}^{JC} = \sum_{n_p, n_p'=0}^{\infty} \sum_{s,s'} |n_p, s\rangle\langle n_p, s|\hat{H}^{JC}|n_p', s'\rangle\langle n_p', s'|. \tag{2.51}$$

Arranging the occurring terms according to the number of polaritons and using the orthogonality relation (2.49), leads to a block-diagonal form of the Hamilton operator. One can see that (2.51) separates into the ground-state contribution and an infinite number of blocks of higher order contributions with fixed polariton number n:

$$\begin{aligned}\hat{H}^{JC} = {} & |0,g\rangle h_{0g,0g}\langle 0,g| \\ & + |1,g\rangle h_{1g,0e}\langle 0,e| + |1,g\rangle h_{1g,1g}\langle 1,g| + |0,e\rangle h_{0e,1g}\langle 1,g| + |0,e\rangle h_{0e,0e}\langle 0,e| \\ & + |2,g\rangle h_{2g,1e}\langle 1,e| + |2,g\rangle h_{2g,2g}\langle 2,g| + |1,e\rangle h_{1e,2g}\langle 2,g| + |1,e\rangle h_{1e,1e}\langle 1,e| \\ & + |3,g\rangle h_{3g,2e}\langle 2,e| + |3,g\rangle h_{3g,3g}\langle 3,g| + |2,e\rangle h_{2e,2e}\langle 2,e| + |2,e\rangle h_{2e,3g}\langle 3,g| \\ & + \ldots,\end{aligned} \tag{2.52}$$

where I used the abbreviation $h_{ns,n's'} = \langle n, s|\hat{H}^{JC}|n', s'\rangle$ for the respective matrix elements. This expression can be transformed to a simpler form by introducing (2×2) matrices for each polariton number, leading to a Hamiltonian of the form

$$\hat{H}^{JC} = \hat{h}_0 + \sum_{n}^{\infty} \hat{h}_n, \tag{2.53}$$

where n labels the number of polaritons and the operators $\hat{h}_0, \hat{h}_n$ are defined as

$$\hat{h}_0 = |0,g\rangle\langle 0,g|\hat{H}^{JC}|0,g\rangle\langle 0,g|, \tag{2.54}$$

$$\hat{h}_n = \begin{pmatrix} \langle n,g|\hat{H}^{JC}|n,g\rangle & \langle n,g|\hat{H}^{JC}|n-1,e\rangle \\ \langle n-1,e|\hat{H}^{JC}|n,g\rangle & \langle n-1,e|\hat{H}^{JC}|n-1,e\rangle \end{pmatrix}. \tag{2.55}$$

Remembering the analysis of Section (2.2.1), one can calculate the matrix entries using the action of the ladder operators on their respective eigenstates. With the properties

$$\begin{gathered} \hat{\sigma}^+\hat{\sigma}^-|n,g\rangle = 0, \quad \hat{\sigma}^+\hat{\sigma}^-|n-1,e\rangle = |n-1,e\rangle, \\ \hat{a}^\dagger\hat{a}|n,g\rangle = n|n,g\rangle, \quad \hat{a}^\dagger\hat{a}|n-1,e\rangle = (n-1)|n-1,e\rangle, \\ \hat{a}\hat{\sigma}^+|n,g\rangle = \sqrt{n}|n-1,e\rangle \quad \hat{a}\hat{\sigma}^+|n-1,e\rangle = 0, \\ \hat{a}^\dagger\hat{\sigma}^-|n,g\rangle = 0, \quad \hat{a}^\dagger\hat{\sigma}^-|n-1,e\rangle = \sqrt{n}|n,g\rangle, \end{gathered} \tag{2.56}$$

one finds, that the sub Hamiltonians (2.55) have the following representation in the bare basis

$$\hat{h}_n = \begin{pmatrix} \hbar\omega n & \hbar g\sqrt{n} \\ \hbar g\sqrt{n} & \hbar\omega n + \hbar\Delta \end{pmatrix}. \tag{2.57}$$

Thus, the problem of finding the eigenvalues of the potentially infinite Hamiltonian (2.37) is reduced to the much simpler task of finding the two eigenvalues of matrix (2.57). Mathematically this is a well known situation, which can be solved in general by evaluating the characteristic polynomial of the respective matrix. In the present case this approach yields the following equation

$$(\hbar\omega n - E_n)(\hbar\omega n + \hbar\Delta - E_n) - \hbar^2 g^2 n = 0, \tag{2.58}$$

which immediately gives the eigenvalues as

$$E_n = \begin{cases} E_{n\pm} = \hbar\omega n + \frac{1}{2}\hbar(\Delta \pm \chi_n(\Delta)) & n \geq 1 \\ E_0 = 0 & n = 0 \end{cases} \tag{2.59}$$

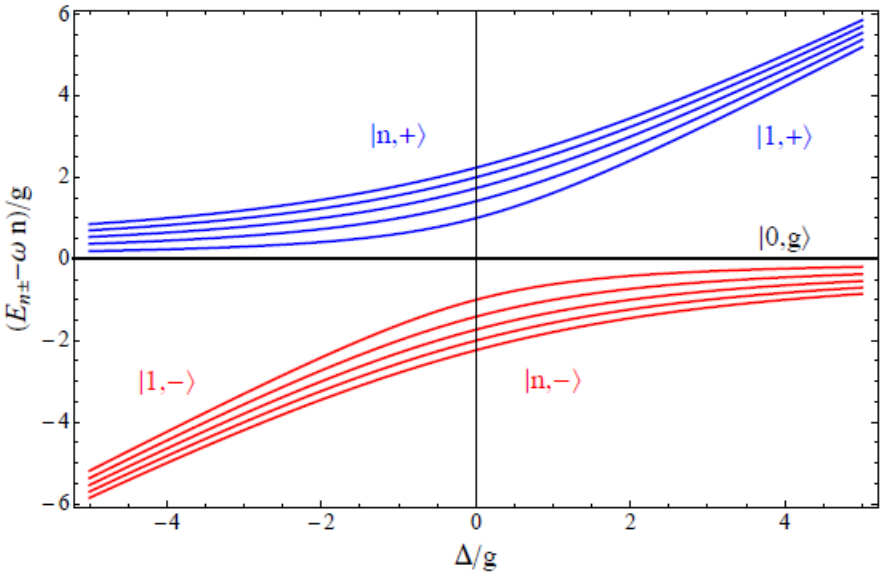

Figure 2.4: Plot of the energy eigenvalues (2.59) of the Jayne's-Cummings model versus the detuning ($\hbar = 1$).

$\chi_n(\Delta)$ is the *generalized Rabi frequency* and has the form

$$\chi_n(\Delta) = \sqrt{\Delta^2 + 4g^2 n}. \tag{2.60}$$

In Figure (2.4), I plot the energy eigenvalues (2.59) versus the detuning. One can see from this picture that for a fixed polariton number n the spectrum contains a set of two non-degenerated energy eigenvalues. One can see clearly that these eigenvalues naturally separate into so called upper and lower polariton branches, with the vacuum state as the only eigenstate belonging to both of them. Furthermore, one finds that the energy eigenvalues remain completely non-degenerated, even if the system is not in resonance. The splitting between states with the same number of polaritons is, according to equation (2.59), given by

$$\delta E_n = E_{n+} - E_{n-} = \chi_n(\Delta) = \sqrt{\Delta^2 + 4g^2 n}. \tag{2.61}$$

Notice, that this splitting does not only depend on the detuning Δ, which is expected, but also on the occupation number n in a non-linear way. Having obtained the energy eigenvalues, I can now calculate the respective eigenvectors by solving

the eigenvalue equation

$$\begin{pmatrix} \hbar\omega n & \hbar g\sqrt{n} \\ \hbar g\sqrt{n} & \hbar\omega n + \hbar\Delta \end{pmatrix} \cdot \begin{pmatrix} \alpha^{\pm} \\ \beta^{\pm} \end{pmatrix} = E_{n\pm} \begin{pmatrix} \alpha^{\pm} \\ \beta^{\pm} \end{pmatrix}. \tag{2.62}$$

To this end, I additionally assume that the eigenvectors are normalized and therefore their coefficients have to satisfy the relation

$$(\alpha^{\pm})^2 + (\beta^{\pm})^2 = 1 \tag{2.63}$$

Hence, the coefficients $\alpha^{\pm}$ and $\beta^{\pm}$ depend on each other. Because of that, I just have to consider one of the equations described by (2.62). Choosing the first one, I find that the coefficients have to satisfy the relation

$$\hbar\omega n\alpha^{+} + \hbar g\sqrt{n}\beta^{+} = E_{n+}\alpha^{+}. \tag{2.64}$$

Inserting (2.63) yields:

$$\begin{aligned} &(\hbar\omega n - E_{n+})\alpha^{+} + \hbar g\sqrt{n}\sqrt{1-(\alpha^{+})^2} = 0, \\ &\frac{1}{2}\hbar(\Delta + \chi_n(\Delta))\alpha^{+} = \hbar g\sqrt{n}\sqrt{1-(\alpha^{+})^2}, \\ &(\Delta + \chi_n(\Delta))^2(\alpha^{+})^2 = 4g^2 n\left[1-(\alpha^{+})^2\right]. \end{aligned} \tag{2.65}$$

Hence, one finds for the first coefficient α^{+}:

$$\alpha^{+} = \frac{2g\sqrt{n}}{\sqrt{(\Delta + \chi_n(\Delta))^2 + 4g^2 n}}. \tag{2.66}$$

Applying (2.63) once more, the second coefficient β^{+} becomes:

$$\beta^{+} = \frac{\Delta + \chi_n(\Delta)}{\sqrt{(\Delta + \chi_n(\Delta))^2 + 4g^2 n}}. \tag{2.67}$$

In principle, one could directly use these expressions. Yet, relation (2.63) implicates, that there should also exist a mapping for the coefficients α^{+} and β^{+} onto the unit circle. This observation justifies the parametrization

$$\sin(\theta_n) := \alpha^{+}, \quad \cos(\theta_n) := \beta^{+}. \tag{2.68}$$

That does not look like a huge improvement, but using some trigonometric addition theorems, one can show that this approach simplifies the form of the eigenstate coefficients. Using an addition theorem for the cosine leads to:

$$\begin{aligned}\cos(2\theta_n) = \cos^2(\theta_n) - \sin^2(\theta_n) &= \frac{(\Delta+\chi_n(\Delta))^2 - 4g^2n}{(\Delta+\chi_n(\Delta))^2 + 4g^2n} \\ &= \frac{2\Delta^2 + 2\Delta\chi_n(\Delta)}{2\chi_n^2(\Delta) + 2\Delta\chi_n(\Delta)} \\ &= \frac{2\Delta(\Delta+\chi_n(\Delta))}{2\chi_n(\Delta)(\chi_n(\Delta)+\Delta)},\end{aligned} \tag{2.69}$$

which, in turn, leads to the neat relation:

$$\cos(2\theta_n) = \frac{\Delta}{\chi_n(\Delta)}. \tag{2.70}$$

A corresponding theorem for the sine yields:

$$\begin{aligned}\sin(2\theta_n) = 2\sin(\theta_n)\cos(\theta_n) &= \frac{4g\sqrt{n}(\chi_n(\Delta)+\Delta)}{(\Delta+\chi_n(\Delta))^2 + 4g^2n} \\ &= \frac{4g\sqrt{n}(\chi_n(\Delta)+\Delta)}{2\chi_n(\Delta)(\chi_n(\Delta)+\Delta)},\end{aligned} \tag{2.71}$$

and thus, one finds:

$$\sin(2\theta_n) = \frac{2g\sqrt{n}}{\chi_n(\Delta)}. \tag{2.72}$$

The combination of equation (2.70) and equation (2.72) results in the concise expression

$$\tan(2\theta_n) = \frac{2g\sqrt{n}}{\Delta}. \tag{2.73}$$

Performing the same calculations for the second set of probability amplitudes, one finds the analogue relations:

$$\alpha^- = \frac{2g\sqrt{n}}{\sqrt{(\Delta-\chi_n(\Delta))^2 + 4g^2n}}, \tag{2.74}$$

$$\beta^- = \frac{(\Delta-\chi_n(\Delta))}{\sqrt{(\Delta-\chi_n(\Delta))^2 + 4g^2n}}. \tag{2.75}$$

In order to determine the corresponding expressions in the parametrized picture, which were introduced in relation (2.68), one needs to take a closer look at the

squared amplitudes. After some transformations:

$$(\alpha^-)^2 = \frac{4g^2n}{(\Delta-\chi_n(\Delta))^2+4g^2n} = \frac{4g^2n(\Delta+\chi_n(\Delta))^2}{(\Delta-\chi_n(\Delta))^2(\Delta+\chi_n(\Delta))^2+4g^2n(\Delta+\chi_n(\Delta))^2} \tag{2.76}$$
$$= \frac{4g^2n(\Delta+\chi_n(\Delta))^2}{(\Delta^2-\chi_n^2(\Delta))^2+4g^2n(\Delta+\chi_n(\Delta))^2} = \frac{4g^2n(\Delta+\chi_n(\Delta))^2}{(-4g^2n)^2+4g^2n(\Delta+\chi_n(\Delta))^2}$$
$$= \frac{4g^2n(\Delta+\chi_n(\Delta))^2}{4g^2n(4g^2n+(\Delta+\chi_n(\Delta))^2)} = \frac{(\Delta+\chi_n(\Delta))^2}{(4g^2n+(\Delta+\chi_n(\Delta))^2)},$$

one gets the important relation:

$$(\alpha^-)^2 = (\beta^+)^2. \tag{2.77}$$

Following the same argumentation for β^-, yields:

$$(\beta^-)^2 = \frac{(\Delta-\chi_n(\Delta))^2}{(\Delta-\chi_n(\Delta))^2+4g^2n} = \frac{4g^2n(\Delta-\chi_n(\Delta))^2}{(\Delta-\chi_n(\Delta))^2 4g^2n+(4g^2n)^2} \tag{2.78}$$
$$= \frac{4g^2n(\Delta-\chi_n(\Delta))^2}{4g^2n(\Delta^2-\chi_n^2(\Delta))^2+(-(\Delta^2-\chi_n^2(\Delta)))^2} = \frac{4g^2n}{4g^2n+\frac{(\Delta^2-\chi_n^2(\Delta))^2}{(\Delta-\chi_n(\Delta))^2}}$$
$$= \frac{4g^2n}{4g^2n+\frac{(\Delta-\chi_n(\Delta))^2(\Delta+\chi_n(\Delta))^2}{(\Delta-\chi_n(\Delta))^2}} = \frac{4g^2n}{4g^2n+(\Delta+\chi_n(\Delta))^2},$$

which indicates that

$$(\beta^-)^2 = (\alpha^+)^2. \tag{2.79}$$

However, there is still an uncertainty left. The calculations of the squared amplitudes $(\alpha^\pm)^2, (\beta^\pm)^2$ harbour the risk to loose some signs. If I consider the case of zero detuning, I find that β^- indeed has to fullfil the relation $\beta^- = -\alpha^+$. Now, I finally arrive at the desired form of the Jaynes-Cummings eigenstates, that for $n \geq 1$ the polariton eigenstates are given by

$$|n,+\rangle := \sin(\theta_n)|n,g\rangle + \cos(\theta_n)|n-1,e\rangle \tag{2.80a}$$
$$|n,-\rangle := \cos(\theta_n)|n,g\rangle - \sin(\theta_n)|n-1,e\rangle, \tag{2.80b}$$

whereas the vacuum state corresponding to $n = 0$ takes on the form

$$|0,\pm\rangle \equiv |0,g\rangle = |0\rangle. \tag{2.81}$$

According to (2.73) the occurring mixing angle θ_n is defined as:

$$\theta_n = \frac{1}{2} \arctan \frac{2g\sqrt{n}}{\Delta}. \tag{2.82}$$

Besides, using the orthogonality relation defined in (2.49), one can show that the Jaynes-Cummings eigenstates are orthonormal and obey the relation

$$\langle m, \alpha | n, \beta \rangle = \delta_{m,n} \delta_{\alpha,\beta}. \tag{2.83}$$

2.3 Dark-polaritons in cavity-QED: Modified Jaynes-Cummings Model

When the two American physicists Edwin Jaynes and Fred Cummings proposed their theoretical model, they were focusing on the interaction of a monochromatic, electromagnetic field mode with a quantum mechanical two-level system. The key feature here is that we have a single field which is quantized itself and the detuning, which describes the depletion of the excited level of the quantum mechanical two-level system through a non-resonant coupling of the atomic transition $|g\rangle \rightarrow |e\rangle$ by the monochromatic electromagnetic field, is arbitrary. However, what happens to the standard JC-model if we add an additional classical field and a further atomic level ? The answer to this question leads us to the modified Jaynes-Cummings (MJC)-model which is going to be discussed within this section. First, we derive the MJC-model Hamiltonian and discuss its properties as well as the properties of the eigenstates. Furthermore, we compare the two models.

2.3.1 Modified Jaynes-Cummings model Hamiltonian

We consider a single photon in a single mode QED cavity in which a Λ three-level atom is embedded. The ground levels are $|g\rangle$ and $|f\rangle$ with their level energies ω_g and ω_f, whereas the excited level $|e\rangle$ with level energy ω_e is detuned by a large, common single photon detuning Δ with respect to two coupling fields. The cavity field with frequency ω_0 couples the transition $|g\rangle \rightarrow |e\rangle$ with strength g_0. Further, a classical control field with frequency ω_c and Rabi-frequency Ω couples the transition $|f\rangle \rightarrow |e\rangle$. Hence, $|\Delta| \gg g_0, \Omega$. The system depicted is shown in Figure (2.5). Our

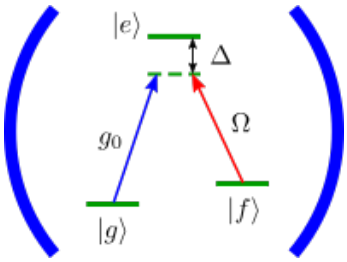

Figure 2.5: Three-level system in a single mode QED cavity with far detuned excited level $|e\rangle$. Blue arrow indicates the cavity mode coupling of the transition $|g\rangle - |e\rangle$ with a strength g_0, while the red arrow indicates the classical control field coupling of the transition $|f\rangle - |e\rangle$ with a Rabi-frequency Ω. Δ represents the common single photon detuning($\hbar = 1$).

bare model Hamiltonian ($\hbar = 1$) has the form

$$\hat{H}_{bare}(t) = \hat{H}_c + \hat{H}_a + \hat{H}_{int}(t), \tag{2.84a}$$

$$\hat{H}_c = \omega_0 \hat{a}^\dagger \hat{a}, \tag{2.84b}$$

$$\hat{H}_a = \omega_g \hat{\sigma}_{gg} + \omega_f \hat{\sigma}_{ff} + \omega_e \hat{\sigma}_{ee}, \tag{2.84c}$$

$$\hat{H}_{int}(t) = -(g_0 \hat{a} \hat{\sigma}_{eg} + g_0^* \hat{a}^\dagger \hat{\sigma}_{ge} + \Omega e^{-i\omega_c t} \hat{\sigma}_{ef} + \Omega^* e^{i\omega_c t} \hat{\sigma}_{fe}), \tag{2.84d}$$

where $\hat{H}_c$ denotes the free field Hamiltonian of the QED cavity, $\hat{H}_a$ stands for the free atomic Hamiltonian and $\hat{H}_{int}(t)$ describes the interaction of the fields with the atom. $\hat{a}^\dagger$ ($\hat{a}$) is the photonic creation (annihilation) operator and $\hat{\sigma}_{\alpha\beta} = |\alpha\rangle\langle\beta|$ ($\alpha, \beta \in \{g, f\}$) are the atomic operators. $\hat{H}_{bare}(t)$ in (2.84) satisfies the time-dependent Schrödinger equation

$$i\partial_t |\Psi(t)\rangle = \hat{H}'(t)|\Psi(t)\rangle. \tag{2.85}$$

We move to a rotating frame in which (2.84) is time-independent. The corresponding gauge transformation [12, 15] has the form ($\hbar = 1$)

$$\hat{H}^T = \hat{U}(t)\hat{H}_{bare}(t)\hat{U}^\dagger(t) + i\partial_t(\hat{U}(t))\hat{U}^\dagger(t), \tag{2.86}$$

where $\hat{U}(t)$ is a unitary transformation. Under the gauge (2.86), $\hat{H}_{bare}(t)$ reads

$$\hat{H}^T_{bare} = \hat{H}_c + \hat{H}_a + \hat{H}_{int}, \tag{2.87a}$$
$$\hat{H}_c = \omega_0 \hat{a}^\dagger \hat{a}, \tag{2.87b}$$
$$\hat{H}_a = \omega_g \hat{\sigma}_{gg} + (\omega_f + \omega_c)\hat{\sigma}_{ff} + \omega_e \hat{\sigma}_{ee}, \tag{2.87c}$$
$$\hat{H}_{int} = -(g_0 \hat{a}\hat{\sigma}_{eg} + g_0^* \hat{a}^\dagger \hat{\sigma}_{ge} + \Omega \hat{\sigma}_{ef} + \Omega^* \hat{\sigma}_{fe}). \tag{2.87d}$$

$\hat{U}(t) = e^{-i\omega_c t \hat{\sigma}_{ff}}$ has been chosen as the unitary transformation in deriving (2.87). Assume that the Λ three-level atom is initially prepared in the state $|g, n\rangle = |g\rangle \otimes |n\rangle$. n represents the arbitrary but fixed number of excitations with $n = 1, 2, 3 \ldots$ and $|n\rangle$ the corresponding number state. Under the action of $\hat{H}^T_{bare}$ onto the state $|g, n\rangle = |g\rangle \otimes |n\rangle$, we get the relations

$$\hat{H}^T_{bare}|g, n\rangle = (\omega_0 n + \omega_g)|g, n\rangle - g_0\sqrt{n}|e, n-1\rangle \tag{2.88a}$$
$$\hat{H}^T_{bare}|e, n-1\rangle = (\omega_0(n-1) + \omega_e)|e, n-1\rangle - g_0^*\sqrt{n}|g, n\rangle - \Omega^*|f, n-1\rangle \tag{2.88b}$$
$$\hat{H}^T_{bare}|f, n-1\rangle = (\omega_0(n-1) + \omega_f + \omega_c)|f, n-1\rangle - \Omega|e, n-1\rangle. \tag{2.88c}$$

In the subspace $\{|g, n\rangle, |e, n-1\rangle, |f, n-1\rangle\}$, $\hat{H}^T_{bare}$ has the matrix-representation

$$h_{bare} = \begin{pmatrix} (\omega_0 n + \omega_g) & -g_0\sqrt{n} & 0 \\ -g_0^*\sqrt{n} & \omega_0(n-1) + \omega_e & -\Omega^* \\ 0 & -\Omega & (\omega_0(n-1) + \omega_f + \omega_c) \end{pmatrix}. \tag{2.89}$$

Under Raman resonance condition $\omega_0 n + \omega_g = \omega_f + \omega_c = \omega_e - \Delta$, we get

$$h_{bare} = \begin{pmatrix} (\omega_0 n + \omega_g) & -g_0\sqrt{n} & 0 \\ -g_0^*\sqrt{n} & (\omega_0(n-1) + \omega_e) & -\Omega^* \\ 0 & -\Omega & (\omega_0(n-1) + \omega_f + \omega_c) \end{pmatrix}. \tag{2.90}$$

Under a rotating-wave approximation, (2.90) is reduced to

$$h^{Raman}_{bare} = \begin{pmatrix} 0 & -g_0\sqrt{n} & 0 \\ -g_0^*\sqrt{n} & \Delta & -\Omega^* \\ 0 & -\Omega & 0 \end{pmatrix}. \tag{2.91}$$

In addition, as we have a far detuned excited state $|e, n-1\rangle$, i.e. $|\Delta| \gg g_0, \Omega$ [66] we can adiabatically eliminate the contribution of the excited state $|e, n-1\rangle$ directly

on the level of (2.91). This yields to

$$h^{(mJC)} = \begin{pmatrix} -\frac{|g_0|^2 n}{\Delta} & -\frac{g_0^* \Omega \sqrt{n}}{\Delta} \\ -\frac{g_0 \Omega^* \sqrt{n}}{\Delta} & -\frac{|\Omega|^2}{\Delta} \end{pmatrix}. \tag{2.92}$$

(2.92) represents the matrix-form of the modified Jaynes-Cummings Hamiltonian (mJC) in the subspace $\{|g,n\rangle, |f,n-1\rangle\}$. The operator form of the modified Jaynes-Cummings Hamiltonian (mJC) reads

$$\hat{H}^{(mJC)} = \hat{H}_S + \hat{H}_{int}, \tag{2.93a}$$

$$\hat{H}_S = -(\frac{|g_0|^2}{\Delta}\hat{a}^\dagger \hat{a}\hat{\sigma}_{gg} + \frac{|\Omega|^2}{\Delta}\hat{\sigma}_{ff}), \tag{2.93b}$$

$$\hat{H}_{int} = -(\frac{g_0^* \Omega}{\Delta}\hat{a}^\dagger \hat{\sigma}_{gf} + \frac{g_0 \Omega^*}{\Delta}\hat{a}\hat{\sigma}_{fg}). \tag{2.93c}$$

The term $\hat{H}_S$ incorporates the influence of Stark shifts of the detuned fields, while $\hat{H}_{int}$ represents the interaction of the cavity field and the atom, where $G = g_0^* \Omega/\Delta$ is the effective atom-photon coupling constant. Hamiltonians $\hat{H}_S$ and $\hat{H}_{int}$ constitute the modified Jaynes-Cummings Hamiltonian. In the sequel, we are going to discuss the eigenstates of $\hat{H}^{(mJC)}$ and look at the effect of the control field Stark shift.

2.3.2 Eigenstates of the modified Jaynes-Cummings model Hamiltonian

In the following, we calculate the eigenenergies and eigenstates of $\hat{H}^{(mJC)}$. We show that dependently on whether one compensates the control field Stark shift by using external fields or not, the eigenenergies, composition of the eigenstates and the mixing angle θ_n differ significantly. First, we consider the case of non-compensated control field Stark shift. $\hat{H}^{(mJC)}$ of (2.93) reduces in the subspace $\{|g,n\rangle, |f,n-1\rangle\}$ as

$$h_n^{(m)} = \begin{pmatrix} -\frac{|g_0|^2 n}{\Delta} & -G\sqrt{n} \\ -G^*\sqrt{n} & -\frac{|\Omega|^2}{\Delta} \end{pmatrix}, \tag{2.94}$$

with $n = 1, 2, 3, \ldots$ the total number of excitations and corresponding number state $|n\rangle$. The eigenenergies are given as

$$E_{+,n}^{(m)} = 0 \tag{2.95}$$

$$E_{-,n}^{(m)} = -(\frac{|g_0|^2 n}{\Delta} + \frac{|\Omega|^2}{\Delta}) \tag{2.96}$$

The eigenstates to the eigenenergies $E_{+,n}^{(m)}$ and $E_{-,n}^{(m)}$ read

$$|n, DP^{(+)}\rangle := \sin(\theta_n)|f, n-1\rangle - \cos(\theta_n)|g, n\rangle \tag{2.97a}$$

$$|n, DP^{(-)}\rangle := \cos(\theta_n)|f, n-1\rangle + \sin(\theta_n)|g, n\rangle \tag{2.97b}$$

with the occurring mixing angle θ_n which is defined as:

$$\theta_n = \frac{1}{2}\arctan(\frac{2|g_0|\sqrt{n}}{\Omega}). \tag{2.98}$$

However, $|n, DP^{(\pm)}\rangle$ are called dark-polaritons. A dark-polariton is a quasiparticle which is a superposition of photonic and atomic excitations, where the atomic excitations have only contributions of ground levels $|g\rangle$ and $|f\rangle$ and not the excited level $|e\rangle$. Such dark-polaritons are very similar to the known dark-state polaritons [10, 11], but with one major difference. Dark-state polaritons are defined at Raman-resonance of two coupling fields and formed independently of the single photon detuning. Instead, dark-polaritons, which are also defined at Raman-resonance, are formed for a large single, common photon detuning Δ of the two coupling fields, i.e. $|\Delta| \gg g_m, \Omega$. The dependence on Δ enables to tune the eigenstate $|n, DP^{(\pm)}\rangle$ from an excited to a ground eigenstate. This follows from the eigenenergy $E_{-,n}^{(m)}$ of the dark-polariton $|n, DP^{(-)}\rangle$. If $\Delta > 0$ ($\Delta < 0$), $|n, DP^{(+)}\rangle$ is an excited (a ground) eigenstate and $|n, DP^{(-)}\rangle$ a ground (an excited) eigenstate. Note that $|n, DP^{(+)}\rangle$ is a degenerate eigenstate because the corresponding eigenenergy $E_{+,n}^{(m)}$ does not depend on the dark-polariton number n. $|n, DP^{(-)}\rangle$ is a degenerate eigenstate as well for $n \geq 2$. Thus, the spectrum is descrete and degenerate in dependence of the dark-polariton number n. Now, we switch to the case of compensated control field Stark shift. Compensation is achieved by using an additional field, which couples the ground state $|f\rangle$ with some far off resonant excited state [67]. Within (2.94) we set the control field Stark shift $\frac{|\Omega|^2}{\Delta}$ to zero. Hence, the new block-matrix representation

$h_n^{(m,comp)}$ in the subspace $\{|g,n\rangle, |f,n-1\rangle\}$ reads

$$h_n^{(m,comp)} = \begin{pmatrix} -\frac{|g_0|^2 n}{\Delta} & -G\sqrt{n} \\ -G^*\sqrt{n} & 0 \end{pmatrix}, \tag{2.99}$$

with $n = 1, 2, 3, \ldots$ the total number of excitations and corresponding number state $|n\rangle$. The block-matrix (2.99) is a 2×2 matrix and can be analitically diagonalized. The eigenenergies are given as

$$\begin{aligned} E_{-,n}^{(comp,m)} &= -\frac{|g_0|^2 n + |g_0|\sqrt{n}\sqrt{|g_0|^2 n + 4|\Omega|^2}}{2\Delta} \\ E_{+,n}^{(comp,m)} &= \frac{-|g_0|^2 n + |g_0|\sqrt{n}\sqrt{|g_0|^2 n + 4|\Omega|^2}}{2\Delta}. \end{aligned} \tag{2.100}$$

The respective eigenstates to the eigenenergies $E_{+,n}^{(comp,m)}$ and $E_{-,n}^{(comp,m)}$ are

$$|n, DP_{comp}^{(+)}\rangle := \sin(\theta_n)|f, n-1\rangle + \cos(\theta_n)|g, n\rangle \tag{2.101a}$$

$$|n, DP_{comp}^{(-)}\rangle := \cos(\theta_n)|f, n-1\rangle - \sin(\theta_n)|g, n\rangle, \tag{2.101b}$$

with the occurring mixing angles θ_n which is defined as:

$$\theta_n = \frac{1}{2}\arctan\left(\frac{A(\Omega, n)}{B(g_0, \Omega, n)}\right). \tag{2.102a}$$

$$A(\Omega, n) = 2\sqrt{2} \times \Omega\sqrt{n} \tag{2.102b}$$

$$B(g_0, \Omega, n) = \sqrt{C(g_0, \Omega, n)} \tag{2.102c}$$

$$C(g_0, \Omega, n) = |g_0|n^2 + 4|\Omega|^2 n + D(g_0, \Omega, n) \tag{2.102d}$$

$$D(g_0, \Omega, n) = |g_0|n\sqrt{n}\sqrt{|g_0|^2 n + 4|\Omega|^2} \tag{2.102e}$$

$|n, DP_{comp}^{(\pm)}\rangle$ are dark-polaritons, but of a different type compared to the case of non-compensated control field Stark shift. First of all, the eigenenergies $E_{s,n}^{(comp,m)}$ with $s = +, -$ depend on the generalized Rabi-frequency $\xi(n) = \sqrt{|g_0|^2 n + 4|\Omega|^2}$. Secondly, $|n, DP_{comp}^{(\pm)}\rangle$ have a common mixing angle θ_n that depends on the generalized Rabi-frequency $\xi(n)$ as well. In addition, the two dark-polariton branches, represented through $|n, DP_{comp}^{(\pm)}\rangle$ are separated by the energy amount

$$E_{-,n}^{(comp,m)} - E_{+,n}^{(comp,m)} = \frac{|g_0|\sqrt{n}\sqrt{|g_0|^2 n + 4|\Omega|^2}}{\Delta}. \tag{2.103}$$

The seperation energy is directly dependent on the generalized Rabi-frequency $\xi(n)$ and the common single photon detuning Δ as well. This seperation is related to the photon-photon repulsion. It is a consequence of the onsite repulsion $U(n)$ which is a measure of the Kerr-nonlinearity [23].

2.3.3 Comparison of the standard and modified Jaynes-Cummings model Hamiltonian

On the level of the individual Hamiltonians, major differences are that at first, in $\hat{H}^{(mJC)}$ the number operator depends on the projection operator $\hat{\sigma}_{gg}$ of the ground level $|g\rangle$ which is not the case in $\hat{H}^{(JC)}$. Second, in $\hat{H}^{(mJC)}$ the atom-cavity field coupling strength $G = g_m\Omega/\Delta$ is rescaled by the common single photon detuning Δ and the Rabi-frequency Ω, where G is chosen to be real. Regarding the eigenstates, a key difference between $\hat{H}^{(mJC)}$ and $\hat{H}^{(JC)}$ is that in the modified Jaynes-Cummings model we have eigenstate dependence on the control field Stark shift. In addition, within the modified Jaynes-Cummings model, we only have a dependence on ground levels, whereas in the standard Jaynes-Cummings model there exists a dependence on the excited level. Hence, these dependency affect the coherences. Namely, the bright-polaritons in the standard Jaynes-Cummings model only consist of optical coherences $\hat{\sigma}_{eg}$ and are explored to spontaneous emission, while in the modified Jaynes-Cummings model, dark-polaritons only consist of spin coherences $\hat{\sigma}_{fg}$ and no exploration to spontaneous emission is present. This enables the usage of dark-polaritons as a quantum memory for photons over their spin-coherences likewise the dark-state polaritons [10, 11, 12, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 15]. Changing the mixing angles in (2.98) and (2.102) over rotations from $0 \rightarrow \frac{\pi}{2}$, which corresponds to an adiabatical change of the Rabi-frequency Ω, photons are transfered to and stored in the spin-coherences in a reversible manner. Optical coherences have shorter coherence times compared to the spin coherences which have longer coherence times. Coherence times of spin-coherences are in the range of μs to ms in dark-state polaritons [10, 11]. Similar is the case for dark-polaritons.

3

Dark-state polaritons in a degenerate two-level system

We investigate the formation of dark-state polaritons in an ensemble of degenerate two-level atoms admitting electromagnetically induced transparency. Using a generalization of microscopic equation-of-motion technique, multiple collective polariton modes are identified depending on the polarizations of two coupling fields. For each mode, the polariton dispersion relation and composition are obtained in a closed form out of a matrix eigenvalue problem for arbitrary control field strengths. We illustrate the algorithm by considering the $F_g = 2 \rightarrow F_e = 1$ transition of the D_1 line in ^{87}Rb atomic vapour. In addition, an application of dark-state polaritons to the frequency and/or polarization conversion, using D_1 and D_2 transitions in cold Rb atoms, is given.

3.1 Degenerate two-level system

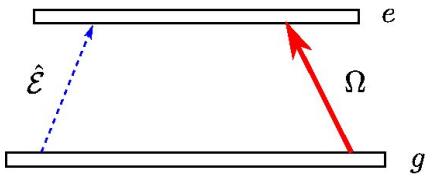

Figure 3.1: Schematic of a degenerate two-level system, having a ground state manifold g and an excited state manifold e, driven by a strong classical control field (thick line) of Rabi frequency Ω and by a weak quantum probe field $\hat{\mathcal{E}}$ (dashed line) of different polarizations.

In this section, we present a general formalism of dark-state polaritons in a degenerate two-level system. It is a generalization of the neat approach of Ref. [12]. We consider a gas sample of N atoms, where N is large. Let us denote by $\mathcal{H}_g$ the Hilbert space of the atomic states in the ground state manifold g and let $\mathcal{H}_e$ be the Hilbert space of atomic excited states in the manifold e. The corresponding ground- and excited-state energies are denoted by $\hbar\omega_g$ and $\hbar\omega_e$, respectively. A strong classical control field of Rabi frequency Ω and a weak quantum probe field $\hat{\mathcal{E}}$, which differ in polarizations and both propagate along the z axis, couple the transition $g \to e$ (see Fig. 3.1). The corresponding raising and lowering operators of the control (probe) field, $\hat{V}_c^\dagger$ and $\hat{V}_c$ ($\hat{V}_p^\dagger$ and $\hat{V}_p$), connect the states in manifold g to the states in manifold e and vice versa. We assume that $\dim \mathcal{H}_g \geq \dim \mathcal{H}_e$ holds, so that the system admits electromagnetically induced transparency [81, 82, 83]. This assures the existence of the Hilbert space $\mathcal{H}_g^d$ of the states in manifold g that are dark to the $g \to e$ transition for the control field [84]. Formally, we can view the raising operator $\hat{V}_c^\dagger$ as a linear mapping $\hat{V}_c^\dagger : \mathcal{H}_g \to \mathcal{H}_e$. The space $\mathcal{H}_g^d$ is then the null space of the mapping $\hat{V}_c^\dagger$

$$\mathcal{H}_g^d = \{|g\rangle \in \mathcal{H}_g \,|\, \hat{V}_c^\dagger |g\rangle = 0\}. \tag{3.1}$$

3.1.1 Model Hamiltonian

We will now present the model Hamiltonian and the dynamics of the lowest energy excitations of the ensemble of degenerate two-level atoms. The free atomic Hamiltonian has the form

$$\hat{H}_{at} = \sum_r (\hbar\omega_g \hat{\mathbb{I}}_g(r) + \hbar\omega_e \hat{\mathbb{I}}_e(r)), \tag{3.2}$$

where the summation index r counts the atomic positions, while $\hat{\mathbb{I}}_g$ and $\hat{\mathbb{I}}_e$ are the projection operators onto the states in the manifolds g and e, respectively. The free photon Hamiltonian, including multiple quantum probe field modes, is

$$\hat{H}_{ph} = \sum_k \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k, \tag{3.3}$$

where $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the creation and annihilation operators of the probe photons with the wave vector k and frequency $\omega_k = c|k| \sim \omega_{eg} \equiv \omega_e - \omega_g$. The atom interaction with the probe field is given through the minimal coupling Hamiltonian

$$\hat{H}_p = -\sum_k \sum_r \hbar g_k \, \hat{a}_k e^{ikr} \, \hat{V}_p^\dagger(r) + \text{H.c.}, \tag{3.4}$$

with coupling constant $\hbar g_k = \sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\, d_{ge}$, where d_{ge} is the effective electric dipole moment of the $g \to e$ transition, ϵ_0 is the vacuum permittivity and V is the quantization volume. The interaction of the atomic ensemble with the classical control field of the carrier frequency $\omega_c \sim \omega_{eg}$ and the wave vector k_c is of the form

$$\hat{H}_c(t) = -\sum_r \hbar\Omega \, e^{-i\omega_c t + ik_c r} \, \hat{V}_c^\dagger(r) + \text{H.c.}. \tag{3.5}$$

For simplicity, we have used the rotating-wave approximation. In addition, for an atomic operator $\hat{A}(r)$ we define a Fourier-transformed operator $\hat{A}(k) = \sum_r \hat{A}(r) e^{ikr}/\sqrt{N}$. Note that $\big(\hat{A}(k)\big)^\dagger = \hat{A}^\dagger(-k)$. Especially, one has $\sum_r \hat{A}(r) = \sqrt{N}\hat{A}(k{=}0)$. In terms of the Fourier-transformed operators, various Hamiltonian parts are

$$\hat{H}_{at} = \hbar\omega_g \sqrt{N}\, \hat{\mathbb{I}}_g(k{=}0) + \hbar\omega_e \sqrt{N}\, \hat{\mathbb{I}}_e(k{=}0), \tag{3.6a}$$

$$\hat{H}_p = -\sum_k \hbar g_k \sqrt{N}\, \hat{a}_k \hat{V}_p^\dagger(k) + \text{H.c.}, \tag{3.6b}$$

$$\hat{H}_c(t) = -\hbar\Omega\sqrt{N} e^{-i\omega_c t} \hat{V}_c^\dagger(k_c) + \text{H.c.} \tag{3.6c}$$

The entire Hamiltonian of the ensemble of degenerate two-level atoms interacting with the probe and the control field is $\hat{H}(t) = \hat{H}_{at} + \hat{H}_{ph} + \hat{H}_p + \hat{H}_c(t)$.

3.1.2 Dark-state polaritons

Now, we focus on the dark-state polaritons in an ensemble of degenerate two-level atoms. Various features of the method in Ref. [12], which are obvious *per se* in the case of a simple Λ system, need to be properly adapted to the degenerate two-level system. Additional complexity of the system we investigate also yields some new inherent requirements.

First of all, we remove the time dependence from the Hamiltonian $\hat{H}(t)$ by performing the following unitary gauge transformation

$$\hat{H}_T = \hat{U}_c(t)\hat{H}(t)\hat{U}_c^\dagger(t) - \hbar\omega_c\big(\sqrt{N}\, \hat{\mathbb{I}}_e(k{=}0) + \sum_k \hat{a}_k^\dagger \hat{a}_k\big), \tag{3.7}$$

where

$$\hat{U}_c(t) = \exp\left[i\omega_c t\left(\sqrt{N}\,\hat{\mathbb{I}}_e(k=0) + \sum_k \hat{a}_k^\dagger \hat{a}_k\right)\right]. \tag{3.8}$$

Eventually, we restate the time-dependent Schrödinger equation $i\hbar\,\partial_t|\phi(t)\rangle = \hat{H}(t)|\phi(t)\rangle$ as

$$i\hbar\,\partial_t\left[\hat{U}_c(t)|\phi(t)\rangle\right] = \hat{H}_T\left[\hat{U}_c(t)|\phi(t)\rangle\right]. \tag{3.9}$$

Solutions of Eq. (3.9) can be obtained by finding the energy eigenstates of the time-independent Hamiltonian $\hat{H}_T$.

Assume that the atomic ensemble is initially prepared in the collective vacuum state with no probe photons $|\boldsymbol{g}_0, 0\rangle = |\boldsymbol{g}_0\rangle \otimes |0\rangle \equiv \otimes_r |g_0\rangle_r \otimes |0\rangle$. Analogously with the Λ system case [11, 12], the atomic ground state $|g_0\rangle$ must be dark with respect to the control field, i.e.

$$\hat{V}_c^\dagger|g_0\rangle = 0, \quad \text{or equivalently} \quad |g_0\rangle \in \mathcal{H}_g^d. \tag{3.10}$$

Additional requirements on the state $|g_0\rangle$ will be specified afterwards.

Dark-state polaritons are particular low-energy, single probe photon driven, collective excitations that do not have a contribution of the excited atomic states. To obtain DSPs, we look for a polariton excitation operator $\hat{\phi}_k^\dagger$ such that in the low energy, single excitation case $\hat{\phi}_k^\dagger|\boldsymbol{g}_0, 0\rangle$ is an eigenstate of $\hat{H}_T$ with the energy $\hbar\omega(k)$. This leads to the following relation

$$\left[\hat{H}_T, \hat{\phi}_k^\dagger\right] = \hbar\omega(k)\hat{\phi}_k^\dagger + \cdots, \tag{3.11}$$

where dots represent the terms that are omitted in the single excitation case and also terms that give zero when acting on the collective vacuum state $|\boldsymbol{g}_0, 0\rangle$. Note that, for notational simplicity, we keep in mind that all subsequent commutators always act on the state $|\boldsymbol{g}_0, 0\rangle$. In agreement with Refs. [11, 12], we neglect Langevin noise effects, which do not influence the adiabatic evolution of the DSPs.

Collective atomic excitations are driven by the probe photons. Hence, we begin by calculating the commutator

$$\left[\hat{H}_T, \hat{a}_k^\dagger\right] = \hbar(\omega_k - \omega_c)\hat{a}_k^\dagger - \hbar g_k\sqrt{N}\,\hat{V}_p^\dagger(k). \tag{3.12}$$

The states that arise from the interaction with the probe field are the pure photon excitation $\hat{a}_k^\dagger|\boldsymbol{g}_0, 0\rangle$, and the collective atomic excitation $\hat{V}_p^\dagger(k)|\boldsymbol{g}_0, 0\rangle$, up to a normalization constant. Hence, in addition to $\hat{a}_k^\dagger$ the operator $\hat{V}_p^\dagger(k)$ is also a member of the polariton excitation operator $\hat{\phi}_k^\dagger$. Next, we determine the commutation relation

$$\begin{aligned}\left[\hat{H}_T, \hat{V}_p^\dagger(k)\right] = \hbar(\omega_{eg} - \omega_c)\hat{V}_p^\dagger(k) - \hbar\Omega^*(\hat{V}_c\hat{V}_p^\dagger)(k-k_c) \\ - \sum_{k'} \hbar g_{k'}^* \hat{a}_{k'}^\dagger(\hat{V}_p\hat{V}_p^\dagger)(k-k').\end{aligned} \tag{3.13}$$

Note that $\sqrt{N}\left[\hat{A}_1(k), \hat{A}_2(k')\right] = \left[\hat{A}_1, \hat{A}_2\right](k+k')$ holds for any two atomic operators $\hat{A}_1$ and $\hat{A}_2$. The new operators, $(\hat{V}_c\hat{V}_p^\dagger)(k-k_c)$ and $\hat{a}_{k'}^\dagger(\hat{V}_p\hat{V}_p^\dagger)(k-k')$, appearing in (3.13) yield the collective states via stimulated emission. The former can readily be included into the polariton excitation operator $\hat{\phi}_k^\dagger$. It creates the spatially dependent coherence among the atomic ground states $|g_0\rangle$ and $\hat{V}_c\hat{V}_p^\dagger|g_0\rangle$, i.e. the ground state coherence wave. When we commute the latter operator with $\hat{H}_T$, we get the operator $\hat{a}_{k''}^\dagger(\hat{V}_p\hat{V}_p^\dagger)(k-k')(\hat{V}_p\hat{V}_p^\dagger)(k'-k'')$. The emergence of such operators of increasing complexity continues and ends with $\hat{a}_{k^{(N)}}^\dagger \prod_{i=1}^N (\hat{V}_p\hat{V}_p^\dagger)(k^{(i)} - k^{(i-1)})$, where $k^{(0)} = k$. This case corresponds to a formidably complex DSP mode that is not tractable. Tractable modes are obtained by imposing one further requirement on the collective vacuum state. Namely, it is crucial that upon action $\hat{V}_p\hat{V}_p^\dagger|g_0\rangle$ we end up with the state $|g_0\rangle$, i.e.,

$$\hat{V}_p\hat{V}_p^\dagger|g_0\rangle = \lambda_p|g_0\rangle, \tag{3.14}$$

where $\lambda_p > 0$ is the corresponding eigenvalue. Thus, one obtains $(\hat{V}_p\hat{V}_p^\dagger)(k-k')$ $|\boldsymbol{g}_0, 0\rangle = \lambda_p\sqrt{N}\delta_{k,k'}|\boldsymbol{g}_0, 0\rangle$, so that the relation (3.13) greatly simplifies to

$$\begin{aligned}\left[\hat{H}_T, \hat{V}_p^\dagger(k)\right] = \hbar(\omega_{eg} - \omega_c)\hat{V}_p^\dagger(k) - \hbar\Omega^*(\hat{V}_c\hat{V}_p^\dagger)(k-k_c) \\ - \hbar g_k^* \lambda_p \sqrt{N}\hat{a}_k^\dagger.\end{aligned} \tag{3.15}$$

To proceed further, we define the excited atomic state $|e\rangle = \hat{V}_p^\dagger|g_0\rangle/\sqrt{\lambda_p}$ associated with the action of the probe field. Clearly, it has the property $\hat{V}_p|e\rangle = \sqrt{\lambda_p}|g_0\rangle$ and it is an eigenstate of $\hat{V}_p^\dagger\hat{V}_p$, i.e. $\hat{V}_p^\dagger\hat{V}_p|e\rangle = \lambda_p|e\rangle$. The eigenstates $|g_0\rangle$ and $|e\rangle$ are "tuned" to the polarization of the probe field. These are so called polarization-dressed states, first introduced and used in [84] for problems of interaction of resonant elliptically polarized light with atomic and molecular energy levels degenerate

in angular momentum projections. Next, let us consider the commutators

$$\big[\hat{H}_T, (\hat{V}_c\hat{V}_p^\dagger)(k-k_c)\big] = -\hbar\Omega(\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k), \tag{3.16}$$

and also

$$\begin{aligned}\big[\hat{H}_T, (\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k)\big] &= \hbar(\omega_{eg}-\omega_c)(\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k)\\ &\quad - \hbar\Omega^*(\hat{V}_c\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k-k_c)\\ &\quad - \sum_{k'}\hbar g_{k'}^*\hat{a}_{k'}^\dagger(\hat{V}_p\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k-k').\end{aligned} \tag{3.17}$$

Similar to the discussion of the relation (3.13), in order to avoid the appearance of probe photons with all wave vectors, we require that $\hat{V}_p\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger|g_0\rangle \propto |g_0\rangle$. That can hold provided that

$$\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger|g_0\rangle = \lambda_c\hat{V}_p^\dagger|g_0\rangle \quad \text{i.e.} \quad \hat{V}_c^\dagger\hat{V}_c|e\rangle = \lambda_c|e\rangle, \tag{3.18}$$

where $\lambda_c > 0$ is the corresponding eigenvalue. Thus, the excited atomic state $|e\rangle$ is a common eigenstate of the operators $\hat{V}_p^\dagger\hat{V}_p$ and $\hat{V}_c^\dagger\hat{V}_c$. Under such a condition, the relation (3.16) becomes

$$\big[\hat{H}_T, (\hat{V}_c\hat{V}_p^\dagger)(k-k_c)\big] = -\hbar\Omega\lambda_c\hat{V}_p^\dagger(k), \tag{3.19}$$

while (3.17) turns into

$$\big[\hat{H}_T, (\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger)(k)\big] = \lambda_c\big[\hat{H}_T, \hat{V}_p^\dagger(k)\big], \tag{3.20}$$

where the last commutator is found in (3.15). Hence, under the previous conditions no new components of the polariton excitation operator $\hat{\phi}_k^\dagger$ appear. Stimulated emission, which is driven by the control field, transfers the atoms from the excited state $|e\rangle$ into the ground state $|f\rangle = \hat{V}_c|e\rangle/\sqrt{\lambda_c}$. The states $|g_0\rangle$ and $|e\rangle$ are coupled by the probe field, while the states $|e\rangle$ and $|f\rangle$ are coupled by the control field. Thus, for each eigenvalue pair (λ_p, λ_c) the three states $|g_0\rangle$, $|e\rangle$ and $|f\rangle$ form an independent Λ system that is related to one independent collective DSP mode. The number of such Λ systems, i.e. tractable DSP modes, can be at most equal to the total number of DSP modes, i.e. to the dimensionality of the dark space $\mathcal{H}_g^d$.

Now, we collect the necessary commutation relations

$$\left[\hat{H}_T, \hat{a}_k^\dagger\right] = \hbar(\omega_k - \omega_c)\hat{a}_k^\dagger - \hbar g_k \sqrt{N}\hat{V}_p^\dagger(k), \tag{3.21a}$$

$$\begin{aligned}\left[\hat{H}_T, \hat{V}_p^\dagger(k)\right] = \hbar(\omega_{eg} - \omega_c)\hat{V}_p^\dagger(k) - \hbar g_k^* \lambda_p \sqrt{N}\hat{a}_k^\dagger \\ - \hbar\Omega^*(\hat{V}_c\hat{V}_p^\dagger)(k-k_c),\end{aligned} \tag{3.21b}$$

$$\left[\hat{H}_T, (\hat{V}_c\hat{V}_p^\dagger)(k-k_c)\right] = -\hbar\Omega\lambda_c\hat{V}_p^\dagger(k), \tag{3.21c}$$

so that the polariton excitation operator is of the form

$$\hat{\phi}_{nk}^\dagger = \alpha_{nk}\hat{a}_k^\dagger + \beta_{nk}\frac{\hat{V}_p^\dagger(k)}{\sqrt{\lambda_p}} + \gamma_{nk}\frac{(\hat{V}_c\hat{V}_p^\dagger)(k-k_c)}{\sqrt{\lambda_p\lambda_c}}, \tag{3.22}$$

where the band index n enumerates the different polariton species. Orthonormal collective excitations $|\boldsymbol{g}_0, 1_k\rangle$, $|\boldsymbol{e}(k), 0\rangle$ and $|\boldsymbol{f}(k-k_c), 0\rangle$ result from the action of the operators $\hat{a}_k^\dagger$, $\hat{V}_p^\dagger(k)/\sqrt{\lambda_p}$ and $(\hat{V}_c\hat{V}_p^\dagger)(k-k_c)/\sqrt{\lambda_p\lambda_c}$ on the collective vacuum state $|\boldsymbol{g}_0, 0\rangle$, respectively,

$$|\boldsymbol{g}_0, 1_k\rangle = \otimes_r |g_0\rangle_r \otimes |1_k\rangle, \tag{3.23a}$$

$$|\boldsymbol{e}(k), 0\rangle = \frac{1}{\sqrt{N}}\sum_r e^{ikr}|e\rangle_r \otimes_{r'\neq r} |g_0\rangle_{r'} \otimes |0\rangle, \tag{3.23b}$$

$$|\boldsymbol{f}(k-k_c), 0\rangle = \frac{1}{\sqrt{N}}\sum_r e^{i(k-k_c)r}|f\rangle_r \otimes_{r'\neq r} |g_0\rangle_{r'} \otimes |0\rangle. \tag{3.23c}$$

Note that the collective states $|\boldsymbol{e}(k), 0\rangle$ and $|\boldsymbol{f}(k-k_c), 0\rangle$ are entangled. This enables the usage of the polariton state

$$|\phi_{nk}\rangle = \alpha_{nk}|\boldsymbol{g}_0, 1_k\rangle + \beta_{nk}|\boldsymbol{e}(k), 0\rangle + \gamma_{nk}|\boldsymbol{f}(k-k_c), 0\rangle \tag{3.24}$$

as a resource for quantum information processing [5].

We determine the c-numbers α_{nk}, β_{nk} and γ_{nk} by inserting (3.22) into (3.11) and make use of (3.21). This leads to three self-consistency equations that we can represent in the basis $\{|\boldsymbol{g}_0, 1_k\rangle, |\boldsymbol{e}(k), 0\rangle, |\boldsymbol{f}(k-k_c), 0\rangle\}$ as

$$\begin{bmatrix} \omega_k - \omega_c & -\widetilde{g}_k^*\sqrt{N} & 0 \\ -\widetilde{g}_k\sqrt{N} & \omega_{eg} - \omega_c & -\widetilde{\Omega} \\ 0 & -\widetilde{\Omega}^* & 0 \end{bmatrix} \begin{bmatrix} \alpha_{nk} \\ \beta_{nk} \\ \gamma_{nk} \end{bmatrix} = \omega_n(k) \begin{bmatrix} \alpha_{nk} \\ \beta_{nk} \\ \gamma_{nk} \end{bmatrix}, \tag{3.25}$$

where $\widetilde{g}_k = g_k\sqrt{\lambda_p}$ and $\widetilde{\Omega} = \Omega\sqrt{\lambda_c}$. Our effective Hamiltonian in Eq. (3.25) is similar to the one in [12], but with a major difference. The effective coupling constant $\widetilde{g}_k$ and the effective Rabi-frequency $\widetilde{\Omega}$ differ from the corresponding one in Ref. [12] because of the inclusion of the eigenvalues λ_p and λ_c. The mentioned difference clearly arises as a consequence of the degenerate two-level atomic system.

The dark-state polaritons are obtained as one of the solutions of the eigenproblem (3.25). The other two solutions are bright-state polaritons, similarly as in [12]. Exactly at the Raman resonance, $\omega_k = \omega_c$, there is an eigenvector $\propto \big[-\frac{\widetilde{\Omega}}{\widetilde{g}_k\sqrt{N}}, 0, 1\big]$. This eigenvector has no contribution of the excited atomic states and represents a stable dark-state polariton that is insensitive to incoherent decay processes acting on the excited atoms. Expansion around the resonance $\omega_k \sim \omega_{eg}$ and $\omega_c \sim \omega_{eg}$ yields a linearized solution for the dark-state polaritons

$$\omega(k) = \frac{|\widetilde{\Omega}|^2}{|\widetilde{g}_k|^2 N + |\widetilde{\Omega}|^2}(\omega_k - \omega_c), \tag{3.26a}$$

$$\alpha_k = -\frac{\widetilde{\Omega}}{\widetilde{g}_k\sqrt{N}}\gamma_k, \quad \beta_k = -\frac{\widetilde{\Omega}(\omega_k - \omega_c)}{|\widetilde{g}_k|^2 N + |\widetilde{\Omega}|^2}\gamma_k. \tag{3.26b}$$

An interesting property of the DSP solution is that it only depends on the Raman detuning $\omega_k - \omega_c$ of the coupling fields and on the coupling parameters $\widetilde{g}_k$ and $\widetilde{\Omega}$. It does not depend on the energy spacing ω_{eg} of the underlying degenerate two-level system.

The algorithm for finding tractable DSP modes in a degenerate two-level system can be summarized as:

(1) Determine the dark space $\mathcal{H}_g^d$ for the operator $\hat{V}_c^\dagger$;

(2) Find all states $|g_0\rangle$ from $\mathcal{H}_g^d$ and pairs of eigenvalues (λ_p, λ_c) such that $\hat{V}_p\hat{V}_p^\dagger|g_0\rangle = \lambda_p|g_0\rangle$ and $\hat{V}_c^\dagger\hat{V}_c\hat{V}_p^\dagger|g_0\rangle = \lambda_c\hat{V}_p^\dagger|g_0\rangle$ hold;

(3) For every such pair of eigenvalues obtain DSPs $|\psi_k(\lambda_p, \lambda_c)\rangle$ from (3.24) and (3.26).

3.2 Dark state polaritons in rubidium vapour

In this section we apply the general formalism to the rubidium vapour. Control and probe fields couple the hyperfine levels $5S_{1/2}$, $F_g = 2$ and $5P_{1/2}$, $F_e = 1$ of ^{87}Rb.

The atomic lowering operators of the control and probe fields are, respectively,

$$\hat{V}_c = \hat{\mathbf{V}} \cdot \mathbf{e}_c, \qquad \hat{V}_p = \hat{\mathbf{V}} \cdot \mathbf{e}_p, \tag{3.27}$$

where $\mathbf{e}_c$ and $\mathbf{e}_p$ are polarizations of the fields. The vector operator $\hat{\mathbf{V}}$ is defined by [84, 85]

$$\begin{aligned} \hat{\mathbf{V}} = (-1)^{F_e+J_g+I+1}\sqrt{(2F_e+1)(2J_g+1)} \begin{Bmatrix} J_e & J_g & 1 \\ F_g & F_e & I \end{Bmatrix} \\ \times \sum_{q=-1}^{1} \sum_{m_g,m_e} \langle F_g, m_g | F_e, m_e; 1, q\rangle |F_g, m_g\rangle \langle F_e, m_e | \mathbf{e}_q^*, \end{aligned} \tag{3.28}$$

where $I = 3/2$ is the nuclear quantum number of ^{87}Rb, $\{:::\}$ is the Wigner $6j$-symbol and $\langle F_g, m_g | F_e, m_e; 1, q\rangle$ is the Clebsch-Gordan coefficient that connects the excited level state $|F_e, m_e\rangle$ to the ground level state $|F_g, m_g\rangle$ via polarization $\mathbf{e}_q^*$,

$$\mathbf{e}_{\pm 1} = \mp \frac{1}{\sqrt{2}}(\mathbf{e}_x \pm i\mathbf{e}_y), \quad \mathbf{e}_0 = \mathbf{e}_z, \tag{3.29}$$

given in some orthonormal basis of polarization vectors. We choose the coordinate system such that the fields propagate along the z axis, and define a basis of Zeeman states relative to this quantization axis. The bases of the individual Hilbert spaces $\mathcal{H}_e$ and $\mathcal{H}_g$ are

$$\mathcal{E} = \{|1, -1\rangle_e, |1, 0\rangle_e, |1, 1\rangle_e\}, \tag{3.30a}$$

$$\mathcal{G} = \{|2, -2\rangle_g, |2, -1\rangle_g, |2, 0\rangle_g, |2, 1\rangle_g, |2, 2\rangle_g\}. \tag{3.30b}$$

We will show that according to the appropriate choice of the polarizations of the coupling fields, one or two DSP modes can be obtained.

3.2.1 Case of orthogonal circular polarizations

Let the control field couples σ^- transitions, while the probe field couples σ^+ transitions, i.e. $\mathbf{e}_c = \mathbf{e}_{+1}$ and $\mathbf{e}_p = \mathbf{e}_{-1}$ (see Fig. 3.2). The lowering operators of the

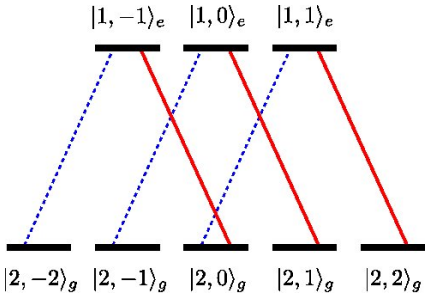

Figure 3.2: Zeeman sublevel scheme of the transition $F_g = 2 \rightarrow F_e = 1$ at the D_1 line of ^{87}Rb. Solid lines denote σ^- transitions coupled by the control field while dashed lines denote σ^+ transitions coupled by the probe field.

coupling fields, $\hat{V}_c$ and $\hat{V}_p$, are represented in the basis $\mathcal{E} \cup \mathcal{G}$ with the matrices

$$\mathbf{V}_c = \begin{bmatrix} & \mathbf{0}_{3,3} & & \mathbf{0}_{3,5} \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ \frac{1}{2\sqrt{3}} & 0 & 0 & \mathbf{0}_{5,5} \\ 0 & \frac{1}{2} & 0 & \\ 0 & 0 & \frac{1}{\sqrt{2}} & \end{bmatrix}, \tag{3.31a}$$

$$\mathbf{V}_p = \begin{bmatrix} & \mathbf{0}_{3,3} & & \mathbf{0}_{3,5} \\ \frac{1}{\sqrt{2}} & 0 & 0 & \\ 0 & \frac{1}{2} & 0 & \\ 0 & 0 & \frac{1}{2\sqrt{3}} & \mathbf{0}_{5,5} \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{bmatrix}, \tag{3.31b}$$

where zeros $\mathbf{0}_{m,n}$ denote rectangular $m \times n$ null matrices. Ground level dark space determined from the null space of $\mathbf{V}_c^\dagger$ is

$$\mathcal{H}_g^d = \{|2,-2\rangle_g, |2,-1\rangle_g\}. \tag{3.32}$$

Both dark states are appropriate as the initial state $|g_0\rangle$. Below we tabulate the corresponding states and eigenvalues of the Λ system:

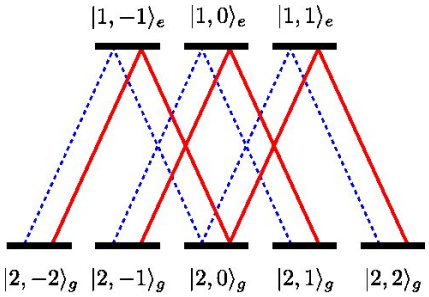

Figure 3.3: Zeeman sublevel scheme of the transition $F_g = 2 \rightarrow F_e = 1$ at the D_1 line of ^{87}Rb. Solid lines denote control field linearly polarized along y axis while dashed lines denote probe field linearly polarized along x axis.

	$\lvert g_0\rangle$	$\lvert e\rangle$	$\lvert f\rangle$	λ_p	λ_c
I	$\lvert 2,-2\rangle_g$	$\lvert 1,-1\rangle_e$	$\lvert 2,0\rangle_g$	1/2	1/12
II	$\lvert 2,-1\rangle_g$	$\lvert 1,0\rangle_e$	$\lvert 2,1\rangle_g$	1/4	1/4

,

that lead to two DSP modes:

$$\omega^I(k) = \frac{|\Omega|^2}{6\,|g_k|^2 N + |\Omega|^2}(\omega_k - \omega_c), \tag{3.33a}$$

$$|\psi_k^I\rangle \propto -\frac{\Omega}{\sqrt{6}\,g_k\sqrt{N}}|\boldsymbol{g}_0^I, 1_k\rangle + |\boldsymbol{f}^I(k-k_c), 0\rangle - \frac{2\sqrt{3}\,\Omega(\omega_k - \omega_c)}{6\,|g_k|^2 N + |\Omega|^2}|\boldsymbol{e}^I(k), 0\rangle, \tag{3.33b}$$

$$\omega^{II}(k) = \frac{|\Omega|^2}{|g_k|^2 N + |\Omega|^2}(\omega_k - \omega_c), \tag{3.34a}$$

$$|\psi_k^{II}\rangle \propto -\frac{\Omega}{g_k\sqrt{N}}|\boldsymbol{g}_0^{II}, 1_k\rangle + |\boldsymbol{f}^{II}(k-k_c), 0\rangle - \frac{2\,\Omega(\omega_k - \omega_c)}{|g_k|^2 N + |\Omega|^2}|\boldsymbol{e}^{II}(k), 0\rangle. \tag{3.34b}$$

We see that for orthogonal circular polarizations of the coupling fields, the maximal number of tractable DSP modes exists. This is the generic case, because relevant independent Λ system(s) can be easily recognized.

3.2.2 Case of orthogonal linear polarizations

Now analyze the case of the control field polarization along the y axis and the probe field polarization along the x axis, i.e. $\mathbf{e}_c = \mathbf{e}_y$ and $\mathbf{e}_p = \mathbf{e}_x$ (see Fig. 3.3). The

matrices representing the atomic lowering operators $\hat{V}_c$ and $\hat{V}_p$ in the basis $\mathcal{E} \cup \mathcal{G}$ are

$$\mathbf{V}_c = i \begin{bmatrix} & \mathbf{0}_{3,3} & & \mathbf{0}_{3,5} \\ \frac{1}{2} & 0 & 0 & \\ 0 & \frac{1}{2\sqrt{2}} & 0 & \\ \frac{1}{2\sqrt{6}} & 0 & \frac{1}{2\sqrt{6}} & \mathbf{0}_{5,5} \\ 0 & \frac{1}{2\sqrt{2}} & 0 & \\ 0 & 0 & \frac{1}{2} & \end{bmatrix}, \tag{3.35a}$$

$$\mathbf{V}_p = \begin{bmatrix} & \mathbf{0}_{3,3} & & \mathbf{0}_{3,5} \\ \frac{1}{2} & 0 & 0 & \\ 0 & \frac{1}{2\sqrt{2}} & 0 & \\ -\frac{1}{2\sqrt{6}} & 0 & \frac{1}{2\sqrt{6}} & \mathbf{0}_{5,5} \\ 0 & -\frac{1}{2\sqrt{2}} & 0 & \\ 0 & 0 & -\frac{1}{2} & \end{bmatrix}. \tag{3.35b}$$

In this case, the ground level dark space is

$$\begin{aligned} \mathcal{H}_g^d = \Big\{ & -\frac{1}{\sqrt{2}}|2,-1\rangle_g + \frac{1}{\sqrt{2}}|2,1\rangle_g, \\ & \frac{1}{\sqrt{8}}|2,-2\rangle_g - \frac{\sqrt{3}}{2}|2,0\rangle_g + \frac{1}{\sqrt{8}}|2,2\rangle_g, \Big\}, \end{aligned} \tag{3.36}$$

but only the first dark state satisfies all necessary conditions for the vacuum state of the tractable mode. The states and eigenvalues of the corresponding Λ system are

$$|g_0\rangle = -\frac{1}{\sqrt{2}}|2,-1\rangle_g + \frac{1}{\sqrt{2}}|2,1\rangle_g, \tag{3.37a}$$

$$|e\rangle = |1,0\rangle_e, \tag{3.37b}$$

$$|f\rangle = \frac{1}{\sqrt{2}}|2,-1\rangle_g + \frac{1}{\sqrt{2}}|2,1\rangle_g, \tag{3.37c}$$

$$\lambda_p = 1/4, \qquad \lambda_c = 1/4. \tag{3.37d}$$

We identify one DSP mode

$$\omega(k) = \frac{|\Omega|^2}{|g_k|^2 N + |\Omega|^2}(\omega_k - \omega_c), \tag{3.38a}$$

$$|\psi_k\rangle \propto -\frac{\Omega}{g_k\sqrt{N}}|\boldsymbol{g}_0, 1_k\rangle + |\boldsymbol{f}(k-k_c), 0\rangle - \frac{2\,\Omega(\omega_k - \omega_c)}{|g_k|^2 N + |\Omega|^2}|\boldsymbol{e}(k), 0\rangle, \tag{3.38b}$$

while the other one is non-tractable.

From the above examples, it can be seen that the choice of the polarization of the coupling fields yields entirely different DSP modes. This is reflected in the composition of the DSP state as well as in the polariton dispersion relation. Note that different polariton dispersion relations would lead to distinct slow light group velocities. In the next section we outline one possible application of DSP modes in degenerate two-level systems for frequency and/or linear polarization conversion.

3.3 Frequency and polarization conversion

Let us consider the DSP modes that can be formed from the states within $5S_{1/2}$, $F_g = 1$ hyperfine level of ^{87}Rb atoms, when the control and the probe field have orthogonal linear polarizations. There are three relevant atomic transitions:

(a) $5S_{1/2},\ F_g = 1 \rightarrow 5P_{1/2},\ F_e = 1,$

(b) $5S_{1/2},\ F_g = 1 \rightarrow 5P_{3/2},\ F_e = 1,$

(c) $5S_{1/2},\ F_g = 1 \rightarrow 5P_{3/2},\ F_e = 0.$

The first belongs to the D_1 line. The last two belong to the D_2 line and can be rendered non-overlapping by using ultracold rubidium atoms.

In the case of orthogonal linear polarizations $\mathbf{e}_c = \mathbf{e}_x$ and $\mathbf{e}_p = \mathbf{e}_y$ of the fields that are resonant to the D_1 line transition (a), we have

$$|g_0\rangle = -\frac{1}{\sqrt{2}}|1,-1\rangle_g + \frac{1}{\sqrt{2}}|1,1\rangle_g, \tag{3.39a}$$

$$|e\rangle = |1,0\rangle_e, \tag{3.39b}$$

$$|f\rangle = \frac{1}{\sqrt{2}}|1,-1\rangle_g + \frac{1}{\sqrt{2}}|1,1\rangle_g, \tag{3.39c}$$

$$\lambda_p = 1/12, \qquad \lambda_c = 1/12. \tag{3.39d}$$

When considering the D_2 line transition (b) with the same polarizations of the coupling fields as in the previous case, $\mathbf{e}_c = \mathbf{e}_x$ and $\mathbf{e}_p = \mathbf{e}_y$, we find

$$|g_0\rangle = -\frac{1}{\sqrt{2}}|1,-1\rangle_g + \frac{1}{\sqrt{2}}|1,1\rangle_g, \tag{3.40a}$$

$$|e\rangle = |1,0\rangle_e, \tag{3.40b}$$

$$|f\rangle = \frac{1}{\sqrt{2}}|1,-1\rangle_g + \frac{1}{\sqrt{2}}|1,1\rangle_g, \tag{3.40c}$$

$$\lambda_p = 5/24, \qquad \lambda_c = 5/24. \tag{3.40d}$$

Finally, for the *swapped linear polarizations*, $\mathbf{e}_c = \mathbf{e}_y$ and $\mathbf{e}_p = \mathbf{e}_x$, of the fields coupling the D_2 line transition (c), we have

$$|g_0\rangle = -\frac{1}{\sqrt{2}}|1,-1\rangle_g + \frac{1}{\sqrt{2}}|1,1\rangle_g, \tag{3.41a}$$

$$|e\rangle = |0,0\rangle_e, \tag{3.41b}$$

$$|f\rangle = \frac{1}{\sqrt{2}}|1,-1\rangle_g + \frac{1}{\sqrt{2}}|1,1\rangle_g, \tag{3.41c}$$

$$\lambda_p = 1/6, \qquad \lambda_c = 1/6. \tag{3.41d}$$

Note, if the polarizations of the fields had not been swapped, the states $|g_0\rangle$ and $f\rangle$ would have been interchanged.

As can be seen from Eqs. (3.39)-(3.41), the DSP modes are formed from the same states $|g_0\rangle$ and $|f\rangle$ in all three cases, but the considered transitions and polarizations of the coupling fields are different. This provides the possibility for frequency [76, 86, 87] and/or polarization conversion [88, 89] of linearly polarized light. First, one can store a pulse of the probe light polarized along the y axis into the atomic coherence

among the states $|g_0\rangle$ and $|f\rangle$ using the transition (a) and the control field polarized along the x axis. The retrieval process, using the transition (b) and the control field polarized along the x axis, would release the pulse at a different frequency, but of the same optical quantum state and polarization along the y axis as the original probe pulse. However, the pulse retrieved using the transition (c) and the control field polarized along the y axis would be in the same optical quantum state as the original probe pulse, but of different carrier frequency and linear polarization along the x axis, i.e. *orthogonal to the original one*. Moreover, this realization does not suffer from the energy loss in the retrieved pulse, since the ratios of the probe and control Clebsch-Gordan coefficients are the same among all three transitions [89].

4

Dark-polariton bound pairs in coupled QED cavity arrays

We investigate a one-dimensional modified Jaynes-Cummings-Hubbard (MJCH) chain of N identical QED cavities with nearest neighbour photon tunnelling and periodic boundary conditions. Each cavity contains an embedded three-level atom which is coupled to a cavity mode and an external classical control field. In the case of two-excitations and common large detuning of two Raman-resonant fields, we show the emergence of two different species of dark-polariton bound pairs (DPBPs) that are mutually localized in their relative spatial coordinates. Due to the high degree of controllability, we show the appearance of either one or two DPBPs, having the energies within the energy gaps between three bands of mutually delocalized eigenstates. Interestingly, in a different parameter regime with negatively detuned Raman fields, we find that the ground state of the system is a DPBP which can be utilized for the photon storage, retrieval and controllable state preparation. Moreover, we propose an experimental realization of our model system.

4.1 Model system and effective model Hamiltonian

An extension of the modified Jaynes-Cummings model to an one-dimensional array of coupled QED cavities is realized. This will lead us to the modified Jaynes-Cummings Hubbard model as our effective model Hamiltonian. It includes the hopping between adjacent cavities. First, we state the model system and second,

present the effective model Hamiltonian.

4.1.1 Model system

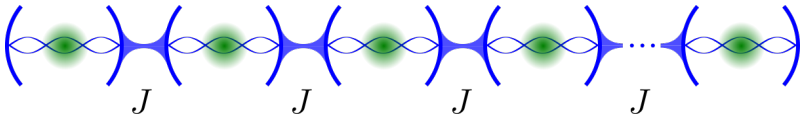

Figure 4.1: One-dimensional array of N coupled, identical QED cavities with uniform inter-cavity photon hopping strength J under periodic boundary conditions. Green transparent sphere in each cavity represents a three-level atom with two ground levels $|g\rangle$ and $|f\rangle$, and an excited level $|e\rangle$. The atomic configuration is given in Figure (2.5) of the theory chapter 2.

The system we consider consists of a one-dimensional array of N coupled QED cavities see Figure (4.1). We assume periodic boundary conditions, i.e., the cavity labelled by $n = N + 1$ corresponds to the cavity $n = 1$. Each cavity embeds a three-level atom with two ground levels $|g\rangle$ and $|f\rangle$, and an excited level $|e\rangle$. The level energies are ω_g, ω_f and ω_e, respectively and the excited level $|e\rangle$ is detuned by the common single photon detuning Δ. In reality, the levels can be either fine or hyperfine levels of alkali-metal atoms. Their D1 or D2 line transitions are nowadays easily accessible via available lasers and optical modes of QED cavities. One mode of a tunable cavity [90, 91] of frequency ω_m, couples the transition $|g\rangle \rightarrow |e\rangle$ with the strength g_m, and the classical control field of frequency ω_c and Rabi-frequency Ω, couples the transition $|f\rangle \rightarrow |e\rangle$. This configuration is known to feature vacuum induced transparency, as first experimentally demonstrated by the group of Vuletić [92]. Both g_m and Ω are typically in MHz range for alkali-metal atoms, which are strongly coupled to QED cavities, and for moderate laser powers.

4.1.2 Effective model Hamiltonian

As we consider a one-dimensional chain of N identical coupled QED cavities, the derived modified Jaynes-Cummings model for a single QED cavity is valid for all QED cavities in the one-dimensional chain. Therefore, our effective model Hamiltonian

(modified Jaynes-Cummings Hubbard model) ($\hbar = 1$) has the form

$$\hat{H}^{(mJCH)} = \hat{H}^{(mJC)} + \hat{H}_{hop}, \tag{4.1a}$$

$$\hat{H}^{(mJC)} = \hat{H}_S + \hat{H}_{int}, \tag{4.1b}$$

$$\hat{H}_S = -\sum_{\mu=1}^{N} \left(\frac{g_m^2}{\Delta} \hat{c}_\mu^\dagger \hat{c}_\mu \hat{\sigma}_{gg}^{(\mu)} + \frac{\Omega^2}{\Delta} \hat{\sigma}_{ff}^{(\mu)} \right), \tag{4.1c}$$

$$\hat{H}_{int} = -G \sum_{\mu=1}^{N} (\hat{c}_\mu^\dagger \hat{\sigma}_{gf}^{(\mu)} + \hat{c}_\mu \hat{\sigma}_{fg}^{(\mu)}), \tag{4.1d}$$

$$\hat{H}_{hop} = -J \sum_{\mu=1}^{N} (\hat{c}_{\mu+1}^\dagger \hat{c}_\mu + \hat{c}_\mu^\dagger \hat{c}_{\mu+1}), \tag{4.1e}$$

where $\hat{c}_\mu^\dagger$ ($\hat{c}_\mu$) is the photonic creation (annihilation) operator and $\hat{\sigma}_{\alpha\beta}^{(\mu)} = |\alpha\rangle_\mu\langle\beta|$ ($\alpha, \beta \in \{g, f\}$) are the atomic operators for the site number μ. The term $\hat{H}_S$ incorporates the influence of Stark shifts of the detuned fields, while $\hat{H}_{int}$ represents the interaction of the cavity field and the atom, where $G = g_m\Omega/\Delta$ is the effective atom-photon coupling constant which is set to be real. Hamiltonians $\hat{H}_S$ and $\hat{H}_{int}$ constitute the modified Jaynes-Cummings Hamiltonian. As will be shown in the sequel, the Stark shifts have profound influence on the energy eigenspectrum. $\hat{H}_{hop}$ describes the photon hopping between adjacent cavities, based on evanescent field coupling, with J as the inter-cavity photon hopping strength. Similar effective Hamiltonian has been previously used to describe a network of fibre coupled cavities, embedded with three-level atoms [67]. However, while that scheme requires the compensation of the level Stark shifts, here we utilize the individual Stark shifts to achieve tunability. Our effective model Hamiltonian (4.1) supports the formation of dark-polariton bound pairs. We will see that the different dark-polaritons, which have been discussed in Section II, are actually involved in the formation of the energy bands and the bound states. Moreover, we show and discuss that the bound states are formed due to the presents of a force called Kerr-nonlinearity which is determined by the onsite repulsion.

4.2 Formation of dark-polariton bound pairs

In the following, we discuss the formation of dark-polariton bound pairs in our system. In order to exploit the invariance of the system under cyclic permutations

of the sites, we introduce the following operators via discrete Fourier transforms

$$\hat{b}_k = \frac{1}{\sqrt{N}} \sum_{\mu=1}^{N} e^{-\frac{2\pi i}{N}\mu k}\, \hat{c}_\mu, \tag{4.2a}$$

$$\hat{s}_{gf}^{(k)} = \frac{1}{\sqrt{N}} \sum_{\mu=1}^{N} e^{-\frac{2\pi i}{N}\mu k}\, \hat{\sigma}_{gf}^{(\mu)}, \tag{4.2b}$$

where $k = 0, 1, \ldots, N-1$ is related to the (discrete) quasi-momentum of the excitation. Similarly to [32], we work in the two-excitation subspace that is spanned by the states $|kj\rangle_F \equiv \hat{b}_k^\dagger \hat{b}_j^\dagger |\Phi_0\rangle$, $|k\rangle_F|j\rangle_A \equiv \hat{b}_k^\dagger \hat{s}_{gf}^{(j)\dagger}|\Phi_0\rangle$ and $|kj\rangle_A \equiv \hat{s}_{gf}^{(k)\dagger}\hat{s}_{gf}^{(j)\dagger}|\Phi_0\rangle$. The subscripts F and A stand for the photonic and atomic excitations respectively. The state $|\Phi_0\rangle = \otimes_{\mu=1}^{N}|g\rangle_\mu|0\rangle_\mu$ is the ground state of the system, where $|0\rangle_\mu$ denotes the vacuum state of the cavity number μ. We note that the excitations (polaritons) are in our case dark in a sense that they do not have the contribution of the excited levels $|e\rangle$ and are not subjected to spontaneous emission. The atomic excitations $|kj\rangle_A$ are in general not orthogonal to each other because of ${}_A\langle k'j'|kj\rangle_A = \delta_{k,k'}\delta_{j,j'} + \delta_{k,j'}\delta_{j,k'} - \frac{2}{N}\delta_{k+j,k'+j'}$. $\hat{b}_k$ and $\hat{b}_j^\dagger$ fulfil the bosonic commutation relation $[\hat{b}_k, \hat{b}_j^\dagger] = \delta_{kj}$, while the atomic operators fulfil the commutation relation $[\hat{s}_{gf}^{(k)}, \hat{s}_{gf}^{(j)\dagger}] = -\frac{1}{N}\sum_{\mu=1}^{N} e^{\frac{2\pi i}{N}\mu(j-k)}\hat{\sigma}_z^{(\mu)}$ with $\hat{\sigma}_z^{(\mu)}$ as the Pauli z matrix for the atom in the μth cavity. Under the action of $\hat{H}$ on the states which form the two-excitation subspace, we get the relations

$$\hat{H}|kj\rangle_F = (\omega_k + \omega_j - 2a)|kj\rangle_F - G(|k\rangle_A|j\rangle_F + |j\rangle_A|k\rangle_F) \tag{4.3a}$$

$$\hat{H}|k\rangle_A|j\rangle_F = (\omega_j - a - b)|k\rangle_A|j\rangle_F - G(|kj\rangle_A + |kj\rangle_F) + \frac{a}{N}\sum_{(k',j')\in S_P}(|k'\rangle_A|j'\rangle_F + |j'\rangle_A|k'\rangle_F) + \frac{2G}{N}\sum_{(k',j')\in S_P}|k'j'\rangle_A \tag{4.3b}$$

$$\hat{H}|j\rangle_A|k\rangle_F = (\omega_k - a - b)|j\rangle_A|k\rangle_F - G(|kj\rangle_A + |kj\rangle_F) + \frac{a}{N}\sum_{(k',j')\in S_P}(|k'\rangle_A|j'\rangle_F + |j'\rangle_A|k'\rangle_F) + \frac{2G}{N}\sum_{(k',j')\in S_P}|k'j'\rangle_A \tag{4.3c}$$

$$\hat{H}|kj\rangle_A = -G(|k\rangle_A|j\rangle_F + |j\rangle_A|k\rangle_F) - 2b|kj\rangle_A, \tag{4.3d}$$

where $\omega_l = -2J\cos(\frac{2\pi l}{N})$ for $l \in \{k, j\}$, $a = g_m^2/\Delta$ and $b = \Omega^2/\Delta$. Within Eq. (4.3b) and Eq. (4.3c) we have a sum over the set $S_P = \{(k,j) \mid 0 \le k < j \le N-1,\ k+j \equiv P \,(\mathrm{mod}\, N)\}$ that is determined by the quasi-momentum P. From Eqs. (4.3a)-(4.3d) we can deduce that the quasi-momentum P is a conserved quantity and hence a good

quantum number. Apart from the quasi-momentum, the total number of excitations (dark polaritons) $\hat{N} = \sum_{\mu=1}^{N}(\hat{c}_\mu^\dagger \hat{c}_\mu + \hat{\sigma}_{ff}^{(\mu)})$ is a conserved quantity.

We can construct the complete set of eigenvectors by solving the eigenproblem within each of the subspaces $P = 0, 1, \ldots, N-1$. Following [32], we restrict the discussion to the case of even N and odd P. A general dark two-polariton eigenvector $|\Psi_P^{(D)}\rangle$ has the form

$$|\Psi_P^{(D)}\rangle = \sum_{(k,j)\in S_P} (\alpha_{kj}|kj\rangle_F + \beta_{kj}|k\rangle_A|j\rangle_F + \beta'_{kj}|j\rangle_A|k\rangle_F + \gamma_{kj}|kj\rangle_A). \tag{4.4}$$

$|\Psi_P^{(D)}\rangle$ satisfies the time-independent Schrödinger equation $\hat{H}|\Psi_P^{(D)}\rangle = \lambda|\Psi_P^{(D)}\rangle$ which yields within each of the subspaces $P = 1, 3, \ldots, N-1$ an eigenproblem that is given by the subsequent set of linear equations

$$\lambda\alpha_{kj} = (\omega_k + \omega_j - 2a)\alpha_{kj} - G(\beta_{kj} + \beta'_{kj}) \tag{4.5a}$$

$$\lambda\beta_{kj} = -G\alpha_{kj} + (\omega_j - a - b)\beta_{kj} - G\gamma_{kj} + \frac{a}{N}\sum_{(k',j')\in S_P}(\beta_{k'j'} + \beta'_{k'j'}) + \frac{2G}{N}\sum_{(k',j')\in S_P}\gamma_{k'j'} \tag{4.5b}$$

$$\lambda\beta'_{kj} = -G\alpha_{kj} + (\omega_k - a - b)\beta_{kj} - G\gamma_{kj} + \frac{a}{N}\sum_{(k',j')\in S_P}(\beta_{k'j'} + \beta'_{k'j'}) + \frac{2G}{N}\sum_{(k',j')\in S_P}\gamma_{k'j'} \tag{4.5c}$$

$$\lambda\gamma_{kj} = -G(\beta_{kj} + \beta'_{kj}) - 2b\gamma_{kj}, \tag{4.5d}$$

where λ is the corresponding eigenvalue. As it was demonstrated in [32], for various values of the quasi-momentum P the majority of eigenvalues are at most distributed among three bands. When all three bands are well resolved, it was shown that each of the two band gaps contains an eigenenergy of the single two-polariton bound state. For sufficiently large inter-cavity photon hopping strength J comparing to the strength of the atom-photon interaction, the bands start to overlap.

However, since we are not dealing with the standard JCH model, but rather with a modified one, we find some important differences and new features. Namely, as opposed to [32] there is only one mutually localized DPBP within one of the existing band gaps, while the other one joins the adjacent outer band. The other DPBP can reappear provided that the Stark shift of the control field is compensated. In both cases, when $\Delta < 0$, $g_m \gg \Omega$ and $g_m^2/|\Delta| \gtrsim 1.5\,J$, the ground state of the system is

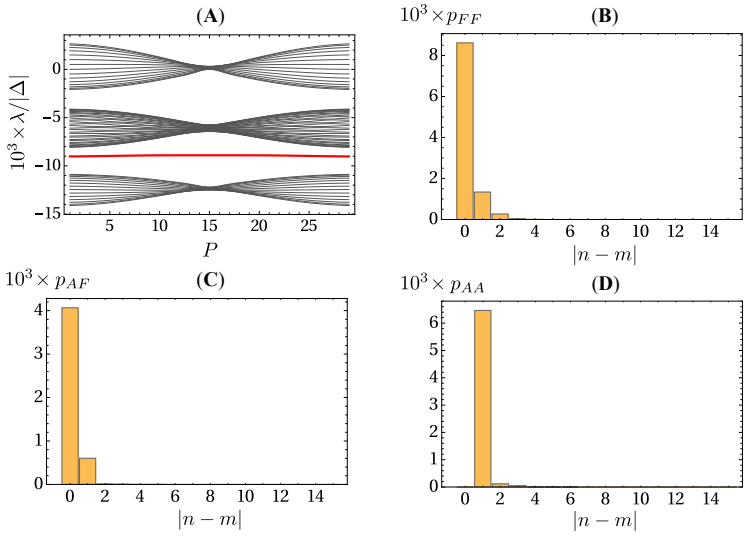

Figure 4.2: (**A**) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Dark-polariton bound pair state (red curve) appears in the low-energy band gap. The eigenvalues are joined by lines for ease of visualization. (**B**)-(**D**) Joint probabilities for different types of double excitations associated to DPBP state for $P = 1$. Used parameters: $\Delta > 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.06\,|\Delta|$ and $J = 0.001\,|\Delta|$.

DPBP of a different type than the aforementioned ones. In the sequel, we report on the state composition of the different DPBP types.

The Kerr-nonlinearity is a known force in light-atom interactions which depends on the atomic level structure as well as on the coupling strength of light-atom interactions. In our case, the strength of light-atom interaction is described by the effective coupling strength $G = g_m\Omega/\Delta$. Tuning g_m and/or Ω directly affects the Kerr-nonlinearity. Compared to [32], we can not only tune and control the Kerr-nonlinearity by the cavity mode coupling strength g_m, but also by the Rabi-frequency Ω. This force can be attractive or repulsive [26, 22, 93, 94, 95]. This force generates the bound state of two dark-polaritons in our case. A measure of the Kerr-nonlinearity is the on site repulsion $U(n)$ which is in general defined as

$$U(n) := (E_+ - E_-)(n+1) - (E_+ - E_-)(n) \tag{4.6}$$

with $E_\pm$ the eigenenergies of the considered eigenstates. In case of the standard Jaynes-Cummings model, the on site repulsion $U(n) = \chi(n+1) - \chi(n)$ is determined by the generalized Rabi-frequency $\chi(n)$ [96]. This will be different in our case as we will see in the following. In our DPBPs we have bound photons and bound atoms. In [97] they have experimentally shown bound states of atoms in coupled QED cavities, when atoms occupy the same site.

4.2.1 Dark-polariton bound pairs in the regime of non-compensated control field Stark shift

We focus on the single DPBP solution of Eqs. (4.5) which is given in red colour within Fig. 4.2(**A**) representing the energy eigenspectrum of the model Hamiltonian $\hat{H}$ in dependence of odd values of quasi-momentum P. Three energy bands are visible for the used parameter values. We define the gap between the two upper energy bands as the high-energy band gap and in accordance the gap between the two lower energy bands as the low-energy band gap. The dark-polaritons, which are involved in the formation of energy bands and the single DPBP in Fig. 4.2(**A**), are given in (2.97). This can be seen by solving Eqs. (4.5) for inter-cavity hopping $J = 0$. Note that the bands are a consequence of repulsively interacting dark-polaritons of different types with respect to the eigenenergies $E^{(m)}_{\pm,n}$. By different types here, we mean that the dark-polariton with eigenenergy $E^{(m)}_{+,n}$ interacts with the dark-polariton of eigenenergy $E^{(m)}_{-,n}$ in a repulsive way at the same site μ. This is a consequence of the on site repulsion $U(n)$. On different sites, dark-polaritons with eigenenergies $E^{(m)}_{+,n}$ and $E^{(m)}_{-,n}$ are non-interacting. Instead, the mentioned Kerr-nonlinearity, expressed through the on site repulsion $U(n) = \frac{g_m^2}{\Delta}$, enables the single DPBP state formation by the two dark-polaritons with eigenenergies $E^{(m)}_{-,n}$ which is placed at the same site μ in case of $\Delta > 0$. There is an additional DPBP, formed by the two dark-polaritons with eigenenergies $E^{(m)}_{+,n}$ in case of $\Delta > 0$, but is not visible in the spectrum as it is attached to the central band. On the contrary, formation of single DPBP interchanges for $\Delta < 0$. Our determined $U(n)$ from [96] is mainly affected by the cavity field coupling strength g_m. By increasing g_m we increase the on site repulsion $U(n)$ which directly enhances the interaction between the two dark-polaritons with eigenenergies $E^{(m)}_{-,n}$ at the same site μ with $\Delta > 0$. Thus, single DPBP is strengthened. Due to the interaction, the single DPBP lies inside the energy band gaps. Depending on the sign of the common single photon detuning Δ, DPBP lies either in the high or low energy band gap. In the case $\Delta > 0$, DPBP lies in the low-energy band gap, whereas in the opposite case it resides within the high-energy band gap. In order to get some information on the inherent state composition of the single DPBP, we calculate, in line with [32], the

joint probabilities

$$p_{FF} = |\langle \Psi_P^{(D)}| \frac{\hat{c}_n^\dagger \hat{c}_m^\dagger}{\sqrt{1+\delta_{nm}}} |\Phi_0\rangle|^2, \tag{4.7a}$$

$$p_{AF} = |\langle \Psi_P^{(D)}| \hat{c}_n^\dagger \hat{\sigma}_{gf}^{(m)\dagger} |\Phi_0\rangle|^2, \tag{4.7b}$$

$$p_{AA} = |\langle \Psi_P^{(D)}| \hat{\sigma}_{gf}^{(n)\dagger} \hat{\sigma}_{gf}^{(m)\dagger} |\Phi_0\rangle|^2, \tag{4.7c}$$

of finding pure photonic, photon-atom and pure atomic excitations, respectively, in cavities at positions n and m. These excitations (pure photonic, pure atomic and photon-atom) reflect the unique property of dark-polaritons in which the superposition of photonic and collective atomic excitations can be tuned by changing Ω in first place. In our case, we can not only change Ω, but also g_m as we use tunable cavities [90, 91]. For a given value of quasi-momentum P, all three joint probabilities only depend on the relative distance $|n-m|$ within the cavities.

In Fig. 4.2(**B**)-4.2(**D**) we present the joint probabilities for the single DPBP state of Fig. 4.2(**A**). We have chosen the number of coupled QED cavities to be $N = 30$, single photon detuning $\Delta > 0$, cavity-mode coupling strength $g_m = 0.05\,|\Delta|$, the control field Rabi-frequency $\Omega = 0.06\,|\Delta|$, inter-cavity photon hopping strength $J = 0.001\,|\Delta|$, and subspace $P = 1$. One can see that the DPBP excitations are well confined together, and all three possible excitation types coexist with roughly equal contributions. The state composition gradually changes by decreasing the contribution of double atomic excitations when P approaches the mid-range values. This regime is roughly characterized by $g_m \approx \Omega$ and $(g_m^2 + \Omega^2)/|\Delta| > 5J$. The energy band gaps close when decreasing the ratio of $(g_m^2 + \Omega^2)/|\Delta|$ and J. At the same time DPBP becomes relatively delocalized, similarly as in [32].

4.2.2 Dark-polariton bound pairs in the regime of compensated control field Stark shift

The tunability of our model enables not only the control of the shape of the energy bands, but also the emergence of an additional DPBP state. Namely, if the control field Stark shift is compensated by using an additional field, which couples the ground state $|f\rangle$ with some far off resonant excited state [67], another DPBP state appears in the formerly empty energy band gap. Such add reflects in the removal of the parameter b from the Eqs. (4.5). The energy bands in Fig. 4.3(**A**), shown for discrete and distinct quasi-momenta P, are formed by the dark-polaritons in (2.101).

This can be seen by solving Eqs. (4.5) for the inter-cavity hopping strength $J = 0$ and set the parameter b equal to zero. The on site repulsion $U(n)$, which assures the formation of the two DPBPs, is given as $U(n) = \frac{g_m\sqrt{n+1}\sqrt{g_m^2(n+1)+4\Omega^2}-g_m\sqrt{n}\sqrt{g_m^2 n+4\Omega^2}}{\Delta}$ for positive and negative single common photon detuning Δ. Thus, the on site repulsion $U(n)$ is invariant under the sign change of Δ. Distinctly to the DPBP formation under non-compensated control field Stark shift, the on site repulsion $U(n)$ apart from the cavity field coupling strength g_m, directly depends on the Rabi-frequency Ω. This gives the opportunity to effectively control and enhance the interaction through g_m and Ω. Further, In Fig. 4.3(**A**) one can observe that each of the two energy band gaps now contain a single DPBP state (blue and red curves). We used the same parameter values as in Fig. (4.2), but with compensated control field Stark shift. In Figs. 4.3(**B**)-4.3(**D**) and Figs. 4.3(**E**)-4.3(**G**) we characterize the state composition of lower and higher energy DPBP state, respectively, by considering the joint probabilities as in the previous subsection. The DPBP in the lower energy band gap is dominantly composed of two-photon excitation, while in the other DPBP state atom-photon excitation prevails. Moreover, higher energy DPBP state is further apart from the outer energy band and it is relatively more localized than the lower energy DPBP state. We checked that the same behaviour persists for other values of quasi-momentum P. Note that the described situation is for $\Delta > 0$, while it interchanges for $\Delta < 0$.

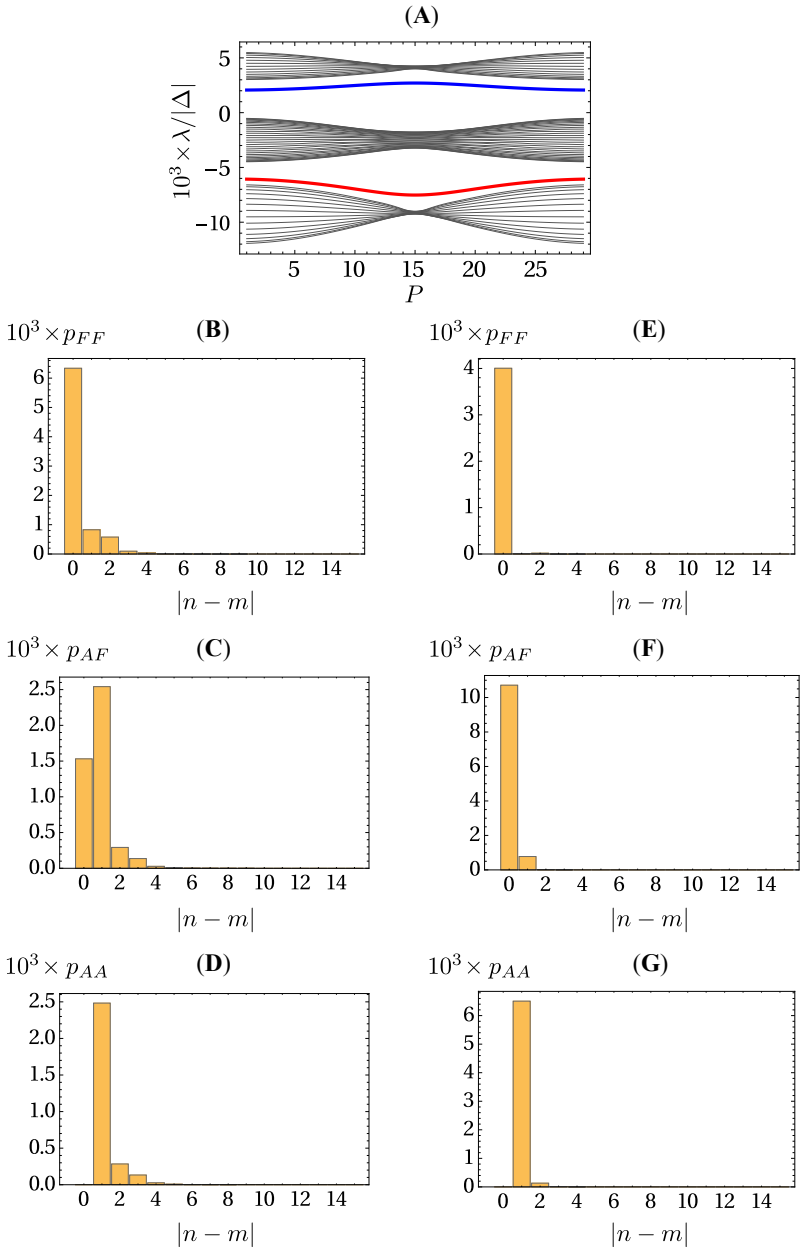

Figure 4.3: (**A**) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Two dark-polariton bound pair states (blue and red curves) appear in both energy band gaps. The eigenvalues are joined by lines for ease of visualization. (**B**)-(**D**) Joint probabilities for different types of double excitations associated to lower energy DPBP state. (**E**)-(**G**) Joint probabilities for different types of double excitations associated to higher energy DPBP state for $P = 1$. Used parameters: $\Delta > 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.06\,|\Delta|$ and $J = 0.001\,|\Delta|$.

4.3 Quantum memory of light in a dark-polariton bound pair

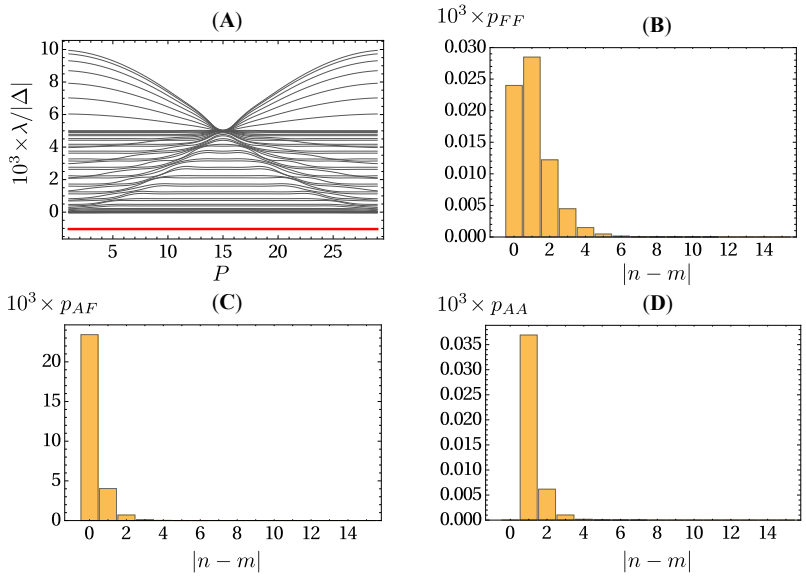

Figure 4.4: (**A**) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Dark-polariton bound pair state (red curve) appears as the ground state. The eigenvalues are joined by lines for ease of visualization. (**B**)-(**D**) Joint probabilities for different types of double excitations associated to DPBP state for $P = 1$. Used parameters: $\Delta < 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.001\,|\Delta|$ and $J = 0.00125\,|\Delta|$.

In the parameter regime where the common single photon detuning Δ is negative and the cavity-atom coupling strength g_m is significantly larger than the control field Rabi frequency Ω, we have a single DPBP state which is the ground state of the system. It is well separated from the rest of the energy spectrum when $g_m^2/|\Delta| \gtrsim 1.5\,J$. This is presented in Fig. 4.4(**A**). DPBP state composition, given in Figs. 4.4(**B**)-4.4(**D**) by the corresponding joint probabilities, reveals that the state is dominantly composed of combined atomic and photonic excitations which are localized in their relative spatial coordinates. Note that this DPBP state is of a completely different type than the ones found in the previous section.

It is important that this state also enables the storage of a single photon in the form of a collective atomic spin coherence excitation to which the other photon is closely bound. Namely, when $\Omega \to 0$ adiabatically, a DPBP becomes a pure combination of an atomic and photonic excitation. From this we can deduce that one photon remains attached to the atomic spin coherence wave. This is reminiscent of the atom-photon molecule.

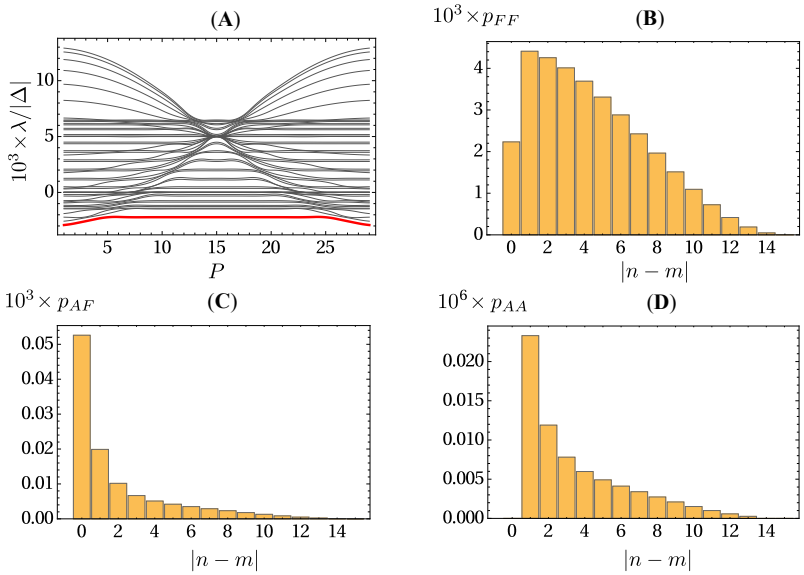

Figure 4.5: (**A**) Normalized eigenvalues dependence on the quasi-momentum P for $N = 30$ cavities. Dark-polariton bound pair state (red curve) appears as the ground state. The eigenvalues are joined by lines for ease of visualization. (**B**)-(**D**) Joint probabilities for different types of double excitations associated to DPBP state for $P = 1$. Used parameters: $\Delta < 0$, $g_m = 0.05\,|\Delta|$, $\Omega = 0.001\,|\Delta|$ and $J = 0.002\,|\Delta|$.

The state composition can be tuned by increasing the relative importance of the inter-cavity photon hopping, e.g., by increasing $|\Delta|$. This is achieved gradually for distinct values of quasi-momentum, starting from the values $P = 1, N-1$ and proceeding towards the mid-range values of P. Figure 4.5(**A**) shows the energy spectrum in such a case. For $P \in \{1, 3, N-3, N-1\}$ the DPBP state is predominantly composed of two-photon excitations which become delocalized in their relative spatial positions, as can be seen in Figs. 4.5(**B**)-4.5(**D**). The reason for such behaviour can be traced back to the emergence of the avoided crossings of the ground state and the first excited state near the edges of the quasi-momentum zone. The crossings shift towards the P-zone centre as the influence of the photon hopping is being increased. For the quasi-momentum values between the crossings, the DPBP state remains dominantly of the atom-photon type. In the case when the control field strength adiabatically reduces to zero, the DPBP state becomes of a pure two-photon type. Therefore, this corresponds to the retrieval procedure of the previously stored photon excitation.

4.4 Experimental realization

Our model system is a large, one-dimensional mJCH chain of N coupled QED cavities. In order to realize it, we need a structure, in which large arrays of coupled QED cavities can be realized. Promising candidates are photonic band gap cavities [44, 45]. It is manageable to produce and position them with high precision and in large numbers. A tempting alternative are photonic crystals as they offer the possibility of fabricating large arrays of QED cavities in one- or two dimensional lattices as well as networks [98, 99, 100]. A third possibility would be the use of toroidal micro-QED cavities that are coupled via tapered optical fibres [101]. Single atoms, embedded in each QED cavity are three-level atoms where the excited level is far detuned by the common single photon detuning with respect to the two coupling fields. In real experiments Cs and ultra cold ^{87}Rb atoms have shown to be very suitable [97, 51, 102]. For Cs in a toroidal micro-QED cavity it has be shown that g_m in the strong coupling regime reaches the value of $\sim 50MHz$ [51]. This fits pretty well with our theoretically chosen value for the formation of individual DPBP inside the energy band gaps, but also for the ground DPBP at $\Delta < 0$ with its potential use as a quantum memory for a single photon.

5

Dark-polariton bound pairs in disordered coupled QED cavity arrays

We investigate a one-dimensional modified Jaynes-Cummings-Hubbard (mJCH) chain of N identical QED cavities with staggered nearest neighbour photon tunnelling and periodic boundary conditions. Each cavity contains an embedded three-level atom which is coupled to a cavity mode and an external classical control field. Through the staggered nearest neighbour photon tunnelling, two sublattices b and c are defined. Moreover, the staggered configuration of nearest neighbour photon tunnelling represents one way of realizing induced disorder which substantially differs from so called random disorder within condensed matter physics, e.g intrinsic imperfections, distortions and defects inside or of the crystal lattice itself. Further, we choose the common large detuning, Rabi frequency and cavity field coupling strength to be equal in and between the two sublattices b and c. In the case of two-excitations and common large detuning of two Raman-resonant fields, we show the emergence of bilocalization within the sublattices and their coupling for the parameter regime $g_m \gg \Omega$. For weak inter-cavity photon hopping strength J_2, bound photons characterize the bilocalization in the individual sublattices, where the sublattice coupling instead is governed by bound spin coherences of the dark-polariton bound pair (DPBP). The appearance of bilocalization does not depend on the entire (not)-compensation of the classical control field Stark shift.

5.1 Model system and effective model Hamiltonian

5.1.1 Model system

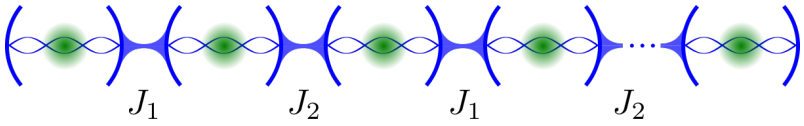

Figure 5.1: One-dimensional array of N coupled QED cavities in staggered configuration with respect to the inter-cavity photon hopping strengths J_1 and J_2 under periodic boundary conditions. Green transparent sphere in each cavity represents a three-level atom with two ground levels $|g\rangle$ and $|f\rangle$, and an excited level $|e\rangle$. The atomic configuration is given in Figure (2.5) of the theory chapter 2.

The system under consideration consists of a one-dimensional array of N coupled QED cavities in a staggered configuration with respect to two inter-cavity photon hopping strengths J_1 and J_2 see Figure (5.1). We assume periodic boundary conditions, i.e., the cavity labelled by $n = N + 1$ corresponds to the cavity $n = 1$. The staggered hopping define two sublattices which we denote by b and c. Each cavity embeds a three-level atom with two ground levels $|g\rangle$ and $|f\rangle$, and an excited level $|e\rangle$. The level energies are ω_g, ω_f and ω_e, respectively. In reality, the levels can be either fine or hyperfine levels of alkali-metal atoms. Their D1 or D2 line transitions are nowadays easily accessible via available lasers and optical modes of QED cavities. One mode of a tunable cavity [90, 91] of frequency ω_m, couples the transition $|g\rangle \rightarrow |e\rangle$ in the sublattice b (c) with the strength g_m^b (g_m^c), while the classical control field of frequency ω_0 couples the transition $|f\rangle \rightarrow |e\rangle$ with Rabi-frequency Ω^b (Ω^c). The fields are detuned from the respective transitions by common single photon detuning Δ^b (Δ^c). This configuration is known to feature vacuum induced transparency, as first experimentally demonstrated by the group of Vuletić [92]. The values of coupling strengths are typically in MHz range for alkali-metal atoms strongly coupled with QED cavities, and for moderate laser powers.

5.1.2 Effective model Hamiltonian

As we consider a one-dimensional, staggered chain of N identical coupled QED cavities, the derived modified Jaynes-Cummings model for a single QED cavity in [103] is valid for all QED cavities in the one-dimensional chain, but has to be

properly adapted to the staggered hopping configuration. Therefore, our effective model Hamiltonian ($\hbar = 1$) has the form

$$\hat{H} = \hat{H}^{(mJC)} + \hat{H}_{hop}, \tag{5.1a}$$

$$\hat{H}^{(mJC)} = \hat{H}_b^{(mJC)} + \hat{H}_c^{(mJC)}, \tag{5.1b}$$

$$\hat{H}_{hop} = \hat{H}_{hop}^{(1)} + \hat{H}_{hop}^{(2)}, \tag{5.1c}$$

where the terms corresponding to modified Jaynes-Cummings model on the two sublattices are

$$\hat{H}_b^{(mJC)} = \hat{H}_S^b + \hat{H}_{int}^b, \tag{5.2a}$$

$$\hat{H}_S^b = -\sum_{\mu=1}^{N} \left(\frac{(g_m^b)^2}{\Delta^b} \hat{b}_\mu^\dagger \hat{b}_\mu \hat{\sigma}_{gg}^{(\mu)} + \frac{(\Omega^b)^2}{\Delta^b} \hat{\sigma}_{ff}^{(\mu)} \right), \tag{5.2b}$$

$$\hat{H}_{int}^b = -G^b \sum_{\mu=1}^{N} (\hat{b}_\mu^\dagger \hat{\sigma}_{gf}^{(\mu)} + \hat{b}_\mu \hat{\sigma}_{fg}^{(\mu)}), \tag{5.2c}$$

and

$$\hat{H}_c^{(mJC)} = \hat{H}_S^c + \hat{H}_{int}^c, \tag{5.3a}$$

$$\hat{H}_S^c = -\sum_{\mu=1}^{N} \left(\frac{(g_m^c)^2}{\Delta^c} \hat{c}_\mu^\dagger \hat{c}_\mu \hat{\tau}_{gg}^{(\mu)} + \frac{(\Omega^c)^2}{\Delta^c} \hat{\tau}_{ff}^{(\mu)} \right), \tag{5.3b}$$

$$\hat{H}_{int}^c = -G^c \sum_{\mu=1}^{N} (\hat{c}_\mu^\dagger \hat{\tau}_{gf}^{(\mu)} + \hat{c}_\mu \hat{\tau}_{fg}^{(\mu)}). \tag{5.3c}$$

Staggered photon hopping between adjacent cavities of the two sublattices, based on evanescent field coupling, with J_1 and J_2 as the inter-cavity photon hopping strengths, is given by

$$\hat{H}_{hop}^{(1)} = -J_1 \sum_{\mu=1}^{N} (\hat{b}_\mu^\dagger \hat{c}_\mu + \hat{c}_\mu^\dagger \hat{b}_\mu), \tag{5.4a}$$

$$\hat{H}_{hop}^{(2)} = -J_2 \sum_{\mu=1}^{N} (\hat{b}_\mu^\dagger \hat{c}_{\mu-1} + \hat{c}_{\mu-1}^\dagger \hat{b}_\mu). \tag{5.4b}$$

$\hat{b}_\mu^\dagger$ ($\hat{b}_\mu$) and $\hat{c}_\mu^\dagger$ ($\hat{c}_\mu$) are the photonic creation (annihilation) operators, $\hat{\sigma}_{\alpha\beta}^{(\mu)} = |\alpha\rangle_\mu\langle\beta|$ and $\hat{\tau}_{\alpha\beta}^{(\mu)} = |\alpha\rangle_\mu\langle\beta|$ ($\alpha, \beta \in \{g, f\}$) are the atomic operators in the sublattices b and c, respectively, for the site number μ. The term $\hat{H}_S^b$ ($\hat{H}_S^c$) incorporates the

influence of Stark shifts of the excited levels of the atoms in the sublattice b (c). $\hat{H}^b_{int}$ ($\hat{H}^c_{int}$) represents the cavity-atom interaction within the sublattice b (c), where $G^b = g^b_m\Omega^b/\Delta^b$ ($G^c = g^c_m\Omega^c/\Delta^c$) is the effective atom-photon coupling constant which is set to be real. Hamiltonians $\hat{H}^{b(c)}_S$ and $\hat{H}^{b(c)}_{int}$ constitute the modified Jaynes-Cummings Hamiltonian in the sublattice $b(c)$. As will be shown in the sequel, the Stark shifts have profound influence on the state composition of the ground state DPBP in the sublattices itself and in the linkage of the two sublattices. Similar effective Hamiltonian has been previously used to describe a network of uniform fibre coupled cavities, embedded with three-level atoms [67]. However, while that scheme requires the compensation of the level Stark shifts, here we utilize the individual level Stark shifts to achieve tunability. Our effective model Hamiltonian (5.1) supports the formation of dark-polariton bound pairs, but now in a staggered configuration. As we concentrate our discussion on the ground state DPBP, which was found in [103], we will see how its state composition and localization property change under various staggered configurations.

5.2 Formation of dark-polariton bound pairs

In the following, we discuss the formation of dark-polariton bound pairs in our system. In order to exploit the invariance of the system under cyclic permutations of the sites, we introduce the following operators via discrete Fourier transforms for each sublattice b and c

$$\tilde{b}_k = \frac{1}{\sqrt{N}}\sum_{\mu=1}^{N} e^{-\frac{2\pi i}{N}\mu k}\,\hat{b}_\mu, \tag{5.5a}$$

$$\tilde{c}_k = \frac{1}{\sqrt{N}}\sum_{\mu=1}^{N} e^{-\frac{2\pi i}{N}\mu k}\,\hat{c}_\mu, \tag{5.5b}$$

$$\tilde{\sigma}^{(k)}_{gf} = \frac{1}{\sqrt{N}}\sum_{\mu=1}^{N} e^{-\frac{2\pi i}{N}\mu k}\,\hat{\sigma}^{(\mu)}_{gf}, \tag{5.5c}$$

$$\tilde{\tau}^{(k)}_{gf} = \frac{1}{\sqrt{N}}\sum_{\mu=1}^{N} e^{-\frac{2\pi i}{N}\mu k}\,\hat{\tau}^{(\mu)}_{gf}, \tag{5.5d}$$

where $k = 0, 1, \ldots, N$–1 is related to the (discrete) quasi-momentum of the excitation and is clearly independent of the individual sublattices. Similarly to [32, 103], we work in an extended two-excitation subspace that is spanned by the states $|kj\rangle^{(b)}_F \equiv$

$\tilde{b}_k^\dagger\tilde{b}_j^\dagger|\Phi_0\rangle$, $|k\rangle_F^{(b)}|j\rangle_A^{(b)} \equiv \tilde{b}_k^\dagger\tilde{\sigma}_{gf}^{(j)\dagger}|\Phi_0\rangle$, $|kj\rangle_A^{(b)} \equiv \tilde{\sigma}_{gf}^{(k)\dagger}\tilde{\sigma}_{gf}^{(j)\dagger}|\Phi_0\rangle$, $|kj\rangle_F^{(c)} \equiv \tilde{c}_k^\dagger\tilde{c}_j^\dagger|\Phi_0\rangle$, $|k\rangle_F^{(c)}|j\rangle_A^{(c)}$ $\equiv \tilde{c}_k^\dagger\tilde{\tau}_{gf}^{(j)\dagger}|\Phi_0\rangle$, $|kj\rangle_A^{(c)} \equiv \tilde{\tau}_{gf}^{(k)\dagger}\tilde{\tau}_{gf}^{(j)\dagger}|\Phi_0\rangle$, $|k\rangle_F^{(b)}|j\rangle_F^{(c)} \equiv \tilde{b}_k^\dagger\tilde{c}_j^\dagger|\Phi_0\rangle$, $|k\rangle_F^{(c)}|j\rangle_A^{(b)} \equiv \tilde{c}_k^\dagger\tilde{\sigma}_{gf}^{(j)\dagger}|\Phi_0\rangle$, $|k\rangle_A^{(c)}|j\rangle_A^{(b)} \equiv \tilde{\tau}_{gf}^{(k)\dagger}\tilde{\sigma}_{gf}^{(j)\dagger}|\Phi_0\rangle$, $|k\rangle_F^{(b)}|j\rangle_A^{(c)} \equiv \tilde{b}_k^\dagger\tilde{\tau}_{gf}^{(j)\dagger}|\Phi_0\rangle$ and $|k\rangle_A^{(b)}|j\rangle_A^{(c)} \equiv \tilde{\sigma}_{gf}^{(k)\dagger}\tilde{\tau}_{gf}^{(j)\dagger}|\Phi_0\rangle$. The subscripts F and A stand for the photonic and atomic excitations respectively, whereas b and c stand for the corresponding sublattices. The state $|\Phi_0\rangle = |\Phi_0\rangle^{(b)}|\Phi_0\rangle^{(c)} = \otimes_{\mu=1}^N|g\rangle_\mu^{(b)}|g\rangle_\mu^{(c)}|0\rangle_\mu$ is the ground state of the system, where $|0\rangle_\mu$ denotes the vacuum state of the cavity number μ. We note that the excitations (polaritons) are in our case dark in a sense that they do not have the contribution of the excited levels $|e\rangle$ and are not subjected to spontaneous emission. The atomic excitations $|kj\rangle_A^{(b),(c)}$ are in general not orthogonal to each other because of ${}_A^{(b),(c)}\langle k'j'|kj\rangle_A^{(b),(c)} = \delta_{k,k'}\delta_{j,j'} + \delta_{k,j'}\delta_{j,k'} - \frac{2}{N}\delta_{k+j,k'+j'}$. $\tilde{b}_k,\hat{b}_j^\dagger$ and $\tilde{c}_k,\tilde{c}_j^\dagger$ fulfil the bosonic commutation relation $[\tilde{b}_k,\tilde{b}_j^\dagger] = [\tilde{c}_k,\tilde{c}_j^\dagger]\delta_{kj}$, while the atomic operators fulfil the commutation relation $[\tilde{\sigma}_{gf}^{(k)},\tilde{\sigma}_{gf}^{(j)\dagger}] = -\frac{1}{N}\sum_{\mu=1}^N e^{\frac{2\pi i}{N}\mu(j-k)}\hat{\sigma}_z^{(\mu)}$ and $[\tilde{\tau}_{gf}^{(k)},\tau_{gf}^{(j)\dagger}] = -\frac{1}{N}\sum_{\mu=1}^N e^{\frac{2\pi i}{N}\mu(j-k)}\hat{\tau}_z^{(\mu)}$ with $\hat{\sigma}_z^{(\mu)}$ and $\hat{\tau}_z^{(\mu)}$ as the Pauli z matrices for the atom in the μth cavity of the two sublattices b and c. Under the action of $\hat{H}$ on the states which form the two-excitation subspace, fort the sublattice b we get the relations

$$\hat{H}|kj\rangle_F^{(b)} = -2a^b|kj\rangle_F^{(b)} - G^b(|k\rangle_A^{(b)}|j\rangle_F^{(b)} + |j\rangle_A^{(b)}|k\rangle_F^{(b)}) + \omega_k|j\rangle_F^{(b)}|k\rangle_F^{(c)} + \omega_j|k\rangle_F^{(b)}|j\rangle_F^{(c)} \tag{5.6a}$$

$$\hat{H}|k\rangle_A^{(b)}|j\rangle_F^{(b)} = -G^b|kj\rangle_F^{(b)} - (a^b+b^b)|k\rangle_A^{(b)}|j\rangle_F^{(b)} - G^b|kj\rangle_A^{(b)} + \frac{a^b}{N}\sum_{(k',j')\in S_P}(|k'\rangle_A^{(b)}|j'\rangle_F^{(b)} + |j'\rangle_A^{(b)}|k'\rangle_F^{(b)}) + \frac{2G^b}{N}\sum_{(k',j')\in S_P}|k'j'\rangle_A^{(b)} \tag{5.6b}$$

$$\hat{H}|j\rangle_A^{(b)}|k\rangle_F^{(b)} = -G^b|kj\rangle_F^{(b)} - (a^b+b^b)|j\rangle_A^{(b)}|k\rangle_F^{(b)} - G^b|kj\rangle_A^{(b)} + \frac{a^b}{N}\sum_{(k',j')\in S_P}(|k'\rangle_A^{(b)}|j'\rangle_F^{(b)} + |j'\rangle_A^{(b)}|k'\rangle_F^{(b)}) + \frac{2G^b}{N}\sum_{(k',j')\in S_P}|k'j'\rangle_A^{(b)} \tag{5.6c}$$

$$\hat{H}|kj\rangle_A^{(b)} = -G^b(|k\rangle_A^{(b)}|j\rangle_F^{(b)} + |j\rangle_A^{(b)}|k\rangle_F^{(b)}) - 2b^b|kj\rangle_A^{(b)}, \tag{5.6d}$$

where $a^b = -\frac{|g_m^b|^2}{\Delta^b}$ and $b^b = -\frac{|\Omega^b|^2}{\Delta^b}$ are the individual Stark-shifts of the cavity and control field, $\omega_l = -J_1 - J_2e^{\frac{i2\pi l}{N}}$ with $l \in \{k,j\}$ and $G^b = \frac{g_m^b\Omega^b}{\Delta^b}$ the effective atom-photon coupling constant within the sublattice b. Moreover, for the sublattice c we

get the relations

$$\hat{H}|kj\rangle_F^{(c)} = -2a^c|kj\rangle_F^{(c)} - G^c(|k\rangle_A^{(c)}|j\rangle_F^{(c)} + |j\rangle_A^{(c)}|k\rangle_F^{(c)}) + \omega_k^*|j\rangle_F^{(b)}|k\rangle_F^{(c)} + \omega_j^*|k\rangle_F^{(b)}|j\rangle_F^{(c)}, \tag{5.7a}$$

$$\hat{H}|k\rangle_A^{(c)}|j\rangle_F^{(c)} = -G^c|kj\rangle_F^{(b)} - (a^c + b^c)|k\rangle_A^{(c)}|j\rangle_F^{(c)} - G^c|kj\rangle_A^{(c)} + \frac{a^c}{N}\sum_{(k',j')\in S_P}(|k'\rangle_A^{(b)}|j'\rangle_F^{(b)} + |j'\rangle_A^{(b)}|k'\rangle_F^{(b)}) + \frac{2G^c}{N}\sum_{(k',j')\in S_P}|k'j'\rangle_A^{(c)}, \tag{5.7b}$$

$$\hat{H}|j\rangle_A^{(c)}|k\rangle_F^{(c)} = -G^c|kj\rangle_F^{(c)} - (a^c + b^c)|j\rangle_A^{(c)}|k\rangle_F^{(c)} - G^c|kj\rangle_A^{(c)} + \frac{a^c}{N}\sum_{(k',j')\in S_P}(|k'\rangle_A^{(c)}|j'\rangle_F^{(c)} + |j'\rangle_A^{(c)}|k'\rangle_F^{(c)}) + \frac{2G^c}{N}\sum_{(k',j')\in S_P}|k'j'\rangle_A^{(c)}, \tag{5.7c}$$

$$\hat{H}|kj\rangle_A^{(c)} = -G^c(|k\rangle_A^{(c)}|j\rangle_F^{(c)} + |j\rangle_A^{(c)}|k\rangle_F^{(c)}) - 2b^c|kj\rangle_A^{(c)}, \tag{5.7d}$$

where $a^c = -\frac{|g_m^c|^2}{\Delta^c}$ and $b^c = -\frac{|\Omega^c|^2}{\Delta^c}$ are the individual Stark-shifts of the cavity and control field, $\omega_l^* = -J_1 - J_2 e^{\frac{-i2\pi l}{N}}$ with $l \in \{k, j\}$ and $G^c = \frac{g_m^c \Omega^c}{\Delta^c}$ the effective atom-photon coupling constant within the sublattice c. Within Eqs. (5.6) and (5.7), which refer to the sublattices b and c, we have a sum over the set $S_P = \{(k, j) \mid 0 \leq k < j \leq N-1,\, k + j \equiv P \,(\mathrm{mod}\, N)\}$ that is determined by the quasi-momentum P. So far, we have described the action of the model Hamiltonian (4.1) within the individual sublattices b and c without considering the coupling of nearest neighbour QED cavities of the sublattices. The coupling between nearest neighbour QED cavities is governed by the inter-cavity hopping of photons which characterizes the appearance of the kets $|k\rangle_F^{(b)}|j\rangle_F^{(c)}$, $|k\rangle_A^{(b)}|j\rangle_F^{(c)}$, $|k\rangle_A^{(c)}|j\rangle_F^{(b)}$ and $|k\rangle_A^{(b)}|j\rangle_A^{(c)}$. As the model

Hamiltonian (5.1) acts on these kets, we end up with the relations

$$\hat{H}|k\rangle_F^{(b)}|j\rangle_F^{(c)} = -(a^b + a^c)|k\rangle_F^{(b)}|j\rangle_F^{(c)} - G^b|k\rangle_A^{(b)}|j\rangle_F^{(c)} - G^c|j\rangle_A^{(c)}|k\rangle_F^{(b)} - \omega_j^*|kj\rangle_F^{(b)} - \omega_k|kj\rangle_F^{(c)}, \tag{5.8a}$$

$$\hat{H}|j\rangle_F^{(b)}|k\rangle_F^{(c)} = -(a^b + a^c)|j\rangle_F^{(b)}|k\rangle_F^{(c)} - G^b|j\rangle_A^{(b)}|k\rangle_F^{(c)} - G^c|k\rangle_A^{(c)}|j\rangle_F^{(b)} - \omega_k^*|kj\rangle_F^{(b)} - \omega_j|kj\rangle_F^{(c)}, \tag{5.8b}$$

$$\hat{H}|k\rangle_A^{(b)}|j\rangle_F^{(c)} = -(a^c + b^b)|k\rangle_A^{(b)}|j\rangle_F^{(c)} - G^b(|k\rangle_F^{(b)}|j\rangle_F^{(c)} + |kj\rangle_F^{(c)}) - G^c|k\rangle_A^{(b)}|j\rangle_A^{(c)} - \omega_j^*|k\rangle_A^{(b)}|j\rangle_F^{(b)}, \tag{5.8c}$$

$$\hat{H}|j\rangle_A^{(b)}|k\rangle_F^{(c)} = -(a^c + b^b)|j\rangle_A^{(b)}|k\rangle_F^{(c)} - G^b(|j\rangle_F^{(b)}|k\rangle_F^{(c)} + |kj\rangle_F^{(c)}) - G^c|j\rangle_A^{(b)}|k\rangle_A^{(c)} - \omega_k^*|j\rangle_A^{(b)}|k\rangle_F^{(b)}, \tag{5.8d}$$

$$\hat{H}|k\rangle_A^{(c)}|j\rangle_F^{(b)} = -(a^b + b^c)|k\rangle_A^{(c)}|j\rangle_F^{(b)} - G^b|k\rangle_A^{(c)}|j\rangle_A^{(b)} - G^c|k\rangle_F^{(c)}|j\rangle_F^{(b)} - \omega_j|k\rangle_A^{(c)}|j\rangle_F^{(c)}, \tag{5.8e}$$

$$\hat{H}|j\rangle_A^{(c)}|k\rangle_F^{(b)} = -(a^b + b^c)|j\rangle_A^{(c)}|k\rangle_F^{(b)} - G^b|j\rangle_A^{(c)}|k\rangle_A^{(b)} - G^c|j\rangle_F^{(c)}|k\rangle_F^{(b)} - \omega_k|j\rangle_A^{(c)}|k\rangle_F^{(c)}, \tag{5.8f}$$

$$\hat{H}|k\rangle_A^{(b)}|j\rangle_A^{(c)} = -(b^b + b^c)|k\rangle_A^{(b)}|j\rangle_A^{(c)} - G^b|j\rangle_A^{(c)}|k\rangle_F^{(b)} - G^c|k\rangle_A^{(b)}|j\rangle_F^{(c)}, \tag{5.8g}$$

$$\hat{H}|j\rangle_A^{(b)}|k\rangle_A^{(c)} = -(b^b + b^c)|j\rangle_A^{(b)}|k\rangle_A^{(c)} - G^b|k\rangle_A^{(c)}|j\rangle_F^{(b)} - G^c|j\rangle_A^{(b)}|k\rangle_F^{(c)}. \tag{5.8h}$$

In case of Eqs. (5.8a)-(5.8h) no sum over the set $S_P = \{(k,j) \mid 0 \leq k < j \leq N-1,\, k+j \equiv P \,(\mathrm{mod}\, N)\}$ appears. This is clearly a consequence of introducing a staggered configuration of the inter-cavity photon hopping J_1 and J_2. Further, from Eqs. (5.6),(5.7) and (5.8) we can deduce that the quasi-momentum P is a conserved quantity and hence a good quantum number. Apart from the quasi-momentum, the total number of excitations (dark polaritons) in each sublattice $\hat{N}^b = \sum_{\mu=1}^N (\hat{b}_\mu^\dagger \hat{b}_\mu + \hat{\sigma}_{ff}^{(\mu)})$ and $\hat{N}^c = \sum_{\mu=1}^N (\hat{c}_\mu^\dagger \hat{c}_\mu + \hat{\tau}_{ff}^{(\mu)})$ is a conserved quantity. From this, it fallows that the total number of excitations (dark polaritons) of the complete system $\hat{N} = \hat{N}^b + \hat{N}^c$ is a conserved quantity as well.

We can construct the complete set of eigenvectors by solving the eigenproblem within each of the subspaces $P = 0, 1, \ldots, N-1$. Following [32, 103], we restrict the discussion to the case of even N and odd P. A general dark two-polariton eigenvector $|\Psi_P^{(D)}\rangle$ in a staggered configuration of coupled QED cavities has the

form

$$|\Psi_P^{(D)}\rangle = \sum_{(k,j)\in S_P} (f_{kj}^{(b)} + f_{kj}^{(c)} + f_{kj}^{(BC)} + f_{kj}^{(CB)}), \tag{5.9a}$$

$$f_{kj}^{(b)} = \alpha_{kj}^{(b)}|kj\rangle_F^{(b)} + \beta_{kj}^{(b)}|k\rangle_A^{(b)}|j\rangle_F^{(b)} + \beta_{kj}^{'(B)}|j\rangle_A^{(b)}|k\rangle_F^{(b)} + \gamma_{kj}^{(b)}|kj\rangle_A^{(b)}, \tag{5.9b}$$

$$f_{kj}^{(c)} = \alpha_{kj}^{(c)}|kj\rangle_F^{(c)} + \beta_{kj}^{(c)}|k\rangle_A^{(c)}|j\rangle_F^{(c)} + \beta_{kj}^{'(C)}|j\rangle_A^{(c)}|k\rangle_F^{(c)} + \gamma_{kj}^{(c)}|kj\rangle_A^{(c)}, \tag{5.9c}$$

$$f_{kj}^{(BC)} = \alpha_{kj}^{(BC)}|k\rangle_F^{(b)}|j\rangle_F^{(c)} + \alpha_{kj}^{'(BC)}|j\rangle_F^{(b)}|k\rangle_F^{(c)} + \beta_{kj}^{(BC)}|k\rangle_A^{(b)}|j\rangle_F^{(c)} + \beta_{kj}^{'(BC)}|j\rangle_A^{(b)}|k\rangle_F^{(c)} + \gamma_{kj}^{(BC)}|k\rangle_A^{(b)}|j\rangle_A^{(c)} + \gamma_{kj}^{'(BC)}|j\rangle_A^{(b)}|k\rangle_A^{(c)}, \tag{5.9d}$$

$$f_{kj}^{(CB)} = \beta_{kj}^{(CB)}|k\rangle_A^{(c)}|j\rangle_F^{(b)} + \beta_{kj}^{'(CB)}|j\rangle_A^{(c)}|k\rangle_F^{(b)}. \tag{5.9e}$$

$|\Psi_P^{(D)}\rangle$ satisfies the time-independent Schrödinger equation $\hat{H}|\Psi_P^{(D)}\rangle = \lambda|\Psi_P^{(D)}\rangle$ which yields within each of the subspaces $P = 1, 3, \ldots, N-1$ an eigenproblem that is given by the subsequent set of linear equations for the sublattice b

$$\lambda\alpha_{kj}^{(b)} = -2a^b\alpha_{kj}^{(b)} - G^b(\beta_{kj}^{(b)} + \beta_{kj}^{'(B)}) + \omega_j\alpha_{kj}^{(BC)} + \omega_k\alpha_{kj}^{'(BC)}, \tag{5.10a}$$

$$\lambda\beta_{kj}^{(b)} = -G^b\alpha_{kj}^{(b)} - (a^b + b^b)\beta_{kj}^{(b)} + \frac{a^b}{N}\sum_{S_p}(\beta_{k'j'}^{(b)} + \beta_{k'j'}^{'(B)}) + \omega_j\beta_{kj}^{(BC)} - G^b\gamma_{kj}^{(b)} + \frac{2G^b}{N}\sum_{S_p}\gamma_{k'j'}^{(b)}, \tag{5.10b}$$

$$\lambda\beta_{kj}^{'(B)} = -G^b\alpha_{kj}^{(b)} - (a^b + b^b)\beta_{kj}^{'(B)} + \frac{a^b}{N}\sum_{S_p}(\beta_{k'j'}^{(b)} + \beta_{k'j'}^{'(B)}) + \omega_k\beta_{kj}^{'(BC)} - G^b\gamma_{kj}^{(b)} + \frac{2G^b}{N}\sum_{S_p}\gamma_{k'j'}^{(b)}, \tag{5.10c}$$

$$\lambda\gamma_{kj}^{(b)} = -G^b(\beta_{kj}^{(b)} + \beta_{kj}^{'(B)}) - 2b^b\gamma_{kj}^{(b)}, \tag{5.10d}$$

and sublattice c

$$\lambda\alpha_{kj}^{(c)} = -2a^c\alpha_{kj}^{(b)} - G^c(\beta_{kj}^{(c)} + \beta_{kj}^{'(c)}) + \omega_k^*\alpha_{kj}^{(bc)} + \omega_j^*\alpha_{kj}^{'(bc)}, \tag{5.11a}$$

$$\lambda\beta_{kj}^{(c)} = -G^c\alpha_{kj}^{(c)} - (a^c + b^c)\beta_{kj}^{(c)} + \frac{a^c}{N}\sum_{S_p}(\beta_{k'j'}^{(c)} + \beta_{k'j'}^{'(c)}) + \omega_j^*\beta_{kj}^{(cb)} - G^c\gamma_{kj}^{(c)} + \frac{2G^c}{N}\sum_{S_p}\gamma_{k'j'}^{(c)}, \tag{5.11b}$$

$$\lambda\beta_{kj}^{'(C)} = -G^c\alpha_{kj}^{(c)} - (a^c + b^c)\beta_{kj}^{'(C)} + \frac{a^c}{N}\sum_{S_p}(\beta_{k'j'}^{(c)} + \beta_{k'j'}^{'(c)}) + \omega_k^*\beta_{kj}^{'(CB)} - G^c\gamma_{kj}^{(c)} + \frac{2G^c}{N}\sum_{S_p}\gamma_{k'j'}^{(c)}, \tag{5.11c}$$

$$\lambda\gamma_{kj}^{(c)} = -G^c(\beta_{kj}^{(c)} + \beta_{kj}^{'(c)}) - 2b^c\gamma_{kj}^{(c)}. \tag{5.11d}$$

Up to now, in (5.10) and (5.11) the eigenproblem of decoupled sublattices b and c have been presented. However, the eigenproblem, which considers the coupling of the individual sublattices b and c through the staggered inter-cavity photon hopping J_1 and J_2, are given by the sequel set of linear equations

$$\lambda\alpha_{kj}^{(bc)} = -(a^b + a^c)\alpha_{kj}^{(bc)} - G^b\beta_{kj}^{(bc)} - G^c\beta^{\prime(cb)} + \omega_j^*\alpha_{kj}^{(b)} + \omega_k\alpha_{kj}^{(c)}, \tag{5.12a}$$

$$\lambda\alpha_{kj}^{\prime(bc)} = -(a^b + a^c)\alpha_{kj}^{\prime(bc)} - G^b\beta_{kj}^{\prime(bc)} - G^c\beta^{(cb)} + \omega_k^*\alpha_{kj}^{(b)} + \omega_j\alpha_{kj}^{(c)}, \tag{5.12b}$$

$$\lambda\beta_{kj}^{(bc)} = -G^b\alpha_{kj}^{(bc)} - (a^c + b^b)\beta_{kj}^{(bc)} - G^c\gamma_{kj}^{(bc)} + \omega_j^*\beta_{kj}^{(b)}, \tag{5.12c}$$

$$\lambda\beta_{kj}^{\prime(bc)} = -G^b\alpha_{kj}^{\prime(bc)} - (a^c + b^b)\beta_{kj}^{\prime(bc)} - G^c\gamma_{kj}^{\prime(bc)} + \omega_k^*\beta_{kj}^{\prime(b)}, \tag{5.12d}$$

$$\lambda\beta_{kj}^{(cb)} = -G^c\alpha_{kj}^{\prime(bc)} - (a^b + b^c)\beta_{kj}^{(cb)} - G^b\gamma_{kj}^{\prime(bc)} + \omega_j\beta_{kj}^{(c)}, \tag{5.12e}$$

$$\lambda\beta_{kj}^{\prime(cb)} = -G^c\alpha_{kj}^{(bc)} - (a^b + b^c)\beta_{kj}^{(cb)} - G^b\gamma_{kj}^{(bc)} + \omega_k\beta_{kj}^{\prime(c)}, \tag{5.12f}$$

$$\lambda\gamma_{kj}^{(bc)} = -G^c\beta_{kj}^{(bc)} - G^b\beta_{kj}^{\prime(CB)} - (b^b + b^c)\gamma_{kj}^{(bc)}, \tag{5.12g}$$

$$\lambda\gamma_{kj}^{\prime(bc)} = -G^c\beta_{kj}^{\prime(bc)} - G^b\beta_{kj}^{(cb)} - (b^b + b^c)\gamma_{kj}^{\prime(bc)}, \tag{5.12h}$$

where λ is the corresponding eigenvalue. Eqs. (5.10), (5.11) and (5.12) represent the most general form of the eigenproblem to our chosen model system. It generalizes the eigenproblem in [103]. We can recover the results in [103] by choosing uniform inter-cavity photon hopping, e.g. $J_1 = J_2$ and equal effective atom-photon coupling constant $G^b = G^c := G$. In the sequel section, we focus on the ground state DPBP as found in [103], discuss staggered induced disorder and its effects on the ground state DPBP.

5.3 Effects of staggered hopping induced disorder on ground state DPBP

In the following, we consider the aspect of induced disorder through the staggered photon hopping [104] in combination with the control field Stark shift. We can basically tune the disorder by an overall or staggered (non-)compensation of the control field Stark shift. Thereby, we keep the effective atom-photon coupling constants G^b and G^c constant and equal. This means that the coupling strengths g_m^b, g_m^c, the individual Rabi frequencies Ω^b, Ω^c and the common single photon detunings Δ^b, Δ^c are individually equal. As we restrict the discussion to the ground state DPBP, $g_m^b = g_m^c := g_m > \Omega^b = \Omega^c := \Omega$, $\Delta^b = \Delta^c := \Delta < 0$ and $G^b = G^c := G$ has to hold as mentioned and introduced in [103].

5.3.1 Staggered photon hopping in the regime of non-compensated and compensated control field Stark shift

We concentrate on the ground state DPBP and its state composition within an array of $N = 20$ coupled QED cavities with periodic boundary conditions and odd discrete quasi-momenta P where the overall control field Stark shift is not compensated. The used parameters for the cavity field coupling strengths in each sublattice b and c are $g_m = 0.05|\Delta|$ with $\Delta = -1000$. The value of g_m fits well with the realization of strong coupling in toroidal microcavities, embedded with Cs-atoms and the discover of ground state DPBP [103, 51]. Further, the Rabi frequencies in each QED cavity within the sublattices b and c have been chosen to be $\Omega = 0.003|\Delta|$, while the inter-cavity photon hopping parameters fulfil the conditions $J_1 \gg J_2$ with J_2 kept fixed and $J_1 > G$ as well as $G > J_2$. In order to get inherent information on the state composition of the ground state DPBP, we follow in line with [32, 103] and calculate the joint probabilities

$$p_{FF}^{(b)} = |\langle\Psi_P^{(D)}|\frac{\hat{b}_n^\dagger \hat{b}_m^\dagger}{\sqrt{1+\delta_{nm}}}|\Phi_0\rangle|^2, \tag{5.13a}$$

$$p_{AF}^{(b)} = |\langle\Psi_P^{(D)}|\hat{b}_n^\dagger \hat{\sigma}_{gf}^{(m)\dagger}|\Phi_0\rangle|^2, \tag{5.13b}$$

$$p_{AA}^{(b)} = |\langle\Psi_P^{(D)}|\hat{\sigma}_{gf}^{(n)\dagger} \hat{\sigma}_{gf}^{(m)\dagger}|\Phi_0\rangle|^2, \tag{5.13c}$$

$$p_{ff}^{(c)} = |\langle\Psi_P^{(D)}|\frac{\hat{c}_n^\dagger \hat{c}_m^\dagger}{\sqrt{1+\delta_{nm}}}|\Phi_0\rangle|^2, \tag{5.13d}$$

$$p_{af}^{(c)} = |\langle\Psi_P^{(D)}|\hat{c}_n^\dagger \hat{\tau}_{gf}^{(m)\dagger}|\Phi_0\rangle|^2, \tag{5.13e}$$

$$p_{aa}^{(c)} = |\langle\Psi_P^{(D)}|\hat{\tau}_{gf}^{(n)\dagger} \hat{\tau}_{gf}^{(m)\dagger}|\Phi_0\rangle|^2, \tag{5.13f}$$

$$p_{Ff}^{(bc)} = |\langle\Psi_P^{(D)}|\frac{\hat{b}_n^\dagger \hat{c}_m^\dagger}{\sqrt{1+\delta_{nm}}}|\Phi_0\rangle|^2, \tag{5.13g}$$

$$p_{Af}^{(bc)} = |\langle\Psi_P^{(D)}|\hat{b}_n^\dagger \hat{\tau}_{gf}^{(m)\dagger}|\Phi_0\rangle|^2, \tag{5.13h}$$

$$p_{Aa}^{(bc)} = |\langle\Psi_P^{(D)}|\hat{\sigma}_{gf}^{(n)\dagger} \hat{\tau}_{gf}^{(m)\dagger}|\Phi_0\rangle|^2, \tag{5.13i}$$

$$p_{fF}^{(cb)} = |\langle\Psi_P^{(D)}|\frac{\hat{c}_n^\dagger \hat{b}_m^\dagger}{\sqrt{1+\delta_{nm}}}|\Phi_0\rangle|^2, \tag{5.13j}$$

$$p_{aF}^{(cb)} = |\langle\Psi_P^{(D)}|\hat{\tau}_n^\dagger \hat{b}_{gf}^{(m)\dagger}|\Phi_0\rangle|^2, \tag{5.13k}$$

where $p_{Aa}^{(bc)} = p_{aA}^{(cb)}$ holds. First of all, we set $J_2 = 0$ and take $J_1 \neq 0$. This configuration corresponds to so called decoupled dimers. Dimers are formed by the nearest neighbour QED cavities of the sublattice b and c with the coupling, given by

the inter-cavity photon hopping strength J_1. The state composition of the ground state DPBP is dominated by the combined excitation of the atom and photon type in each sublattice, but equally distributed. Sublattice linkage is also dominated by the combined excitation of the atom and photon type. From this we can deduce that one photon remains attached to the atomic spin coherence wave. This is reminiscent of the atom-photon molecule. The state composition can be tuned by increasing the relative importance of the inter-cavity photon hopping J_1, e.g., by increasing $|\Delta|$. This is achieved gradually for distinct values of quasi-momentum, starting from the values $P = 1, N - 1$ and proceeding towards the mid-range values of P. As we reach the critical value of the inter-cavity photon hopping $J_c = 0.0039|\Delta|$, we have a change from bound spin coherences to bound photons within the individual sublattices as well as at the sublattice connection. The arrangement changes as we include J_2. Namely, if we switch on J_2, keep it fixed and fulfil the requirement that $J_1 \gg J_2$, ground state DPBP in the individual sublattices b and c appears as a two-photon bound state while at the sublattice connection as a bound state of spin coherences. Hence, the weak inter-cavity photon hopping strength J_2 introduces a so called bi-localization as seen in [104], but in our case it slightly differs. We have bound photons and bound spin coherences at the same time, whereas [104] only has bound photons or free spin coherences. If we compensate the control field Stark shift, the arrangement of excitations and the composition of the ground state DPBP remains the same as in the case of non-compensated Stark shift. Thus, in the staggered configuration of inter-cavity hopping strengths J_1 and J_2, control field Stark shift does not have any affect on the arrangement of excitations and its composition regarding the ground state DPBP, which was not the case in the uniform configuration of inter-cavity hopping within our work [103]. By switching off the Rabi frequency adiabatically, i.e. $\Omega \to 0$, increase J_1 equidistantly by $0.0003|\Delta|$ as it has been previously done and keep J_2 fixed, we see a transfer of pure bound spin coherences between the sublattices independently of (not)-compensating the control field Stark shift.

6

Conclusions

To sum up, we have investigated the formation of dark-state polaritons in an ensemble of degenerate two-level atoms with ground state Hilbert space $\mathcal{H}_g$ and excited state Hilbert space $\mathcal{H}_e$, where $\dim \mathcal{H}_g \geq \dim \mathcal{H}_e$ holds. We elaborated an algorithm, which is a generalization of the Sawada-Brout-Chong approach [12]. Under suitable conditions, the polariton mode dispersion relation and composition can be stated in a closed form. Such DSPs do not depend on the energy spacing of the two-level system, but rather on the Raman detuning of the coupling fields. For each polariton mode, the effective field coupling parameters depend on the appropriate eigenvalues of the atomic operators $\hat{V}_p^\dagger \hat{V}_p$ and $\hat{V}_c^\dagger \hat{V}_c$ that determine the eigenproblem for the polariton species. The application of the general procedure is given for ^{87}Rb atomic transition $F_g = 2 \to F_e = 1$ of the D_1 line. Two cases of polarizations of the control and probe field are analysed, when the two fields have orthogonal circular polarizations and when both are linearly polarized in the orthogonal directions. In the former case, two DSP modes are identified, while in the latter case, only one DSP mode can be determined. The formation of the modes as well as their dispersion relation critically depend on the polarizations chosen. Possible application of DSP modes in ultracold ^{87}Rb atoms for frequency and/or linear polarization conversion without energy loss in the retrieved pulse is presented. Our algorithm can be extended to degenerate systems with more levels and might have applications in quantum information processing as a building block for a preparation and read out schemes with the DSPs as qubit states.

Moreover, we have derived a modified Jaynes-Cummings model from the bare model under two conditions: (i) two-photon Raman resonance of the cavity mode and classical control field (ii) common single photon detuning $|\Delta| \gg g_m, \Omega$. We

have shown that the eigenstates on one hand depend on the common single photon detuning and on the other hand, their composition differs with respect to the control field Stark shift. Furthermore, we have extended the modified Jaynes-Cummings model to a modified Jaynes-Cummings-Hubbard model where an array of N coupled QED cavities, each having an embedded single three-level atom, is considered. The modified Jaynes-Cummings-Hubbard model supports DPBPs. The formation of two different species of spatially localized dark-polariton bound pairs (DPBPs) has been elaborated when there are exactly two excitations in the system. It was shown that the onsite repulsion $U(n)$ as a consequence of the Kerr-nonlinearity represents the attractive force between interacting dark-polaritons and enables the existence of DPBP states. Furthermore, it is demonstrated that our model system offers a high degree of tunability that can affect both quantitative and qualitative behaviour. In particular, the number of DPBP states can be controlled by (not) compensating the Stark shift due to the control field. Further, in the regime when cavity-atom coupling overwhelms the influence of the control field, and the common single photon detuning of the fields is negative, we obtained a ground DPBP eigenstate on which the storage and read out of a single photon can be effectively performed. An experimental realization is proposed for our model system. Cs atom has been mentioned as a promising candidate as its value of the cavity mode coupling strength g_m fits very well with our theoretically chosen and determined one.

In addition, disorder has been introduced into the modified Jaynes-Cummings-Hubbard model by a staggered arrangement of inter-cavity photon tunnelling strengths J_1 and J_2. This staggered arrangement leads to a formation of two sublattices b and c. Under the assumptions (i) $g_m \gg \Omega$, (ii) common single photon detuning $\Delta < 0$, (iii) inter-cavity photon hopping strength $J_1 \gg J_2$ as well as equality of the common single photon detuning, Rabi frequency and cavity mode coupling strength in and between the sublattices, bilocalization occurs. Within the sublattices, bilocalization is given by bound photons while the sublattice coupling is provided by bound spin coherences of the ground state DPBP. However, if the the Rabi frequency $\Omega \to 0$ adiabatically and we increase J_1 equidistantly, bound spin coherences of the ground state DPBP are transferred between the sublattices. Depending on the Rabi frequency, our system can be used to transfer quantum states either of bound photons or dark-polariton bound pairs given by bound spin coherences.

7

Outlook

In case of dark-polariton bound pairs several open questions remain for future investigations. If we look at the uniform configuration, which means that the inter-cavity hopping strength is equal within the array of coupled QED cavities, the number of excitations can be increased. Due to the cyclic symmetry in k-space, which is determined through the quasi-momentum P as a good quantum number, up to seven excitations can be studied. Furthermore, a more sophisticated atomic level structure can be considered, e.g a tripod configuration with contra-propagating fields. These kind of structures are interesting because spinor-like polaritons have been observed in these level structures. So, an effective spin bound state might be able to realize. On the other hand, the one-dimensional modified Jaynes-Cummings-Hubbard model could be extended to two dimensions and optical lattices could be studied with respect to the inter-cavity photon hopping strength J_1 or J_2. A realization of a frustrated Heisenberg spin system might be possible. Frustration could be introduced through the non-uniform (staggered) inter-cavity photon hopping strengths J_1 and J_2.

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БИОГРАФИЈА

Мађити Анђело, рођен 4. септембра 1977 године у Базелу(Швајцарска) је завршио основне студије нано науке на Филозофском-природно математичком факултету универзитета у Базелу. Након основне студије, наставио је мастер студије такође на Универзитету у Базелу. Правац усмеравање и специајлизацију на мастеру је било нанофизика.

Током мастер студије, извршене су три научна пројеката од која је једна мастер рад. Први научни пројекат под називом "Оптимизоване структуре $Mg^{+}(Ne)_{m}$-кластера ($1+m\leq 8$) симулиране методом Фурије интеграла по трајекторијама" [Fourier Path Integral Simulations and Optimized Structures of $Mg^{+}(Ne)_{m}$-Clusters($1+m\leq 8$)] припадао је области теоријске квантне хемије и спроведен је под руководством проф. др Маркус Мојвли (Prof. Dr. Markus Meuwly). Други научни пројекат о наномеханичким резонаторима на бази јонских замки [Towards ion trap transducers of nanomechanical resonators] био је из области теоријске наномеханике и спроведен је под руководством проф. др Мартино Пођоа (Prof. Dr. Martino Poggio).

Мастер рад под називом "О квантованој проводности у двослојном графену" [Towards quantized conductance in graphene bilayer] је био изведен на Техничком Универзитетом у Делфту у Холандији под руководством проф. др Ливен Вандерсајпена (Prof. Dr. Lieven Vandersypen) као ментора и коментора проф. др Мартино Пођоа (Prof. Dr. Martino Poggio). За мастер рад Мађити Анђело је био награђен стипендијом Универзитета у Базелу и учешћем у Еразмус програму.

Докторске студије је уписао на Физичком факултету Универзитета у Београду 2011. године, на смеру Квантна, математичка и нанофизика. Упис је уследен јавном конкурсу и препоруком од Универзитета у Базелу. Докторска дисертација под називом "ФОРМИРАЊЕ ТАМНИХ ПОЛАРИТОНА И ДВО-ПОЛАРИТОНСКИХ ВЕЗАНИХ СТАЊА У НИЗОВИМА АТОМА И ОПТИЧКИХ МИКРОРЕЗОНАТОРА" је урађена у институту за физику унутар центар за фотонику под руководством др Милана Радоњића ментор и коментора др Бранислава Јеленковића.

Прилог 1.

Изјава о ауторству

Потписани-а Анђело Мађити (Angelo Maggitti)

број индекса 31-D/2010

Изјављујем

да је докторска дисертација под насловом

FORMATION OF DARK-STATE POLARITONS AND TWO-POLARITON BOUND STATES IN ARRAYS OF ATOMS AND OPTICAL CAVITIES

- резултат сопственог истраживачког рада,
- да предложена дисертација у целини ни у деловима није била предложена за добијање било које дипломе према студијским програмима других високошколских установа,
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A. Maggitti

Прилог 2.

Изјава о истоветности штампане и електронске верзије докторског рада

Име и презиме аутора Анђело Мађити (Angelo Maggitti)

Број индекса 31-D/2010

Студијски програм Квантна, Математичка и Нанофизика

Наслов рада FORMATION OF DARK-STATE POLARITONS AND TWO-POLARITON BOUND STATES IN ARRAYS OF ATOMS AND OPTICAL CAVITIES

Ментор др Милан Радоњић

Потписани/а Анђело Мађити (Angelo Maggitti)

Изјављујем да је штампана верзија мог докторског рада истоветна електронској верзији коју сам предао/ла за објављивање на порталу **Дигиталног репозиторијума Универзитета у Београду.**

Дозвољавам да се објаве моји лични подаци везани за добијање академског звања доктора наука, као што су име и презиме, година и место рођења и датум одбране рада.

Ови лични подаци могу се објавити на мрежним страницама дигиталне библиотеке, у електронском каталогу и у публикацијама Универзитета у Београду.

Потпис докторанда

У Београду, 24.10.2015

A. Maggitti

Прилог 3.

Изјава о коришћењу

Овлашћујем Универзитетску библиотеку „Светозар Марковић“ да у Дигитални репозиторијум Универзитета у Београду унесе моју докторску дисертацију под насловом:

FORMATION OF DARK-STATE POLARITONS AND TWO-POLARITON BOUND STATES IN ARRAYS OF ATOMS AND OPTICAL CAVITIES

која је моје ауторско дело.

Дисертацију са свим прилозима предао/ла сам у електронском формату погодном за трајно архивирање.

Моју докторску дисертацију похрањену у Дигитални репозиторијум Универзитета у Београду могу да користе сви који поштују одредбе садржане у одабраном типу лиценце Креативне заједнице (Creative Commons) за коју сам се одлучио/ла.

1. Ауторство

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- Ауторство –без прераде
- Ауторство –делити под истим условима

(Молимо да заокружите само једну од шест понуђених лиценци, кратак опис лиценци дат је на полеђини листа).

Потпис докторанда

У Београду, 24.10.2015

A. Majkić

1. Ауторство - Дозвољавате умножавање, дистрибуцију и јавно саопштавање дела, и прераде, ако се наведе име аутора на начин одређен од стране аутора или даваоца лиценце, чак и у комерцијалне сврхе. Ово је најслободнија од свих лиценци.

2. Ауторство – некомерцијално. Дозвољавате умножавање, дистрибуцију и јавно саопштавање дела, и прераде, ако се наведе име аутора на начин одређен од стране аутора или даваоца лиценце. Ова лиценца не дозвољава комерцијалну употребу дела.

3. Ауторство - некомерцијално – без прераде. Дозвољавате умножавање, дистрибуцију и јавно саопштавање дела, без промена, преобликовања или употребе дела у свом делу, ако се наведе име аутора на начин одређен од стране аутора или даваоца лиценце. Ова лиценца не дозвољава комерцијалну употребу дела. У односу на све остале лиценце, овом лиценцом се ограничава највећи обим права коришћења дела.

4. Ауторство - некомерцијално – делити под истим условима. Дозвољавате умножавање, дистрибуцију и јавно саопштавање дела, и прераде, ако се наведе име аутора на начин одређен од стране аутора или даваоца лиценце и ако се прерада дистрибуира под истом или сличном лиценцом. Ова лиценца не дозвољава комерцијалну употребу дела и прерада.

5. Ауторство – без прераде. Дозвољавате умножавање, дистрибуцију и јавно саопштавање дела, без промена, преобликовања или употребе дела у свом делу, ако се наведе име аутора на начин одређен од стране аутора или даваоца лиценце. Ова лиценца дозвољава комерцијалну употребу дела.

6. Ауторство - делити под истим условима. Дозвољавате умножавање, дистрибуцију и јавно саопштавање дела, и прераде, ако се наведе име аутора на начин одређен од стране аутора или даваоца лиценце и ако се прерада дистрибуира под истом или сличном лиценцом. Ова лиценца дозвољава комерцијалну употребу дела и прерада. Слична је софтверским лиценцама, односно лиценцама отвореног кода.