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DIAGNOSIS OF DYNAMIC BEHAVIOR OF STRUCTURES USING THE DISTRIBUTION OF KINETIC AND POTENTIAL ENERGY

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DIJAGNOSTIKA DINAMIČKOG PONAŠANJA STRUKTURA PRIMENOM RASPODELE KINETIČKIH I POTENCIJALNIH ENERGIJA

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To my family

DIAGNOSIS OF DYNAMIC BEHAVIOR OF STRUCTURES USING THE DISTRIBUTION OF KINETIC AND POTENTIAL ENERGY

Abstract:

In most structures vibration is undesirable. This is because vibration creates dynamic stresses and strains which can cause fatigue and failure of the structure. The response of the structure to excitation depends upon the method of application and the location of the exciting force or motion, and the dynamic characteristics of the structure such as its natural frequencies and inherent damping level. The structural response can be improved by changing the mass or stiffness of the structure, by moving the source of excitation to another location, or by increasing the damping in the structure.

Structural Dynamics Modification (SDM) is a very effective and reliable technique which is extensively used to improve structure's dynamic characteristics such as natural frequency, mode shape and frequency response functions (FRFs). The dynamic behavior of the structure can be improved by predicting the modified behavior making some modifications parts like rigid links, beams, lumped masses, dampers etc. Many times it happens that the structure does not meet the required design constraints and the design has to be modified numerous times before it meets all the design constraints. This repeated analysis for each such modification becomes very expensive and time consuming, especially if there are lots of degrees of freedoms. The main point of improving dynamic behavior of a structure is increasing its natural frequencies and maximizing the interval between adjacent natural frequencies. This request can be achieved by changing the design parameters of the structure.

The procedures used in this thesis are concerned with the analysis of the distribution of potential and kinetic energy and the differences between them in elements of the structure. Study of distribution of potential and kinetic energy in main oscillation modes of structure gives obvious prediction which elements need some modifications to achieve the best dynamic characteristics. The aim of developed the proposed method of reanalysis and diagnostic of structure behavior is to determine real behavior of the construction in exploitation.

Reanalysis technique can be done for the structure using finite element methods (FEM). Information about the structure like material, geometry and boundary conditions should be prepared before making FE model.

In this thesis, Structural Dynamics Modification procedures, in process of reanalysis, have been numerically applied on well known simple structures, such as trusses, beams, and plates, as well as on complex real structures to improve its dynamic response.

FE models, using ABQUS and KOMPIS softwares, were created for each case. Then, by studying the distribution of kinetic and potential energy and the difference between them through whole elements of the structure one can predict the appropriate structural dynamic modification needed to avoid problems caused by resonance. Furthermore, in order to verify the results which were obtained numerically using finite element analysis, laboratory experiments were conducted to prototype model which simulate a case study of a real structure.

The obtained results of the FE model were in reasonably good agreement with the measured results of the prototype model. Accordingly, successful investigation was conducted on the real structure model, which emphasis that the proposed method provides effective results.

Keywords:

Dynamic, Finite element method, diagnosis, energy, distribution, behavior

Scientific field: Doctor Science, Mechanical engineering

Narrow scientific filed: Strength of structures

UDC number: 539.4.014(043.3)

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DIJAGNOSTIKA DINAMIČKOG PONAŠANJA STRUKTURA PRIMENOM RASPODELE KINETIČKIH I POTENCIJALNIH ENERGIJA

Rezime:

U većini objekata vibracije su nepoželjne. To je zato što vibracije stvaraju dinamičke sile i udarce koji mogu izazvati zamor i otkaz strukture. Odgovor strukture na njenu pobudu zavisi od načina primene i lokacije pobudne sile, kao i dinamičke karakteristike strukture kao što su prirodne frekvencije i nivo prigušenja. Strukturalni odgovor se može poboljšati promenom raspodele masa ili krutosti strukture, pomeranjem izvora pobude na drugu lokaciju , ili povećanjem prigušenja u strukturi .

Strukturalna dinamička modifikacija (SDM) je veoma efikasna i pouzdna tehnika koja se intenzivno koristi za poboljšanje dinamičkih karakteristika strukture kao što su prirodne frekvencije, glavnih oblika i funkcija frekventnih odziva (FRFs) . Dinamičko ponašanje konstrukcije može se poboljšati čineći modifikacije delova kao što su kruta mesta, masa , prigušenja itd. Mnogo puta se desi da struktura ne ispunjava potrebne ograničenja dizajna i da dizajn mora da bude modifikovan nekoliko puta pre nego što ona ispuni sve uslove projektovanja. Suština poboljšanja dinamičko ponašanje objekta jeste povećanje prirodnih frekvencija i povećanje intervala između susednih prirodnih frekvencija. Ovaj zahtev se može postići promenom dizajna parametara strukture .

Procedura koje se koriste u ovom radu jesu analize distribucije potencijalne i kinetičke energije i razlike između njih u elementima strukture. Studija distribucije potencijalne i kinetičke energije na glavnim oblicima oscilacija strukture daje očigledno predviđanje koje elemente i kako treba izmeniti da se postigne najbolje dinamičko ponašanje . Cilj predloženog razvijenog metoda reanalise i dijagnostike ponašanja struktura je da se utvrdi stvarno ponašanje konstrukcije u eksploataciji.

Tehnika reanalise strukture se izvodi primenom metode konačnih elemenata (MKE) . Informacije o strukturi kao materijal, geometrija i granični uslovi treba da budu spremni pre nego što generiše model.

U ovom radu , procedure strukturne dinamičke modifikacija, u procesu reanalise, brojčano su primenjene na poznate jednostavne strukture, kao što su rešetke, grede, i ploče, kao i na složene realne strukture sa ciljem da se poboljša dinamičko ponašanje.

Proračun je izveden sa programima ABAKUS i Komips. Proučavajući raspodelu kinetičke i potencijalne energija i razlika između njih kroz sve elemente strukture može se predvideti odgovarajuće strukturne dinamičke modifikacije potrebne da bi izbegli problemi izazvani rezonancom . Osim toga, da bi se proverili rezultati koji su dobijeni numerički pomoću konačnih elemenata, laboratorijski eksperimenti su sprovedeni na prototipu modela koji simuliraju pravu strukturu.

Dobijeni rezultati proračuna modela su u prilično dobroj saglasnosti sa rezultatima merenja prototipa modela.

Ključne reči:

Dinamika, Metod konačnih elemenata, Dijagnostika, energije, distribucija, ponašanje

Naučna oblast: Doktor nauka, Mašinsko inženjerstvo

Uža naučna oblast : Otpornost konstrukcija

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Author
Belgrade, November 2013

Contents

Nomenclature

A – cross section area of the element

E – Young's modulus of material

FRF – Frequency Response Function

L, l – length

Q – nodal displacement.

$\dot{Q}$ – nodal velocity

R – dissipation function

I_z – area moment of inertia

E_k – Kinetic energy

E_p – Potential energy

e_k – Kinetic energy of element

e_p – Potential energy of element

E_r, $E_{k,r}$, $E_{p,r}$ – total, kinetic and potential energies of a structure in *r-th* main oscillation mode

$(e_k)_e$, $(e_p)_e$ – kinetic and potential energies of *e-th* element in *r-th* main oscillation mode

$V^{(e)}$ – volume of element

$\{\overrightarrow{U}(x, y, z, t)\}$ – vector of displacement

$\{\dot{\vec{U}}(x, y, z, t)\}$ – vector of velocity

$[N(x, y, z)]$ – the matrix of the shape functions

$\{\vec{Q}^{(e)}\}$ – vector of nodal displacements

$\left\{\underset{\sim}{\vec{Q}}\right\}$ – global nodal displacement vector

$\left\{\underset{\sim}{\dot{\vec{Q}}}\right\}$ – global nodal velocity vector

$\{\underset{\sim}{\overrightarrow{P_c}}\}$ – vector of concentrated nodal forces of the structure or body
$[P_s]_e$ – vector of element nodal forces produced by surface forces
$[P_b]_e$ – vector of element nodal forces produced by body forces
$\{q_r^s\}_e$– corresponding *r-th* eigenvector, of *e-th* element with *s* degrees of freedom
$\{\underline{Q}_r\}$ – eigenvector
[B] – matrix relating strains and nodal displacements
[D] – elasticity matrix
$[N]$ – matrix of shape function
$[C]_e$ – damping matrix of element *e* in local coordinate system
$[K]_e$ – stiffness matrix of element *e* in local coordinate system
$[M]_e$ – mass matrix of element *e* in local coordinate system
[C]– global damping matrix of the structure
[K]– global stiffness matrix of the structure
$[M]$– global mass matrix of the structure
$[T]$ – transformation matrix from local to global coordinate
$[\Delta K]$, $[\Delta M]$ – changes in stiffness and mass matrices
$[K]'$ – stiffness matrix of the modified structure
$[M]'$ – mass matrix of the modified structure
λ_r – eigenvalue
$\Delta\lambda$, $\left\{\Delta\underline{Q}\right\}_r$ – changes of eigenvalues and eigenvectors
α_e, β_e – values that define the modification of *e-th* element
ρ – density of material
ω_r – natural frequency
f_{0r} – frequency of structure in *r-th* main oscillation mode
$\vec{\varepsilon}$ – strain vector
$\vec{\sigma}$ – stress vector

Chapter 1

INTRODUCTION

1.1 General Introduction

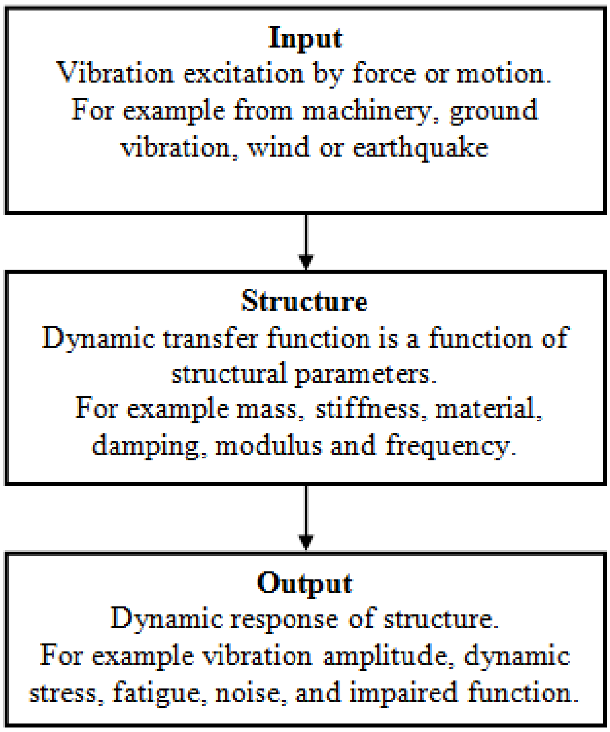
Nowadays structure design requirements have broad definitions because of high technology industry. For example, the development of materials with superior properties in exploitation conditions leads to extend the design requirements to involve structural integrity, reliability and life specification, in order to increase the life of structure. Structures which have a complicated design require massive efforts in analyzing and diagnosing the defects. Thus, one should deal carefully with the factors affecting the structure. The external load is one of the important factors that have big influence on the structure and its response. Moreover, in the static analysis, strength and deformation of structure are governed by the value of the external load. Therefore, the strength and deformation should be always under control in the case of static load. Although the static analysis is very important, the complete significant solution requires a dynamic analysis to reach the best results especially when the structure is subjected to the dynamic load or under high revolution rates such as complex manufacturing systems in mines and power plants, aircrafts, ground vehicles, rail-road vehicles, etc.

Dynamic analysis is more complex than static analysis, and the design requirements must include dynamic properties such as vibration level, resonance range, response properties, eigenvalues, dynamic stability and modal forms. The vibration that occurs in most machines, structures and dynamic systems is undesirable, not only because of the resulting unpleasant motions, the noise and the dynamic stresses which may lead to fatigue and failure of the structure or machine, but also because of the energy losses and the reduction in performance that accompany the vibrations. It is therefore essential to carry out a vibration analysis of any proposed structure. There have been very many cases of systems failing or not meeting performance targets because of resonance, fatigue or excessive vibration of one component or another. It is usually much easier to analyze and modify a structure at the design stage than it is to modify a structure with undesirable vibration characteristics after it has been built [1].

1.1.1 The Causes and Effects of Structural Vibration

There are two factors that control the amplitude and frequency of vibration in a structure: the excitation applied and the response of the structure to that particular excitation. Changing either the excitation or the dynamic characteristics of the structure will change the vibration stimulated. The excitation arises from external sources such as ground or foundation vibration, cross winds, waves and currents, earthquakes and sources internal to the structure such as moving loads and rotating or reciprocating engines and machinery. These excitation forces and motions can be periodic or harmonic in time, due to shock or impulse loadings, or even random in nature. The response of the structure to excitation depends upon the method of application and the location of the exciting force or motion, and the dynamic characteristics of the structure such as its natural frequencies and inherent damping level.

In most structures vibration is undesirable. This is because vibration creates dynamic stresses and strains which can cause fatigue and failure of the structure, fretting corrosion between contacting elements and noise in the environment; also it can impair the function and life of the structure or its components (see Fig. 1.1).



Input
Vibration excitation by force or motion.
For example from machinery, ground vibration, wind or earthquake

Structure
Dynamic transfer function is a function of structural parameters.
For example mass, stiffness, material, damping, modulus and frequency.

Output
Dynamic response of structure.
For example vibration amplitude, dynamic stress, fatigue, noise, and impaired function.

Figure 1.1 Causes and effects of structural vibration.

1.1.2 The Reduction of Structural Vibration

The level of vibration in a structure can be attenuated by reducing either the excitation or the response of the structure to that excitation or both (see Fig. 1.2). It is sometimes possible, at the design stage, to reduce the exciting force or motion by changing the equipment responsible, by relocating it within the structure or by isolating it from the structure so that the generated vibration is not transmitted to the supports. The structural response can be altered by changing the mass or stiffness of the structure, by moving the source of excitation to another location, or by increasing the damping in the structure.

Naturally, careful analysis is necessary to predict all the effects of any such changes, whether at the design stage or as a modification to an existing structure [1].

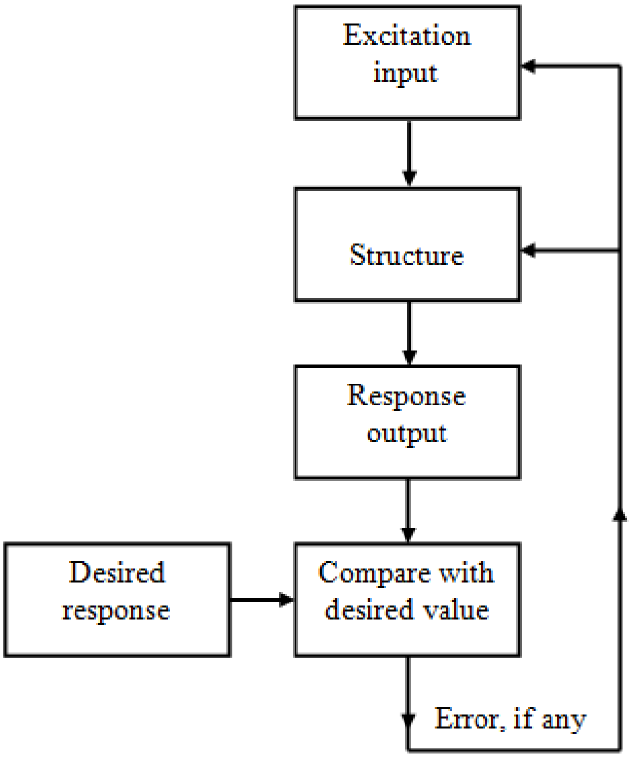


Figure 1.2 Reduction of structural vibration.

1.1.3 Structural Dynamics Modification (SDM)

Structural Dynamics Modification (SDM) is a very effective and reliable technique which is extensively used to improve structure's dynamic characteristics such as natural frequency, mode shape and frequency response functions (FRFs). Although this topic has been widely studied in the previous decades, the methodology of modification (reanalysis) of constructions is still under intense development. Predicting the change of natural frequencies and mode shapes is not effortless, because of the complexity of the structure. The dynamic behavior of the structure can be improved by predicting the modified behavior making some modifications parts like rigid links, beams, lumped masses, dampers etc. Many times it happens that the structure does not meet the required design constraints and the design has to be modified numerous times before it meets all the design constraints. This repeated analysis for each such modification becomes very expensive and time consuming, especially if there are lots of degrees of freedoms. To avoid dynamic problems, some modification will be done for structure in process of reanalysis. Reanalysis is a technique through which the dynamic response of the structure is improved. The main purpose of reanalysis techniques is to analyze the modified structure without performing the complete analysis of the structure but give a reasonably accurate solution within the allowable tolerance limits. This helps in reducing both the computational time and cost. Finite element (FE) is a powerful method to perform these processes using simple procedures.

Due to advances in numerical methods and the availability of powerful computing facilities, FE analysis has become the most popular technique in structural dynamic analysis. Modeling of complex structures using finite elements method is a helpful approach in solving problems in short time with reliable results. The fundamental principle of the FE method is to divide a complicated structure into many small elements such as plates, beams, shells, etc. The mass and stiffness matrices of an individual element, which is a simple, homogeneous element, can be obtained easily. The global mass and stiffness matrices of the structure can be assembled using these element matrices by considering connectivity and all the boundary conditions. Once the mathematical model has been built (or the mass and stiffness matrices have been constructed), the equations of motion can be solved by using various algorithms to obtain a description of the dynamic behavior of the structure.

Anew user of finite element analysis is unlikely to start writing a computer program. The reason for this is that there are large numbers of general purpose finite element programs which can be obtained commercially. All are available on a wide range of powerful desktop computers. There is also an increasing number available for running on personal computers. These tend to be a subset of the desktop version. They can be used to analyze small scale structures and also prepare the input data for large scale structures which are to be analyzed on a powerful desktop [2].

The main point of improving dynamic behavior of a structure is increasing its natural frequencies and maximizing the interval between adjacent natural frequencies. This request can be achieved by changing the design parameters of the structure. The procedures used in this thesis are concerned with the analysis of the distribution of potential and kinetic energy and the differences between them in elements of the structure, which gives prediction for which elements need a modification.

1.2 OBJECTIVES

The main purpose of the proposed thesis is, therefore, to diagnosis and investigates the dynamic behavior of some real structures. Structural Dynamics Modification procedures will be applied on well known simple structures, such as trusses, beams, and plates, as well as on complex real structures to improve its dynamic response. To this end, FE model will be created for each case. Then, by studying the distribution of kinetic and potential energy and the difference between them through whole elements of the structure one can predict the appropriate structural dynamic modification needed to avoid problems caused by resonance. Furthermore, in order to validate the theoretical results which are obtained using finite element analysis, laboratory experiments will be conducted to a prototype model which simulates the real structure.

1.3 OUTLINE OF THE THESIS

In order to fulfill the above objectives, this Thesis is organized as follows:

In Chapter 2, a literature review on the structural dynamic modification is given.

In Chapter 3, the procedures involved in deriving the finite element equation of dynamic problems are presented. Stiffness and mass matrices of some structural element are derived.

In Chapter 4, the procedures of reanalysis which are used in this thesis are proposed. The proposed method depends on the concept of energy distribution through the structure. The process of analysis is done using a computer program, based on the using of finite element methods and the implementation of structure energy distributions.

Chapters 5 and 6 are devoted to the application of the proposed method on some cases: in Chapter 5, numerical examples of some different structures (one Dimensional (1D) and two

Dimensional (2D)) subjected to structural modifications are presented in order to assess numerically the effectiveness of the method proposed in this thesis. In Chapter 6, the reanalysis procedures are applied to real complex structures. Finite Element Analysis is made in order to diagnosis of dynamic behavior of some real complex structures. Furthermore, a case study for a complex real structure (Bucket wheel excavators) is presented. In order to validate the effectiveness of the proposed method, numerical analysis and experiments are done on a prototype model which simulates the Bucket wheel excavators, and then the reanalysis procedures are applied to the real structure.

In Chapter 7, the major conclusions are drawn.

Chapter 2

LITERATURE REVIEW

This chapter gives brief literature reviews as it pertains to this research.

Structural dynamic modification is aimed to improve the dynamic behavior of a structure, based on the available information for the original structure. There are two opposite approaches for structural modifications. The first one is direct structural modification, and the second is inverse structural modification. Several studies have been addressed to the subject of modal reanalysis and structure dynamic modifications. Also, a few surveys have been conducted [3,4,5,6].

2.1 Direct Structural Modification

The direct structural modification problem is treated as prediction problem which is concerned with determining the dynamic response of a structure brought about by modification. The Rayleigh quotient can be considered as the basic of direct dynamic modification [7]. Rayleigh showed that the smallest natural frequency is the global minimum and the largest natural frequency the global maximum of the quotient. A consequence of this minimal property is that any stiffness increase or mass decrease will generally result in an increase the system natural frequencies, except when a mass or stiffness is added at a vibration node when there is no change in that particular natural frequency. Wittrick [8] concluded that any small change to an eigenvalue should be attributed to small parameter changes only and not to any small changes to the modes shape, because of the stationarity of the Rayleigh quotient. Fox and Kapoor [9] showed that expressions of both eigenvalue and eigenvector rate of change may be written in terms of only the corresponding unmodified eigenvector and eigenvalue. The importance of knowing the rates of change of eigenvectors and eigenvalues with respect to structural changes is that they can be used to obtain a first-order approximation of the actual modified eigenvalues and eigenvectors. Structural modification techniques based on the Rayleigh Quotient, and in general on techniques that rely on the estimation of rates of change of eigenvalues and eigenvectors with respect to structural parameters, are suitable only for infinitesimal modifications. Weissenberg [10,11] treated lumped mass and stiffness modifications as a symmetric unit rank perturbation on the eigenvalue problem of the unmodified structure. For example, for a point mass modification this perturbation is given by

$$(\mathbf{K} - \bar{\omega}_i^2\mathbf{M} - \bar{\omega}_i^2\delta m\mathbf{u}\mathbf{u}^{\mathrm{T}})\bar{\varphi}_i = 0, \tag{2.1}$$

Where $\delta\, m$ is the mass modification and $\mathbf{u}$ a unit vector that indicates the position of the structural modification. He obtained the expressions

$$\frac{1}{\delta m} = \sum_{j=1}^{n} \frac{\omega^2\eta_j^2}{\omega_j^2 - \omega^2}\,, \tag{2.2}$$

$$z_{ji} = \beta_i \frac{\eta_j}{\omega_j^2 - \bar{\omega}^2}\,, \tag{2.3}$$

Where ω_j and $\bar{\omega}_i$ are, respectively, the *j*th natural frequency of the original system and the *i*th natural frequency of the modified system. The latter is one of several frequencies $\omega = \bar{\omega}_i$ that satisfies Eq. (2.2). z_{ji} the *j*th component of $\mathbf{z}_i = \mathbf{M}\mathbf{\Phi}^{\mathrm{T}}\bar{\varphi}_i$ where $\mathbf{\Phi}$ is the modal matrix having the eigenvectors of the original system in its columns. M is the mass matrix and $\bar{\varphi}_i$ is the *i*th eigenvector of the modified system. η_j is the *j*th component of the vector $\eta = \mathbf{\Phi u}$ and β_i is a scaling constant. This approach was extended by Pomazal and Snyder [12] and Hallquist [13] by taking damping into consideration and adding DOFs. An exact method for calculating the eigenvalues and eigenvectors of the modified structure using a local modification based on the complete eigensolution of the original structure was presented by Hirai et al [14]. Although the accurate results can be obtained from only several eigenvalues and eigenvectors of the original structure by using this method, it lacks applicability when only a few lower eigenvalues and eigenvectors of the original structure are known, or when higher ordered eigenvalues of the modified structure need to be determined from a limited number of eigenvalues and eigenvectors of the original structure

Crowley, J. et al. [15] presented the basic theory behind direct Structural Modification using experimental frequency response functions. He also examined some of the applications, advantages and disadvantages of the technique, and presented examples illustrating its usage. Wang et al. [16] experimentally investigated the effects of local modification on the dynamic characteristics of an existing structure. The frequency response functions (FRFs) of the modified structure were computed based on the frequency response data of the original structure and the characteristics of the local modification. Hence, the dynamic characteristics of the modified structure could be identified using the calculated transfer functions. This method was applied to a free-free plate with local mass modification and without additional DOFs. Wallack [17] extended the local modification method, which must be used repeatedly, i.e., for each local modification, to handle general matrix modification which can solve multiple modifications simultaneously. Özgüven[18] developed a general method, using either theoretically calculated or experimentally measured FRFs, to analyze a structure subjected to lumped structural modification in two different cases: with and without additional DOFs. In this method, the frequency response functions (FRFs) of the modified structure are calculated from those of the original system and system matrices of the modifying structure. This method has been successfully applied to several simple structures, e.g., a plate and a propeller by Tahtali[19]. The technique of Structural Modifications Using Frequency Response Functions SMURF involves the direct manipulation of frequency response functions, or FRFs, of component systems to yield the FRFs of the modified system. This is advantageous since FRFs from sources such as experimental modal tests, finite element models, or analytical models may be combined to produce a modified set of FRFs for a system. The theoretical formulation of the SMURF technique is well established and has been documented on many occasions [20-25]. This technique was employed by D.S. Massey and C.P. Constancon [26] for the prediction of the modified dynamic characteristics of beam-like structures, with experimentally derived FRFs serving as a basis. The underlying principles of these limitations Pinned and rigid modification types were employed, which made it necessary to consider FRFs which related rotational excitation and response. The results indicated that the (SMURF) technique was able to predict the dynamic characteristics of the modified structures with a high degree of accuracy. Aforementioned studies were dealt with problems of lumped modification. In the direct structural dynamic modification, the number of studies on distributed structural modification problems is limited although most modifications in engineering practice being continuous. D'Ambrogio [27] studied the prediction of the frequency response function (FRF) of the modified structure subjected to structural modifications. This approach depends on knowledge of the FRF of the original structure in addition to a physical description of the modified structure. He demonstrated that is difficult to combining the theoretical model of the modifying structure, which includes rotational DOF with the frequency response model of the original structure system, derived from measurements based on translational dofs. The proposed solution is based on the condensation of mass and stiffness matrices of the modifying structure to eliminate rotational dofs. B. J. Schwarz and M. H. Richardson [28] demonstrated how all of the most commonly used elements of finite element analysis (FEA) can also be used to model structural modifications(SDM). These include rods, bars, triangular and quadrilateral plate and shell elements, and tetrahedron, prism, and brick solid elements. They modeled and tested a flat plate structure with a rib stiffener attached to its centerline using SDM, with both plate and bar elements. The modal data for the unmodified structure (plate without rib) and the element properties were used as input data to the SDM method. The modes of the modified structure (plate with rib) were calculated by SDM. To evaluate the structural modification results objectively, they used the FEA modes of the unmodified plate, and compared the SDM results with the FEA modes of the plate with rib. Likewise they used the experimental modes of the unmodified plate, and compared the SDM results with the experimental results for the plate with rib. The obtained results were very useful. In his doctoral thesis H. Grafe [29] investigated the fundamental concepts of FRF model updating methods. The use of component mode synthesis methods for FE model reduction was proposed. Two new FRF correlation functions were introduced, the shape- and amplitude-correlation coefficients. Both correlation measures may be used across the full measured frequency range and uniquely map any complex response to a real scalar between zero and unity. An analytical closed-form solution of the derivatives of the correlation functions was used to formulate the predictor- corrector model updating formulation. This correlation-based technique resolves problems associated with incomplete measurements and updating frequency point selection and is also robust against measurement noise. As a result of this new philosophy of FRF model updating, modal damping coefficients may also be identified. . D'Ambrogio, and A. Sestieri [30, 31, 32] presented a procedure for creating a consistent model for distributed modifications. They proposed a solution for the problem of computing the dynamic stiffness matrix [ΔB] for distributed structural dynamic modifications by employing its quasi-local characteristics for no change in DOFs. They extend their work in [33] by applying it to a real structure consisting of a complex plate. Three different techniques to tackle appropriately the structural modification problem were considered which are a condensation technique, an expansion technique providing the rotational DOFs from translational measurements and a modal synthesis technique providing the rotation/moment elements from measured translations and rotations. A comparison among these methods was presented and discussed. M. Corus and E. Balm'es [34] proposed a method to predict the effects of distributed modifications of structures. The method is an evolution of the classical formulation but uses distinct measurement and coupling points; it includes a smoothed expansion procedure and two indicators to estimate the quality of the result. Two examples were presented to illustrate some advantages of the proposed approach. Starting from the original relationship developed by Özgüven [18] for structural modifications and the description method for distributed modifications developed by D’Ambrogio and Sestieri [30], Hang et al. [35] proposed an approach to predict the effects of distributed structural modifications with additional DOFs. Also, Hang et al. [36-38] developed the distributed structural modification theory to include distributed structural modifications with reduced DOFs. In order to include the non-zero effects after the condensation procedure, known as Guyan reduction, the definition of the interface DOFs is extended from physical to numerical. G. Canbaloglu and H. N. Özgüven [39] presented an approach for predicting the dynamic response of a structure with distributed modifications from the response of the original structure itself and dynamic flexibility matrix of the modifying structure. The performance of this method was investigated by applying it to a real structure. They compared the receptances calculated by using the structural modification method with measured ones. Successful results were obtained. Accordingly, they concluded that the structural reanalysis method proposed can be successfully and efficiently used for structures with distributed modifications. H.P. Chen [40] proposed an improved iterative procedure for efficiently determining the eigenvalues and the corresponding eigenvectors for a dynamic system with large modifications of structural parameters and a large number of DOFs present. Only a limited knowledge of original modes is required to provide correct predictions of the modified modal parameters, and the knowledge of the original or modified stiffness and mass matrices may not be needed. He demonstrated that the proposed high order approximation approach can give good predictions of the modified modal parameters even in the cases where relatively large modifications of structural parameters are present

2.2 Inverse structural modification

The inverse structural dynamic modification is treated as an optimization procedure which is used to determine necessary modifications in order to achieve the desired dynamic behavior of structure usually in terms of the desired values for natural frequencies and mode shapes. Since the 1970s, there has been considerable development of this topic, as summarized in the review by Venkayya [41]. K. Saitou et al. [42] reviewed and summarized a brief history of structural analyses and optimization from an industry perspective. The general inverse structural modification problem was first considered by Weissenburger [43], and Pomazal and Snyder [44]. Due to the development of commercial softwares which are running on desktop computers and with increased use of finite element analysis, perturbation methods have become popular for inverse modification problems. peturbation is a technique used to investigate the solution of a modified structure by considering the modification as a perturbation of the linear baseline system. Stetson [45] proposed a first-order perturbation method based on the assumption that the new mode shapes could be expressed as admixtures of the baseline mode shapes. In a subsequent work, Stetson et al [46-48] casted this technique in terms of finite elements and applied it to several problems. Kim et al [49] presented an analytical method for the automated redesign of the modal characteristics of undamped mechanical systems. The method is based on a perturbation of the eigensystem, and all nonlinear terms were considered. A penalty function method was employed in this work in which the original objective function is a minimum weight condition and the penalty term is a properly normalized set of residual nodal force errors. A nonlinear incremental inverse perturbation method for structural redesign was presented by Hoff et al [50, 51]. The method uses a single finite element analysis of an undamped baseline structural system, and can be applied to large or small natural frequency and/or mode shape changes. Done and Rangacharyulu [52] described an application of a mathematical optimization process to helicopter vibration control by structural modification. Their attention was focused on the reduction of vibration in the crew area. Forced vibration response was used to identify the most effective parameters for controlling vibration in the crew area. The method was applied to a simple two-dimensional beam-element helicopter fuselage model. As a result, the exercise demonstrated what can and cannot be done in controlling vibration by using optimization structural modification. Wang et al. [53] developed numerical methods for the structural modification of a fuselage structure and for the analysis and design of appendant structures. These were applied to the problem of alleviation of helicopter vibration while design techniques were developed to achieve the desired anti-resonance through structural modification. A local modification method was used to analyze both the appendant structures and the modified system. Sestieri and D'Ambrogio [54] developed a method which uses the raw data of measured FRFs and lumped modifications to achieve the best structural modifications in order to fulfill the need to vary the structural dynamic characteristics. Bucher and Braun [55] developed a theory to show how the necessary mass and stiffness modifications can be computed using modal test results only, even when only a partial set of eigensolutions is available from such tests. Their method relied upon extracting the left eigenvectors from noisy data in order to assign the mode shapes of the modified system. S.G. Hutton [56] presented analytical procedures for modifying the vibration characteristics of structures. It has been shown that once a baseline analysis has been conducted the calculated modes can be effectively used in further computation to determine how to modify the structure to obtain the prescribed frequencies. Ram and Elhay [57] studied the undamped multi-degree-of-freedom vibration absorber and treated the problem of determining the parameters of the secondary system as an inverse eigenvalue problem. This is the problem of determining the terms in the stiffness and mass matrices that will produce specified natural frequencies and mode shapes. G. Gladwell [58] discussed some inverse problems relating to finite-element formulations for simple chain-like structures. Li and He [59] developed a new approach based on the solution of linear equations for structural modification of a dynamic system. It determines mass and stiffness modifications of an undamped structural system needed to change the dynamic characteristics of the system. This approach formulates the structural modification problem using a set of linear equations and does not require an eigenvalue solution. The data it needs are only the frequency response function data at designated modification points. A structural modification method based on frequency response functions was presented by Park et al. [60, 61]. The design objective in this method is to derive multiple lumped mass, damper and stiffness modifications needed to reallocate eigenvalues and specify eigenvectors of an existing structure. Based on FRFs, a substructure coupling concept is used to derive the system dynamic equations. Finally a linear algebraic equation for identifying the necessary structural modifications is obtained. W.H.Tong et al [62] presented the basic theory to find out a solution existence for the structure optimization with frequency constraints. Based on this theory, natural frequencies do not change with uniform frame modification and key limitation for determination of optimal dynamic solution of frame structure modification is mostly that of eigenfrequencies. Mottershead et al. [63] presented an inverse method for the assignment of natural frequencies and nodes of normal modes of vibration by the addition of grounded springs and concentrated masses. The method relies entirely on measured receptances at the coordinates of the nodes and the modifications. Measurements at other locations are not needed and it does not require an analytical model. The stiffness and mass parameters are determined by an analysis of the null space of a matrix containing the measured receptances. An optimization procedure was presented in [64, 65] for the minimum weight optimization with discrete design variables for truss structures subjected to constraints on stresses, natural frequencies and frequency responses. An efficient relationship between geometric and material properties of pin-jointed truss structures and their eigenvalues was established by Djoudi et al. [66]. The problem is formulated as an inverse eigenvalue problem. This formulation allows the determination of the required modifications on the structural members to achieve specified eigenfrequencies. In addition to the modification of the existing structural elements, the formulation also allows addition of new structural elements to obtain the desired frequencies. Bahai et al. [67] and Bahai and Aryana [68] presented formulations for inverse optimisation of vibration behaviour of finite element models of both truss and continuous structures. The proposed algorithms determine the required modifications on truss and continuous structures to achieve specified natural frequencies. The modification can be carried out globally or locally on the structures stiffness and matrices and the formulation can also be used to add new structural members to achieve the desired response. The problem of assigning natural frequencies to a multi-degree-of-freedom undamped system by an added mass connected by one or more springs was addressed by Kyprianou et al. [69]. The added mass and stiffnesses are determined using receptances from the original system. The modifications required to assign a single natural frequency may be obtained by the non-unique solution of a polynomial equation. If more than one frequency is to be assigned, then a system of non-linear multivariate polynomial equations must be solved. Such a modification involves not only an added mass and one or more stiffness terms, but also an added coordinate. It is demonstrated that it is impossible to assign more than two natural frequencies independently. In order to assign a third natural frequency independently, using the same oscillator, a second spring is connected between the added mass and another coordinate. The proposed method uses Groebner bases [70] as a tool for finding the simultaneous solutions that define the modification parameters. Farahani and Bahai [71] proposed an inverse strategy for relocation of structural natural frequencies using first order formulation and solution algorithm. The proposed technique incorporates the design constraints or objective functions in the system equations in such a way that a square system of equations is always preserved. The formulations are general and applicable to all finite element structures. This technique was extended in [72] to second order methods for relocation of structural natural frequencies from their initial design values to new modified frequencies. Kyprianou et al. [73] solved the inverse structural modification problem in order to determine the dimensions of the cross-section of a beam that when added to an original structure will assign natural frequencies or anti-resonances as specified. In order for this to be accomplished rotational receptances must be measured as presented in the companion paper [74]. When added beam is cast as an additional forcing term on the original structure a system of multivariate polynomials in the parameters of the beam cross-section are revealed. The solution of this system gave the beam cross-section dimensions that assigned the desired natural frequencies and anti-resonances. Hua-Peng Chen [75] presented efficient methods for determining the modified modal parameters (natural frequencies and mode shapes) in a structural dynamic modification analysis when structural modifications are relatively large. An improved iterative procedure was proposed for efficiently determining the eigenvalues and the corresponding eigenvectors for a dynamic system with large modifications of structural parameters and a large number of DOFs present. A high order approximation approach is also presented without iterative procedures involved. He concluded that even in the cases with a large modification of structural parameters the proposed iterative procedure can provide exact predictions of the modified modal parameters after only a few iterations, and the high order approximation approach can give excellent estimates. The computation of the modified modal parameters does not require the knowledge of the original or modified structural parameters, and only a limited knowledge of the original modal data may be sufficient in a dynamic reanalysis for complex structures. Olsson and Lidström [76] studied the inverse structural modification using constraints. The undamped natural vibrations of a constrained linear structure were calculated by solving a generalized eigenvalue problem derived from the equations of motion for the constrained system involving Lagrangian multipliers. The non-symmetric constraint formulation which is given in [77] was used with the constraint matrix elements as design variables. The procedure was applied numerically to a few simple problems in order to illustrate the methodology of this method.

The methods of sensitivity analysis of the structure's eigenvalues and eigenvectors with respect to modification parameters have proved to be a powerful tool leading to achieve the objective of inverse dynamic modification problems. For a thorough review of the research in sensitivity methods, one may refer to the survey presented by Haftka and Adelman [78]. An approach for real symmetric eigensystems was first presented by Fox, R. and Kapoor [9]. Hallquist [79] proposed a method for determining the effects of mass modification in viscously damped vibratory systems. Nelson [80] proposed a method to calculate the first-order derivatives of eigenvectors with distinct eigenvalues for the general real eigensystems. This method was developed by Ojalvo [81], Mills-Curran [82], Dailey [83] and Wuetal [84] for calculating the first-order derivatives of eigensolutions of structures with repeated eigenvalues. Yoshimura [85,86] presented a new optimization method for increasing or decreasing the natural frequency at an arbitrary degree of natural mode and maximizing the frequency interval between adjacent natural frequency, in order to improve dynamic characteristics with respect to forced vibration, noise, and feed drive performance of machine structures. The optimization is performed in two steps. First, the evaluation of the energy distribution through the complete structure at the objective natural frequency provides design modifications continuing until roughly optimum design points are obtained and reduction of design parameters taking part in the optimization are determined. Secondary, a mathematical direct search method identifies the true optimum design point. Zimoch [87] presented a method, which is applied to conservative as well as non-conservative systems, for the analysis of the sensitivity dynamical characteristics of mechanical linear systems to variations in the parameters. In his doctoral thesis, Jung [88] discussed the advantages and disadvantages (or limitations) of various model updating methods. One of the advantages of model updating using eigensensitivity analysis is that mode expansion is not required. However, this method requires large computational effort because of the repeated solution of the eigendynamic problem and repeated calculation of the sensitivity matrix. Jung developed a sensitivity method using arbitrarily chosen macro elements at the error location stage in order to reduce the computational time and to reduce the number of experimental modes required. Accordingly, the model updating problem which is generally under determined can be transformed into an over-determined one and the updated analytical model can be a physically meaningful model. Lee and Jung [89, 90] proposed methods for the computation of eigenpair derivatives for the real symmetric eigenvalue problem with distinct eigenvalues. Lee et al. [91, 92] extended the previous work to the proportionally and non-proportionally damped systems with multiple natural frequencies. Aryana and Bahai [93] proposed a method which is based on the second order approximation in order to determine the required geometrical and material modifications for pre-defined natural frequency values of structures. The advantage of this method is that the modification is conducted only on small parts of the global stiffness and mass matrices, requiring minimal computational processing time and memory. Therefore, the modifications can be conducted on the stiffness and mass matrices in their assembled form. Choi et al. [94] presented a method for the derivatives of eigenvalues and eigenvectors of non-conservative systems. In this method, contrary to previous methods, the eigenvalue and eigenvector sensitivities can be obtained simultaneously from one equation. Chen and Tan [95] presented a new method to compute the eigensolution variability of asymmetric damped systems. In order to reduce the condition number of the coefficient matrix, some weight constants were introduced. The method is well-conditioned since elements of the coefficient matrix are all of the same order of magnitude and several special cases can be presented based on the similar idea of the proposed method.

2.3 Energy methods

In general, Finite Element Method is used to predict vibration levels of structures at low frequency. However, as the frequency increases, this method needs higher order shape functions or more numbers of elements [96]. Consequently, this leads to a time consuming and unreliable results. At higher frequencies, the response of the system becomes sensitive to small details in its construction. Various Energy-based approaches have been presented in order to overcome this limitation of frequencies. One of these methods is Statistical Energy Analysis (SEA), which has been developed by Lyon and Dejong [97]. Another analysis method for high frequency is Energy Flow Analysis (EFA), which was introduced by Belov et al [98]. Nefske and Sung [99] developed a finite element method to predict the energy flow and the vibrational response of the Euler–Bernoulli beam. Wohlever and Bernhard [100] established the energy flow model for the rod and Euler– Bernoulli beam. The Energy Finite Element Analysis (EFEA) has been developed for high-frequency structural simulations [101,102] and constitutes an alternative formulation to the established SEA method. Cho and Bernhard [103] used Energy Finite Element Analysis formulation to predict the frequency-averaged vibrational response of a frame structure with a three-dimensional joint.

The proposed method which is used in this thesis is concerned with distribution of potential and kinetic energy in all elements of the structure. Accordingly, elements which is needed modifications can be easily determined. Then, through reanalysis procedures one can predict and develop the behavior of the structure. The methodology of this method has been proposed by Ki, I. K. [121], and later developed by Maneski, T [104]. This methodology was applied numerically by Trisovic,N. [105] to investigate the dynamic behavior of some real structures using procedures of re-analysis. Trisovic, N. etc [106,107] extended this work by applying it to complex real structures. The obtained results were very useful.

Chapter 3

FINITE ELEMENT METHOD FOR DYNAMIC PROBLEMS

The first step in the analysis of any structural vibration problem is the formulation of the equations of motion. If the structure has a simple geometric shape, such as uniform axial, torque and beam elements, then a partial differential equation of motion can be used to describe its dynamics behavior. Complex structures, however, consist of an assemblage of components of different types (beams, plates, shells and solids). Furthermore, in many real structures, the shape of the boundaries cannot be described in terms of known functions. Therefore, it is impossible to obtain analytical solutions, which describe the dynamics behavior of these complex structures. Finite element method is a powerful technique which used to solve real complex structures problems. The principal advantage of the finite element method is its generality; it can be used to calculate the natural frequencies and mode shapes of any linear elastic system.

3.1 Dynamic Equation of motion

In dynamic problems the displacements, velocities, strains, stresses, and loads are all time dependent. The procedure involved in deriving the finite element equation of dynamic problems can be stated by the following steps [108]:

Step 1: dividing the structure into E finite elements.

Step 2: assume the displacement model of element e as:

$$\overrightarrow{U}(x,y,z,t) = \begin{Bmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \end{Bmatrix} = [N(x,y,z)]\vec{Q}^{(e)}(t) \qquad (3.1)$$

Where:

$\overrightarrow{U}(x,y,z,t)$: is the vector of displacement.

$[N(x,y,z)]$: is the matrix of the shape functions.

$\vec{Q}^{(e)}(t)$: is the vector of nodal displacements that assumed to be a function of time (t).

Step 3: Derive the stiffness and mass matrices and load vector.

From Eq.(3.1), the strains can be expressed in form

$$\vec{\varepsilon} = [B]\vec{Q}^{(e)} \tag{3.2}$$

and the stresses as

$$\vec{\sigma} = [D]\vec{\varepsilon} = [D][B]\vec{Q}^{(e)} \tag{3.3}$$

Where:

[B]: is the matrix relating strains and nodal displacements, and

[D]: is the elasticity matrix

Differentiating equation (3.1) with respect to time leads to obtain the velocity as:

$$\dot{\vec{U}}(x, y, z, t) = [N(x, y, z)]\dot{\vec{Q}}^{(e)}(t) \tag{3.4}$$

Where:

$\dot{\vec{Q}}^{(e)}(t)$: is the vector of nodal velocity.

The dynamic equations of motion of a structure can be derived by applying Lagrange equations as following:

$$\frac{d}{dt}\left\{\frac{\partial L}{\partial \dot{Q}}\right\} - \left\{\frac{\partial L}{\partial Q}\right\} + \left\{\frac{\partial R}{\partial \dot{Q}}\right\} = \{0\} \tag{3.5}$$

Where:

$$L = E_k - E_p \tag{3.6}$$

is called the Lagrangian function. E_k and E_p are the kinetic and potential energy of the system respectively.

R : is the dissipation function.

Q: is the nodal displacement.

$\dot{Q}$: is the nodal velocity.

The kinetic and potential energies of an element “e” can be expressed, respectively, as

$$(e_k)_e = \frac{1}{2}\iiint_{V(e)} \rho \left\{\dot{\vec{U}}\right\}^T \left\{\dot{\vec{U}}\right\} dV \tag{3.7}$$

and

$$\left(e_p\right)_e = \frac{1}{2}\iiint_{V(e)} \{\vec{\varepsilon}\}^T \{\vec{\sigma}\} dV - \iint_{S_1^{(e)}} \left\{\vec{U}\right\}^T \left\{\vec{\overline{\Phi}}\right\} dS_1 - \iiint_{V(e)} \left\{\vec{U}\right\}^T \left\{\vec{\overline{\emptyset}}\right\} dV \tag{3.8}$$

Where

$V\,(e)$ is the volume, ρ is the density, and $\dot{\vec{U}}$ is the vector of velocities of the element *e*.

The dissipation function $R^{(e)}$ of the element *e* can be expressed as:

$$R^{(e)} = \frac{1}{2}\iiint_{V(e)} \mu \left\{\dot{\vec{U}}\right\}^T \left\{\dot{\vec{U}}\right\} dV \tag{3.9}$$

Where μ is called the damping coefficient. In equations (3.7) – (3.9) the volume integral has to be taken over the volume of the element. And in Eq. (3.8) the surface integral has to be taken over that portion of the surface of the element on which distributed surface forces are prescribed. By using Equations (3.1)-(3.3), the expressions for E_k, E_p and R can be written as

$$E_k = \sum_{e=1}^{E} (e_k)_e = \frac{1}{2}\left\{\underset{\sim}{\dot{\vec{Q}}}\right\}^T \left[\sum_{e=1}^{E} \iiint_{V^{(e)}} \rho [N]^T [N] dV\right] \left\{\underset{\sim}{\dot{\vec{Q}}}\right\} \tag{3.10}$$

$$\begin{aligned} E_p = \sum_{e=1}^{E} \left(e_p\right)_e &= \frac{1}{2}\left\{\underset{\sim}{\vec{Q}}\right\}^T \left[\sum_{e=1}^{E} \iiint_{V^{(e)}} [B]^T [D][B] dV\right] \left\{\underset{\sim}{\vec{Q}}\right\} \\ &- \left\{\underset{\sim}{\vec{Q}}\right\}^T \left(\sum_{e=1}^{E} \iint_{S_1^{(e)}} [N]^T \vec{\overline{\Phi}}(t) dS_1 + \iiint_{V^{(e)}} [N]^T \vec{\overline{\phi}}(t) dV\right) - \left\{\underset{\sim}{\vec{Q}}\right\}^T \left\{\underset{\sim}{\overrightarrow{P_c}}\right\}(t) \end{aligned} \tag{3.11}$$

$$R = \sum_{e=1}^{E} (R)_e = \frac{1}{2} \left\{ \underset{\sim}{\dot{\vec{Q}}} \right\}^T \left[\sum_{e=1}^{E} \iiint_{V^{(e)}} \mu [N]^T [N] dV \right] \left\{ \underset{\sim}{\dot{\vec{Q}}} \right\} \qquad (3.12)$$

Where

$\left\{ \underset{\sim}{\vec{Q}} \right\}$ is the global nodal displacement vector

$\left\{ \underset{\sim}{\dot{\vec{Q}}} \right\}$ is the global nodal velocity vector.

$\left\{ \underset{\sim}{\overrightarrow{P_c}} \right\}$..is the vector of concentrated nodal forces of the structure or body.

From equations (3.10) – (3.12) the next expressions can be defined as:

$[M]_e$ = element mass matrix

$$= \iiint_{V^{(e)}} \rho [N]^T [N] dV \qquad (3.13)$$

$[K]_e$ = element stiffness matrix

$$= \iiint_{V^{(e)}} [B]^T [D][B] dV \qquad (3.14)$$

$[C]_e$ = element damping matrix

$$= \iiint_{V^{(e)}} \mu [N]^T [N] dV \qquad (3.15)$$

$[P_s]_e$ = vector of element nodal forces produced by surface forces

$$= \iint_{S_1^e} [N]^T \overrightarrow{\overline{\Phi}} \, dS_1 \qquad (3.16)$$

$[P_b]_e$ = vector of element nodal forces produced by body forces

$$= \iiint_{V^{(e)}} [N]^T \overset{\rightrightarrows}{\phi}\, dV \tag{3.17}$$

Step 4: Derive the equations of motion of the whole system by assembling the elements matrices and vectors of equations (3.13)-(3.17). Thus equations (3.10)-(3.12) can be written as:

$$E_k = \frac{1}{2}\left\{\underset{\sim}{\dot{\vec{Q}}}\right\}^T [M]\left\{\underset{\sim}{\dot{\vec{Q}}}\right\} \tag{3.18}$$

$$E_p = \frac{1}{2}\left\{\underset{\sim}{\vec{Q}}\right\}^T [K]\left\{\underset{\sim}{\vec{Q}}\right\} - \left\{\underset{\sim}{\vec{Q}}\right\}^T \left\{\underset{\sim}{\vec{P}}\right\} \tag{3.19}$$

$$R = \frac{1}{2}\left\{\underset{\sim}{\vec{Q}}\right\}^T [K]\left\{\underset{\sim}{\vec{Q}}\right\} - \left\{\underset{\sim}{\vec{Q}}\right\}^T \left\{\underset{\sim}{\overrightarrow{P_c}}\right\} \tag{3.20}$$

Where:

$[M] =$ master (global) mass matrix of the structure $= \sum_{e=1}^{E} [m]_e$,

$[K] =$ master (global) stiffness matrix of the structure $= \sum_{e=1}^{E} [k]_e$,

$[C] =$ master (global) damping matrix of the structure $= \sum_{e=1}^{E} [c]_e$

$\left\{\underset{\sim}{\vec{P}}\ (t)\right\}$ = global load vector of the structure $= \sum_{e=1}^{E} (P_s^{(e)}(t) + P_b^{(e)}(t) + \underset{\sim}{\vec{P_c}}(t)$

The desired dynamic equations of motion of the structure can be obtained by substituting equations (3.18)-(3.20) into equation (3.5) as:

$$[M]\left\{\underset{\sim}{\ddot{Q}}\,(t)\right\} + [C]\left\{\underset{\sim}{\dot{Q}}\,(t)\right\} + [K]\left\{\underset{\sim}{Q}(t)\right\} = \{\underset{\sim}{P}(t)\} \tag{3.21}$$

Where:

$\left\{ \underset{\sim}{\ddot{Q}}\,(t) \right\}$ is the vector of nodal acceleration in the global system.

When damping is not taken into account the equations of motion become as

$$[M]\left\{ \underset{\sim}{\ddot{Q}}\,(t) \right\} + [K]\left\{ \underset{\sim}{Q}(t) \right\} = \{\underset{\sim}{P}(t)\} \tag{3.22}$$

Step 5and 6: Appling the boundary and initial conditions to solve the system equations of motion. Equations (3.21) or (3.22) can be solved numerically.

3.2 Consistent and Lumped Mass Matrices

Equation (3.13) for the mass matrix was first derived by Archer [109] and is called *the consistent mass matrix* of the element. It is called consistent because the same displacement model that is used for deriving the element stiffness matrix is used for the derivation of mass matrix. It is of interest to note that several dynamic problems have been and are being solved with simpler forms of mass matrices. The simplest form of mass matrix that can be used is that obtained by placing point (concentrated) masses m_i at node points i in the directions of the assumed displacement degrees of freedom. The concentrated masses refer to translational and rotational inertia of the element and are calculated by assuming that the material within the mean locations on either side of the particular displacement behaves like a rigid body while the remainder of the element does not participate in the motion. Thus, this assumption excludes the dynamic coupling that exists between the element displacements, and hence the resulting element mass matrix is purely diagonal and is called *the lumped mass matrix*. The lumped mass matrices will lead to nearly exact results if small but massive objects are placed at the nodes of a lightweight structure. The consistent mass matrices will be exact if the actual deformed shape (under dynamic conditions) is contained in the displacement shape functions [N]. Since the deformed shape under dynamic conditions is not known, frequently the static displacement distribution is used for [N]. Hence, the resulting mass distribution will only be approximate; however, the accuracy is generally adequate for most practical purposes. Since lumped element matrices are diagonal, the assembled or overall mass matrix of the structure requires less storage space than the consistent mass matrix. Moreover, the diagonal lumped mass matrices greatly facilitate the desired computations [108].

3.3 Stiffness matrix and Mass matrix of some structural element

In order to do dynamic finite element analysis, one needs a mass matrix to pair with the stiffness matrix. The mass and stiffness matrices of an individual element, which is a simple, homogeneous element, can be obtained easily. The global mass and stiffness matrices of the structure can be assembled using these element matrices by considering connectivity and all the boundary conditions. Once the mass and stiffness matrices have been constructed, the equations of motion can be solved to obtain a description of the dynamic behavior of the structure.

3.3.1 Stiffness and Mass matrix of bar element

Consider a uniform elastic bar of length *l*, figure 3.1, with elastic modulus *E* and cross sectional area *A*.

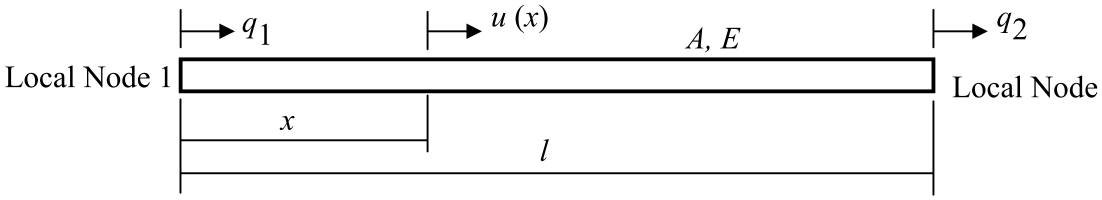


Figure 3.1 two nodes bar element

For a linear displacement model

$$u(x) = [N]\vec{q}^{(e)} \tag{3.23}$$

where:

$$[N] = \left[\left(1 - \frac{x}{l}\right) \cdot \left(\frac{x}{l}\right)\right] \tag{3.24}$$

$$\vec{q}^{(e)} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}^{(e)} \tag{3.25}$$

Where q_1 and q_2 represent the nodal degrees of freedom in the local coordinate system (unknowns) and the superscript e denotes the element number. The axial strain can be expressed as

$$\varepsilon_{xx} = \frac{\partial u(x)}{\partial x} = \frac{q_2 - q_1}{l} \tag{3.26}$$

or

$$\{\varepsilon_{xx}\} = [B]\, \vec{q}^{(e)} \tag{3.27}$$

where

$$[B] = \begin{bmatrix} -\frac{1}{l} & \frac{1}{l} \end{bmatrix} \tag{3.28}$$

The stiffness matrix of the element (in the local coordinate system) can be obtained, from Eq. (3.14) for [*D*] = [*E*], as

$$[k^{(e)}] = \iiint\limits_{V^{(e)}} [B]^T[D][B]dV = A \int\limits_{x=0}^{l} \begin{Bmatrix} -\frac{1}{l} \\ \frac{1}{l} \end{Bmatrix} E \begin{Bmatrix} -\frac{1}{l} & \frac{1}{l} \end{Bmatrix} dx = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{3.29}$$

Similarly, from Eq. (3.13), the consistent mass matrix of the element is

$$[m^{(e)}] = \iiint\limits_{V^{(e)}} \rho[N]^T[N]dV = \frac{\rho A l}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{3.30}$$

In general, the consistent mass matrices are fully populated. While, the lumped mass matrix of the element is a diagonal matrix, and can be obtained (by dividing the total mass of the element equally between the two nodes) as

$$[m^{(e)}] = \frac{\rho A l}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.31}$$

3.3.2 Stiffness and Mass matrix of beam element

Consider a beam element of length l in the xy plane as shown in Figure 3.2. The four degrees of freedom in the local (xy) coordinate system are indicated as q_1, q_2, q_3, and q_4. Because there are four nodal displacements, we assume a cubic displacement model for $v(x)$ as (Figure 3.2)

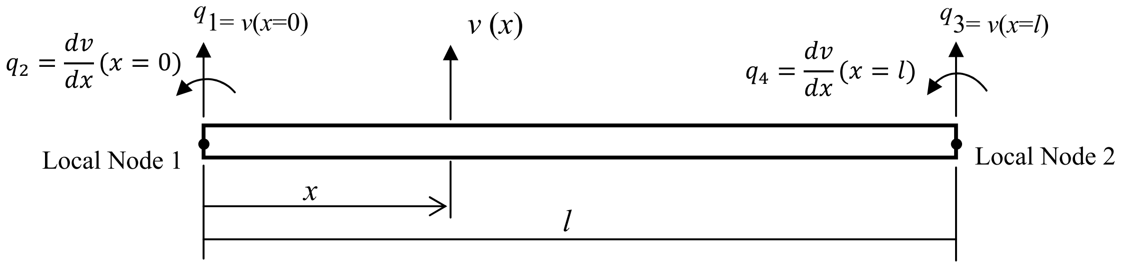


Figure 3.2 two nodes beam element

$$v(x) = \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \alpha_4 x^3 \tag{3.32}$$

where the constants α_1 - α_4 can be found by using the conditions

$$v(x) = q_1 \quad and \quad \frac{dv}{dx} = q_2 \quad at \quad x = 0$$

and

$$v(x) = q_3 \quad and \quad \frac{dv}{dx} = q_4 \quad at \quad x = l$$

Equation (3.32) can thus be rewritten as

$$v(x) = [N]\vec{q}^{(e)} \tag{3.33}$$

Where $[N]$ is given by

$$[N] = [N_1 \quad N_2 \quad N_3 \quad N_4]$$

with

$$N_1(x) = \frac{2x^3 - 3lx^2 + l^3}{l^3}$$

$$N_2(x) = \frac{x^3 - 2lx^2 + l^2x}{l^2}$$

$$N_3(x) = -\frac{2x^3 - 3lx^2}{l^3}$$

$$N_4(x) = \frac{x^3 - lx^2}{l^2} \tag{3.34}$$

and

$$\vec{q} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} \tag{3.35}$$

Based on simple beam theory, plane sections of the beam remain plane after deformation and hence the axial displacement *u* due to the transverse displacement *v* can be expressed as (Figure 3.3) [108]

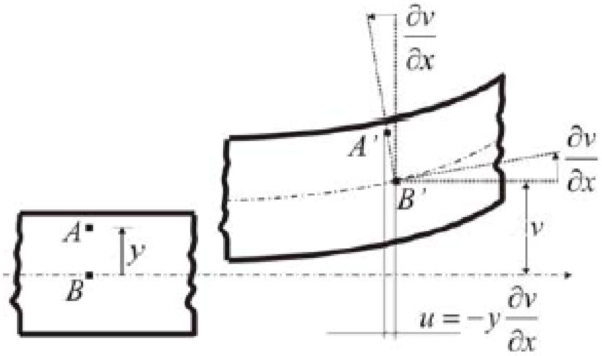


Figure 3.3 Deformation of an Element of Beam in *xy* Plane

$$u = -y\frac{\partial v}{\partial x}$$

where *y* is the distance from tile neutral axis. Thus, the axial strain is

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -y\frac{\partial^2 v}{\partial x^2} = [B]\vec{q} \tag{3.36}$$

where the matrix [B] is

$$[B] = -\frac{y}{l^3}\{(12x - 6l) \quad l(6x - 4l) \quad -(12x - 6l) \quad l(6x - 2l)\} \tag{3.37}$$

The stiffness matrix of the element can be obtained, from Eq. (3.14) for [*D*] = [*E*], as

$$\left[k^{(e)}\right] = \iiint\limits_{V^{(e)}} [B]^T[D][B]dV = E\int\limits_{x=0}^{l} dx \iint\limits_{A} [B]^T[B]dA$$

$$= \frac{EI_z}{l^3}\begin{array}{c c} \begin{array}{cccc} q_1 & q_2 & q_3 & q_4 \end{array} & \\ \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} & \begin{array}{c} q_1 \\ q_2 \\ q_3 \\ q_4 \end{array} \end{array} \tag{3.38}$$

where $I_z = \int\int_A y^2 dA$ is the area moment of inertia of the cross section about the z axis. The consistent mass matrix of the beam element is

$$\left[m^{(e)}\right] = \iiint\limits_{V^{(e)}} \rho[N]^T[N]dV = \frac{\rho Al}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{3.39}$$

Also, the lumped mass matrix for the beam element is

$$\left[m^{(e)}\right] = diag\,\frac{\rho Al}{2}[1 \quad 0 \quad 1 \quad 0] \tag{3.40}$$

3.3.3 Stiffness and Mass matrix of space frame element

A space frame element is a straight bar of uniform cross section that is capable of resisting axial forces, bending moments about the two principal axes in the plane of its cross section, and twisting moment about its centroidal axis. The corresponding displacement degrees of freedom are shown in Figure 3.3.

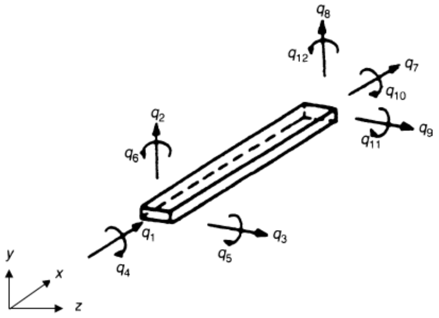


Figure 3.4 Space frame element with 12 degrees of freedom

In order to obtain the stiffness matrix and mass matrix of space frame element, the displacements can be separated into four groups, each of which can be considered independently of the others. The stiffness matrix and mass matrix of each independent set of displacement can be obtained separately and then the total stiffness matrix and mass matrix of the element can be obtained by superposition.

3.3.3.1 Axial displacement

Similar to steps which were done in section 3.3.1, for bar element, the stiffness matrix and consistent mass matrix of the space frame element, corresponding to the axial displacement, are, respectively:

$$[k^{(e)}] = \iiint\limits_{V^{(e)}} [B]^T[D][B]dV$$

$$= \frac{AE}{l}\begin{array}{cc} \begin{array}{cc} q_1 & q_7 \end{array} & \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & \begin{array}{c} q_1 \\ q_7 \end{array} \end{array} \tag{3.41}$$

$$[m^{(e)}] = \iiint_{V^{(e)}} \rho [N]^T [N] dV = \frac{\rho A l}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{3.42}$$

3.3.3.2 Torsional displacements

Figure 3.4(b) shows the torsional degrees of freedom for the space frame element. For the linear variation of the torsional displacement or twist angle, the displacement model can be expressed as:

$$\theta(x) = [N]\, \vec{q}_t \tag{3.43}$$

where

$$[N] = \left[\left(1 - \frac{x}{l}\right) \left(\frac{x}{l}\right) \right] \tag{3.44}$$

and

$$\vec{q}_t = \begin{Bmatrix} q_4 \\ q_{10} \end{Bmatrix} \tag{3.45}$$

The shear strain in the element, considering a circular cross section of the frame, is:

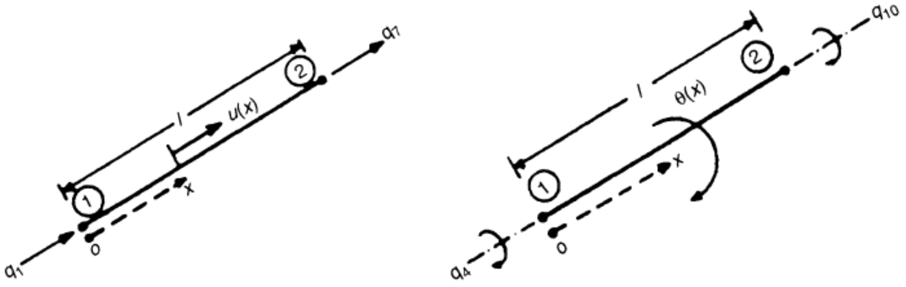
$$\varepsilon_{\theta x} = r \frac{d\theta}{dx} \tag{3.46}$$

where r is the distance of the fiber from the centroidal axis of the element.

Thus,

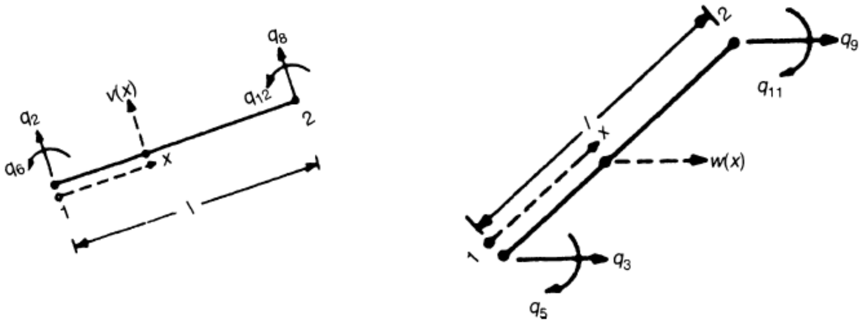
$$\{\varepsilon_{\theta x}\} = [B]\, \vec{q} \tag{3.47}$$

where $[B] = \left[-\frac{r}{l} \quad \frac{r}{l} \right]$



(a) Axial degrees of frrdom

(b) Torsional degrees of freedom



(c) Bending degrees of freedom in *xy* plane

(d) Bending degrees of freedom in *xz* plane

Figure 3.5 Degrees of freedom of a space frame element

Consequently; for [D] = [G], G is the shear modulus of material, the stiffness matrix of the element corresponding to torsional displacement degrees of freedom can be derived as:

$$\left[k_t^{(e)}\right] = \iiint\limits_{V^{(e)}} [B]^T[D][B]dV$$

$$= G \int\limits_{x=0}^{l} dx \iint_A r^2\, dA \begin{Bmatrix} -\dfrac{1}{l} \\ \dfrac{1}{l} \end{Bmatrix} \begin{Bmatrix} -\dfrac{1}{l} & \dfrac{1}{l} \end{Bmatrix}$$

$$= \frac{GJ}{l} \begin{array}{cc} \begin{array}{cc} q_4 & q_{10} \end{array} & \\ \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} & \begin{array}{c} q_4 \\ q_{10} \end{array} \end{array} \tag{3.48}$$

where $J = \int\int_A r^2 dA$ is the polar moment of inertia of the cross section, and $\frac{GJ}{l}$. is the torsional stiffness of the frame element.

The consistent mass matrix of the element corresponding to torsional displacement degrees of freedom is

$$[m^{(e)}] = \iiint\limits_{V^{(e)}} \rho[N]^T[N]dV = \frac{\rho J l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{3.49}$$

3.3.3.3 Bending displacements in the Plane xy

Similar to the section 3.3.2, the stiffness matrix and consistent mass matrix of the space frame element, corresponding to the Bending displacement in the xy plane (figure 3.4(c)), are, respectively:

$$\left[k_{xy}^{(e)}\right] = \iiint\limits_{V^{(e)}} [B]^T[D][B]dV$$

$$= \frac{EI_z}{l^3}\begin{array}{c} \\ \left[\begin{array}{cccc} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{array}\right] \end{array}\!\!\begin{array}{c} \\ q_2 \\ q_6 \\ q_8 \\ q_{12} \end{array}$$

$$\begin{array}{cccc} q_2 & q_6 & q_8 & q_{12} \end{array} \tag{3.50}$$

$$\left[m_{xy}^{(e)}\right] = \iiint\limits_{V^{(e)}} \rho[N]^T[N]dV = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{3.51}$$

3.3.3.4 Bending displacements in the Plane xz

By replacing the degrees of freedom q_3, q_5, q_9, and q_{11} instead of q_2, q_6, q_8, and q_{12} in the previous section, the stiffness matrix and consistent mass matrix of the space frame element, corresponding to the bending displacement in the xz plane (figure 3.4(d)), can be obtained as:

$$\left[k_{xz}^{(e)}\right] = \frac{EI_y}{l^3}\begin{array}{c} \begin{array}{cccc} q_3 & q_5 & q_9 & q_{11} \end{array} \\ \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \end{array} \begin{array}{c} \\ q_3 \\ q_5 \\ q_9 \\ q_{11} \end{array} \tag{3.52}$$

$$\left[m_{xz}^{(e)}\right] = \iiint\limits_{V^{(e)}} \rho[N]^T[N]dV = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{3.53}$$

Now, the complete stiffness matrix and complete mass matrix of the space frame can be obtained (by superposition), respectively, as:

$$[k^{(e)}]\begin{bmatrix}
\frac{EA}{l} & & & s & & & & & & & & \\
0 & \frac{12EI_z}{l^3} & & & y & & & & & & & \\
0 & 0 & \frac{12EI_y}{l^3} & & & m & & & & & & \\
0 & 0 & 0 & \frac{GJ}{l} & & & m & & & & & \\
0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & & & e & & & & \\
0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l} & & & t & & & \\
-\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{l} & & & r & & \\
0 & -\frac{12EI_z}{l^3} & 0 & 0 & 0 & -\frac{6EI_z}{l^2} & 0 & \frac{12EI_z}{l^3} & & & i & \\
0 & 0 & \frac{12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} & 0 & 0 & 0 & \frac{12EI_y}{l^3} & & & c \\
0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l} & & \\
0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & \\
0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{2EI_z}{l} & 0 & -\frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l}
\end{bmatrix} \tag{3.54}$$

$$[m^{(e)}] = \rho AL\begin{bmatrix}
\frac{1}{3} & & & s & & & & & & & & \\
0 & \frac{13}{35} & & & y & & & & & & & \\
0 & 0 & \frac{13}{35} & & & m & & & & & & \\
0 & 0 & 0 & \frac{J}{3A} & & & m & & & & & \\
0 & 0 & -\frac{11l}{210} & 0 & \frac{l^2}{105} & & & e & & & & \\
0 & \frac{11l}{210} & 0 & 0 & 0 & \frac{l^2}{105} & & & t & & & \\
\frac{1}{6} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & & & r & & \\
0 & \frac{9}{70} & 0 & 0 & 0 & \frac{13l}{420} & 0 & \frac{13}{35} & & & i & \\
0 & 0 & \frac{9}{70} & 0 & -\frac{13l}{420} & 0 & 0 & 0 & \frac{13}{35} & & & c \\
0 & 0 & 0 & \frac{J}{6A} & 0 & 0 & 0 & 0 & 0 & \frac{J}{3A} & & \\
0 & 0 & \frac{13l}{420} & 0 & -\frac{l^2}{140} & 0 & 0 & 0 & \frac{11l}{210} & 0 & \frac{l^2}{105} & \\
0 & -\frac{13l}{420} & 0 & 0 & 0 & -\frac{l^2}{140} & 0 & -\frac{11l}{210} & 0 & 0 & 0 & \frac{l^2}{105}
\end{bmatrix} \tag{3.55}$$

For planer frame analysis, the element is assumed to lie in the XZ plane, the stiffness matrix and mass matrix can be determined respectively, as

$$[k^{(e)}] = \frac{EI_y}{l^3}\begin{bmatrix} \frac{Al^2}{I_y} & & & & & \\ 0 & 12 & & symmitric & & \\ 0 & 6l & 4l^2 & & & \\ -\frac{Al^2}{I_y} & 0 & 0 & \frac{Al^2}{I_y} & & \\ 0 & -12 & -6l & 0 & 12 & \\ 0 & 6l & 2l^2 & 0 & -6l & 4l^2 \end{bmatrix} \tag{3.56}$$

$$[m^{(e)}] = \rho AL\begin{bmatrix} \frac{1}{3} & & & & & \\ 0 & \frac{13}{35} & & & & \\ 0 & \frac{11l}{210} & \frac{l^2}{105} & & & \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & & \\ 0 & \frac{9}{70} & \frac{13l}{420} & 0 & \frac{13}{35} & \\ 0 & -\frac{13l}{420} & -\frac{l^2}{140} & 0 & -\frac{11l}{210} & \frac{l^2}{105} \end{bmatrix} \tag{3.57}$$

3.3.4 Stiffness and Mass matrix of Triangular Membrane element

Consider a triangular membrane element which lies in the xy plane of a local xy coordinate system as shown in Figure 4.5. For a linear displacement variation inside the element, the displacement model can be expressed as:

$$u(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y \tag{3.58}$$

$$v(x,y) = \alpha_4 + \alpha_5 x + \alpha_6 y \tag{3.59}$$

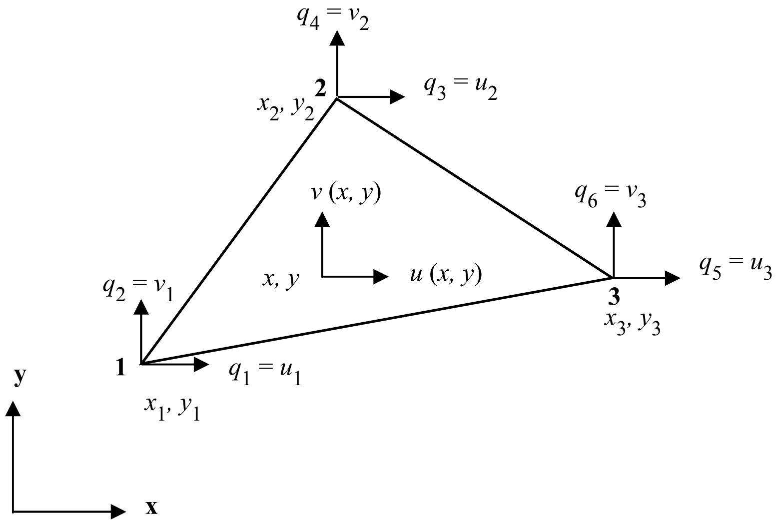


Figure 3.6 Degrees of freedom of Triangular Membrane element

Thus, the displacement model is;

$$\vec{U} = \begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix} = [N]\vec{q}^{(e)} \tag{3.60}$$

where

$$[N(x,y)] = \begin{bmatrix} N_1(x,y) & 0 & N_2(x,y) & 0 & N_3(x,y) & 0 \\ 0 & N_1(x,y) & 0 & N_2(x,y) & 0 & N_3(x,y) \end{bmatrix} \tag{3.61}$$

$$\left.\begin{aligned} N_1(x,y) &= \frac{1}{2A}[y_{32}(x-x_2) - x_{32}(y-y_2)) \\ N_2(x,y) &= \frac{1}{2A}[-y_{31}(x-x_3) - x_{31}(y-y_3)) \\ N_3(x,y) &= \frac{1}{2A}[y_{21}(x-x_1) - x_{21}(y-y_1)) \end{aligned}\right\} \tag{3.62}$$

A is the area of triangle

$$A = \frac{1}{2}(x_{32}y_{21} - x_{21}y_{32}) \tag{3.63}$$

$$\left.\begin{matrix} x_{ij} = x_i - x_j \\ x_{ij} = y_i - y_j \end{matrix}\right\} \tag{3.64}$$

$$\vec{q}^{(e)} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix}^{(e)} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}^{(e)} \tag{3.65}$$

By using the relations

$$\vec{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial_u}{\partial_x} \\ \frac{\partial_v}{\partial_y} \\ \frac{\partial_u}{\partial_y} + \frac{\partial_v}{\partial_x} \end{Bmatrix} = [B]\,\vec{q}^{(e)} \tag{3.66}$$

where

$$[B] = \frac{1}{2A}\begin{bmatrix} y_{32} & 0 & -y_{31} & 0 & y_{21} & 0 \\ 0 & -x_{32} & 0 & x_{31} & 0 & -x_{21} \\ -x_{32} & y_{32} & x_{31} & -y_{31} & -x_{21} & y_{21} \end{bmatrix} \tag{3.66}$$

For a constant plate thickness (t), the stiffness matrix of the triangular membrane element can be obtained using Eq. (3.14) as following;

$$\left[k^{(e)}\right] = \iiint\limits_{V^{(e)}} [B]^T[D][B]dV = [B]^T[D][B]t \iint\limits_{A} dA = tA[B]^T[D][B] \tag{3.67}$$

where

$$[D] = \frac{E}{1 - v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1 - v}{2} \end{bmatrix} \tag{3.68}$$

Eq.(3.67) can be solved numerically. For simplicity, the matrix $\left[k^{(e)}\right]$ can be separated into two parts: one due to normal stresses $\left[k_n^{(e)}\right]$, and the other due to shear stresses $\left[k_s^{(e)}\right]$. Thus, the components of the matrices $\left[k_n^{(e)}\right]$ and $\left[k_s^{(e)}\right]$ are, respectively:

$$\left[k_n^{(e)}\right] = \begin{bmatrix} y_{32}^2 & & & & & \\ -\nu y_{32}x_{32} & x_{32}^2 & & symmetric & & \\ -y_{32}y_{31} & \nu x_{32}y_{31} & y_{31}^2 & & & \\ \nu y_{32}x_{31} & -x_{32}x_{31} & -\nu y_{31}x_{31} & x_{31}^2 & & \\ y_{32}y_{21} & -\nu x_{32}y_{21} & -y_{31}y_{21} & \nu x_{31}y_{21} & y_{21}^2 & \\ -\nu y_{32}x_{21} & x_{32}x_{21} & \nu y_{31}x_{21} & -x_{31}x_{21} & -\nu y_{21}x_{21} & x_{21}^2 \end{bmatrix} \tag{3.69}$$

$$\left[k_s^{(e)}\right] = \begin{bmatrix} x_{32}^2 & & & & & \\ -x_{32}y_{32} & y_{32}^2 & & symmetric & & \\ -x_{32}x_{31} & y_{32}x_{31} & x_{31}^2 & & & \\ x_{32}y_{31} & -y_{32}y_{31} & -x_{31}y_{31} & y_{31}^2 & & \\ x_{32}x_{21} & -y_{32}x_{21} & -x_{31}x_{21} & y_{31}x_{21} & x_{21}^2 & \\ -x_{32}y_{21} & y_{32}y_{21} & x_{31}y_{21} & -y_{31}y_{21} & -x_{21}y_{21} & y_{21}^2 \end{bmatrix} \tag{3.70}$$

Also, the consistent mass matrix of a triangular membrane element can be derived as

$$\left[m^{(e)}\right] = \iiint\limits_{V^{(e)}} \rho[N]^T[N]dV = \frac{\rho At}{12}\begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix} \tag{3.71}$$

and, the lumped mass matrix is

$$\left[m^{(e)}\right] = \frac{\rho At}{3} diag\,[1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] = \frac{\rho At}{3}\,\mathbf{I_6} \tag{3.72}$$

3.3.5 Stiffness and Mass matrix of a Plate Bending Element

For the triangular plate bending element shown in figure 3.5, the displacement model can be described as

$$\begin{aligned} w(x,y) &= \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 \\ &\quad + \alpha_7 x^3 + \alpha_8 (x^2 y + xy^2) + \alpha_9 y^3 = [\eta]\vec{\alpha} \end{aligned} \tag{3.73}$$

the constants $\alpha_1, \alpha_2 \ldots, \alpha_9$ can be determined from the nodal conditions as

$$\left.\begin{array}{l} w(x,y) = q_1 \ , \dfrac{\partial w}{\partial y}(x,y) = q_2 \ , -\dfrac{\partial w}{\partial x}(x,y) = q_3 \quad at \ (x_1,y_1) = (0,0) \\ w(x,y) = q_4 \ , \dfrac{\partial w}{\partial y}(x,y) = q_5 \ , -\dfrac{\partial w}{\partial x}(x,y) = q_6 \quad at \ (x_2,y_2) = (0,y_2) \\ w(x,y) = q_7 \ , \dfrac{\partial w}{\partial y}(x,y) = q_7 \ , -\dfrac{\partial w}{\partial x}(x,y) = q_9 \quad at \ (x_2,y_2) \end{array}\right\} \tag{3.74}$$

Thus,

$$\vec{q}^{(e)} = \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_9 \end{Bmatrix}^{(e)} = [\underset{\sim}{\eta}]\vec{\alpha} \tag{3.75}$$

where

$$[\underset{\sim}{\eta}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & y_2 & 0 & 0 & y_2^2 & 0 & 0 & y_2^3 \\ 0 & 0 & 1 & 0 & 0 & 2y_2 & 0 & 0 & 3y_2^2 \\ 0 & -1 & 0 & 0 & -y_2 & 0 & 0 & -y_2^2 & 0 \\ 1 & x_3 & y_3 & x_3^2 & x_3 y_3 & y_3^2 & x_3^3 & (x_3^2 y_3 + x_3 y_3^2) & y_3^3 \\ 0 & 0 & 1 & 0 & x_3 & 2y_3 & 0 & (2x_3 y_3 + x_3^2) & 3y_3^2 \\ 0 & -1 & 0 & -2x_3 & -y_3 & 0 & -3x_3^2 & (-3y_3^2 + 2x_3 y_3) & 0 \end{bmatrix} \tag{3.76}$$

For plate bending, the stress-strain relations are

$$\vec{\sigma} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} [D] \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}$$

$$\left.\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} \\ \varepsilon_{yy} &= \frac{\partial v}{\partial y} = -z\frac{\partial^2 w}{\partial y^2} \\ \varepsilon_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z\frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\} \tag{3.77}$$

By using Eqs. (3.73) and (3.75) ,Eq.(3.77) can be written in the form

$$\vec{\varepsilon} = \left[\underset{\sim}{B}\right]\vec{\alpha} = [B]\vec{q}^{(e)} \tag{3.78}$$

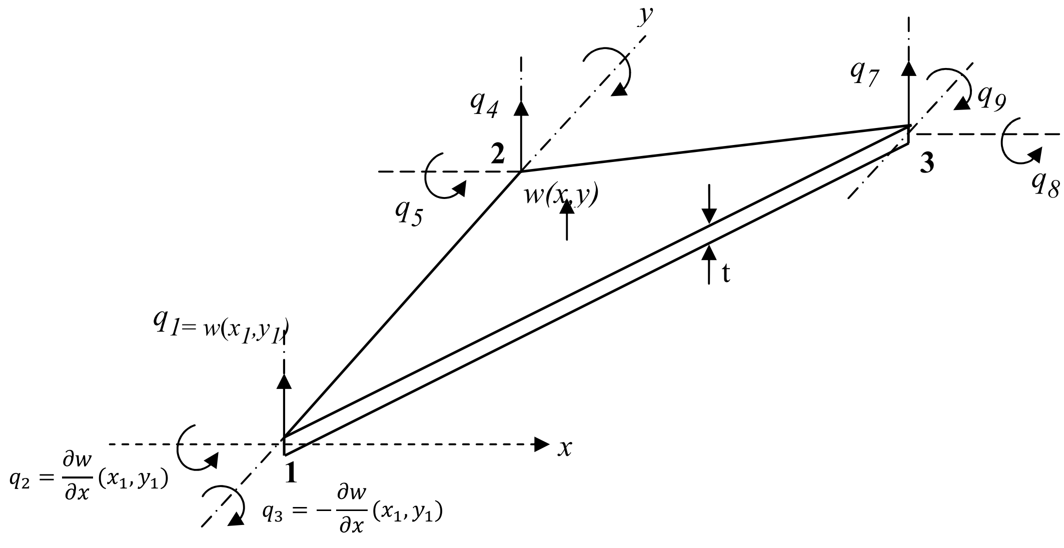


Figure 3.7 Degrees of freedom of Triangular plate bending element

For a constant plate thickness (t), the stiffness matrix of the triangular plate bending element can be obtained using Eq. (3.14) as

$$\left[k^{(e)}\right] = \iiint\limits_{V^{(e)}} [B]^T [D][B] dV$$

$$= \left([\underset{\sim}{\eta}]^{-1}\right)^T \left\{ \iint\limits_{area} dA \left(\int_{-t/2}^{t/2} \left[\underset{\sim}{B}\right]^T [D] \left[\underset{\sim}{B}\right] dz \right) \right\} [\underset{\sim}{\eta}]^{-1} \qquad (3.79)$$

where

$$\left[\underset{\sim}{B}\right] = -z \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 6x & 2y & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2x & 6y \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 4(x+y) & 0 \end{bmatrix} \qquad (3.80)$$

and

$$[B] = \left[\underset{\sim}{B}\right] [\underset{\sim}{\eta}]^{-1} \qquad (3.80)$$

and

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}$$

For the triangular bending element, the consistent mass matrix of a triangular membrane element can be derived as

$$w(x,y) = [\eta]\vec{\alpha} \qquad (3.81)$$

From Eq.(3.75)

$$\vec{\alpha} = [\underset{\sim}{\eta}]^{-1} \, \vec{q}^{(e)} \qquad (3.82)$$

Substituting Eq.(3.82) in to Eq.(3.81) leads to

$$w(x,y) = \left([\eta][\underset{\sim}{\eta}]^{-1}\right) \vec{q}^{(e)} \tag{3.83}$$

Based on the principle of thin plate theory, the displacements parallel to the undeformed middle surface are given by [1]

$$\left.\begin{aligned} u &= -z\,\frac{\partial w}{\partial x} \\ u &= -z\,\frac{\partial w}{\partial x} \end{aligned}\right\} \tag{3.84}$$

Therefore, using Eqs.(3.83) and (3.84), one can get

$$\vec{U}(x,y) = \begin{Bmatrix} u(x,y) \\ v(x,y) \\ w(x,t) \end{Bmatrix} = \begin{bmatrix} -z\,\dfrac{\partial[\eta]}{\partial x} \\ -z\,\dfrac{\partial[\eta]}{\partial y} \\ [\eta] \end{bmatrix} [\underset{\sim}{\eta}]^{-1}\,\vec{q}^{(e)} = [N_1][\underset{\sim}{\eta}]^{-1}\,\vec{q}^{(e)} \equiv [N]\vec{q}^{(e)} \tag{3.85}$$

where

$$[N_1] = \begin{bmatrix} 0 & -z & 0 & -2xz & -yz & 0 & -3x^2z & -z(y^2+2xy) & 0 \\ 0 & 0 & -z & 0 & -xz & -2yz & 0 & -z(2xy+x^2) & -3y^2z \\ 1 & x & y & x^2 & xy & y^2 & x^3 & (x^2y+xy^2) & y^3 \end{bmatrix} \tag{3.86}$$

Thus, the consistent mass matrix of the element can be obtained using Eq. (3.13) as

$$[m^{(e)}] = \iiint\limits_{V^{(e)}} \rho[N]^T[N]dV = \iiint\limits_{V^{(e)}} \rho\left([\underset{\sim}{\eta}]^{-1}\right)^T [N_1]^T[\underset{\sim}{\eta}]^{-1}dV \tag{3.87}$$

For more details of plate bending, and derivation of more stiffness and mass matrix of some structural different elements, one can see references [1],[3], and [108].

3.3.6 Transformation to global coordinates

The stiffness and mass matrices derived previously used a system of local coordinates as the reference plane. Transformation of coordinates to common global system is necessary to assemble the elements in global matrix.

If the element nodal displacements and nodal velocities are denoted as $\vec{Q}^{(e)}$ and $\dot{\vec{Q}}^{(e)}$ in the global system, the transformation relations can be written as

$$\vec{q}^{(e)} = [T]\vec{Q}^{(e)} \tag{3.88}$$

and

$$\dot{\vec{q}}^{(e)} = [T]\dot{\vec{Q}}^{(e)} \tag{3.89}$$

where *T* is the transformation matrix[108].

The kinetic energy associated with the motion of the element in the local coordinate system can be expressed as

$$(e_k)_e = \frac{1}{2}\dot{\vec{q}}^{(e)^T}\left[m^{(e)}\right]\dot{\vec{q}}^{(e)} \tag{3.90}$$

From Eqs.(3.89) and (3.90), one can get

$$(e_k)_e = \frac{1}{2}\dot{\vec{Q}}^{(e)T}[T]^T\left[m^{(e)}\right][T]\,\dot{\vec{Q}}^{(e)} \tag{3.91}$$

Also, the kinetic energy associated with the motion of the element in the global coordinate system can be expressed as

$$(E_k)_e = \frac{1}{2}\dot{\vec{Q}}^{(e)T}\left[M^{(e)}\right]\dot{\vec{Q}}^{(e)} \tag{3.92}$$

Since kinetic energy is a scalar quantity, it must be independent of the coordinate system.

Thus, the consistent mass matrix of the element in the global system can be obtained by equating Eqs.(3.91) and (3.92) as

$$\left[M^{(e)}\right] = [T]^T\left[m^{(e)}\right][T] \tag{3.93}$$

The previous procedure can be used to determine the stiffness matrix of the element in the global system [108].

3.4 Analysis of Free Vibration

When an elastic structure is disturbed initially at time t = 0, the structure can be made to oscillate harmonically. This oscillatory motion is a characteristic property of the structure and it depends on the distribution of mass and stiffness in the structure. If damping is present, the amplitudes of oscillations will decay progressively and if the magnitude of damping exceeds a certain critical value, the oscillatory character of the motion will cease altogether. On the other hand, if damping is absent, the oscillatory motion will continue indefinitely, with the amplitudes of oscillations depending on the initially imposed disturbance or displacement. The oscillatory motion occurs at certain frequencies known as natural frequencies or characteristic values, and it follows well defined deformation patterns known as mode shapes or characteristic modes. The study of such free vibrations (free because the structure vibrates with no external forces after t = 0) is very important in finding the dynamic response of the elastic structure.

For no external force (P=0), and harmonic motion, the displacements can be described as

$$\vec{Q} = \underline{\vec{Q}}.e^{i\omega t} \tag{3.94}$$

Then, Eq.(3.22) becomes

$$\left[[K] - \omega^2[M]\right]\underline{\vec{Q}} = \{0\} \tag{3.95}$$

where

$\underline{\vec{Q}}$ is the amplitudes of the displacements Q (the mode shape or eigenvector)

ω is the natural frequency of vibration .

The natural frequency is a very important characteristic of the structure carrying dynamic loads. It has been found that if a structure is excited by a load with a frequency of one of the structure's natural frequencies, the structure can undergo extremely violent vibration, which often leads to catastrophic failure of the structural system. Such a phenomenon is called resonance. Therefore, an eigenvalue analysis has to be performed in designing a structural system that is to be subjected to dynamic loadings.

Also analysis of the eigenvalue equation gives very important information on possible vibration modes experienced by the structure when it undergoes a vibration. Vibration modes of a structure are therefore another important characteristic of the structure. Mathematically, the eigenvectors can be used to construct the displacement fields. It has been found that using a few of the lowest modes can obtain very accurate results for many engineering problems. Modal analysis techniques have been developed to take advantage of these properties of natural modes.

Equation (3.95) is called a "linear" algebraic eigenvalue problem since neither [K] nor [M] is a function of the circular frequencyω, and it will have a nonzero solution for $\underline{\vec{Q}}$ provided that the determinant of the coefficient matrix $([K] - \omega^2[M])$ is zero that is.

$$|[K] - \omega^2[M]| = 0 \tag{3.96}$$

Two general types of methods, namely, transformation methods and iterative methods, are available for solving eigenvalue problems Eq. (3.96). The transformation methods such as Jacobi, Givens and Householder schemes are preferable, when all the eigenvalues and eigenvectors are required and the dimension of the eigenvalue problem is small. The iterative methods such as the power method, subspace iteration and Lanczos methods are preferable, when few eigenvalues and eigenvectors are required only and the eigenvalue problem has a large dimension. These methods are described in more details in [1], and [117].

In general, all the eigenvalues of Eq. (3.96) will be different, and hence the structure will have *n* different natural frequencies. Only for these natural frequencies, a nonzero solution can be obtained for $\underline{\vec{Q}}$ from Eq. (3.95).

Once the natural frequencies and its mode shapes have been determined, the dynamic behavior of the structure can be easily predictable. Accordingly, the dynamic structural modification can be done accurately.

Chapter 4

Procedures of Reanalysis Technique

4.1 Reanalysis Procedure

The procedures of reanalysis which are used in this thesis depend on the concept of energy distribution through the structure. Study of the energy distribution leads to find out the right place, which will be conducted by some modifications to improve the eigenvalues of the structure. Therefore, determination of distribution of kinetic and potential energies on the elements of whole structure is the main step in the reanalysis procedure. Complex structures need several steps during the analysis to reach the most accurate results. Starting with initial rough analysis of a structure which is followed by the precise analysis based on the sensitivity of each element of the structure. The improvement of dynamic characteristics, during the reanalysis steps, can be achieved by making some adjustment to the structure such as geometrical modifications, material properties and boundary conditions. The process of analysis is done using a computer program, based on the using of finite element methods and the implementation of structure energy distributions. The distributions of potential and kinetic energies of elements of the whole structure give a clear view to the problem, which help to make appropriate decision for structure modifications. The decision of the final modification can be made according to the structure dynamic behavior during reanalysis steps and its obtained results. Several studies have been addressed to the subject of modal reanalysis and structure dynamic modifications (see, e.g. [118]).

4.2 Potential and kinetic energy

Free vibration of systems involves the cyclic interchange of kinetic and potential energy [119]. In undamped free vibrating systems, no energy is dissipated or removed from the system. The kinetic energy K_E is stored in the mass by virtue of its velocity and the potential energy K_P is stored in the form of strain energy in elastic deformation. Since the total energy in the system is constant, the principle of conservation of mechanical energy applies. Since the mechanical energy is conserved, the sum of the kinetic energy and potential energy is constant and its rate of change is zero. This principle can be expressed as

$$K_E+K_P = \text{Constant} \tag{4.1}$$

Or $$\frac{d}{dt}\left(K_E + K_P\right) = 0 \tag{4.2}$$

The principle of conservation of energy can be restated by

$$K_E 1 + K_P 1 = K_E 2 + K_P 2 \tag{4.3}$$

Where the subscripts 1 and 2 denote two different instances of time when the mass is passing through its static equilibrium position and select $K_P 1 = 0$ as reference for the potential energy.

Subscript 2 indicates the time corresponding to the maximum displacement of the mass at this position, we have then $K_E = 0$, and

$$K_E 1 + 0 = 0 + K_P 2 \tag{4.4}$$

If the system is undergoing harmonic motion, then K_E 1 and K_P 2 denote the maximum values of K_E and K_P, respectively and therefore last equation becomes

$$K_E \max = K_P \max \tag{4.5}$$

It is quite useful in calculating the natural frequency directly.

4.3 Potential and kinetic energy distribution over the principal modes of oscillation

For the system with no damping and no external force, the equation of motion in the matrix form is:

$$[M]\cdot\{\ddot{Q}(t)\}+[K]\cdot\{Q(t)\}=\{0\} \tag{4.6}$$

Then, the eigenvalues of the previous differential equation for r-th mode can be expressed as:

$$[K]\cdot\{Q_r\}-\lambda_r[M]\cdot\{Q_r\}=\{0\} \tag{4.7}$$

Where λ_r - is the r-th eigenvalue, and Q_r - is the r-th eigenvector for the structure.
Now, by multiplying the left side of equation (4.7) by transposed value of *r*-th eigenvector and divided by 2 one can get:

$$\frac{1}{2}\{Q_r\}^T[K]\{Q_r\}=\frac{1}{2}\lambda_i\{Q_r\}^T[M]\cdot\{Q_r\} \tag{4.8}$$

Equation (4.8) is the balance equation of potential and kinetic energy for a structure in main modes of oscillation. Furthermore, the potential energy of a structure on *r*-th main oscillation mode, having in mind the previous equation, can be rewritten as:

$$E_{p,r}=\frac{1}{2}\{Q_r\}^T[K]\{Q_r\}. \tag{4.9}$$

In the same way, the kinetic energy is:

$$E_{k,r}=\frac{1}{2}\lambda_r\{Q_r\}^T[M]\{Q_r\}, \tag{4.10}$$

Theoretically, the total energy conservation on main oscillation modes is:

$$E_{p,r} = E_{k,r} = E_r . \tag{4.11}$$

The kinetic and potential energy of the structure on r-th main oscillation mode is the sum energy of all elements structure modeling and can be represented as:

$$E_{k,r} = \sum_{e=1}^{N} \left(e_{k,r}\right)_e = \frac{1}{2}\sum_{e=1}^{N} \omega_r^2 \left\{q_r^s\right\}_e^T [m]_e \left\{q_r^s\right\}_e$$

$$E_{p,r} = \sum_{e=1}^{N} \left(e_{p,r}\right)_e = \frac{1}{2}\sum_{e=1}^{N} \left\{q_r^s\right\}_e^T [k]_e \left\{q_r^s\right\}_e \tag{4.12}$$

Where are:

$\left(e_{p,r}\right)_e = \frac{1}{2}\left\{q_r^s\right\}_e^T [k]_e \left\{q_r^s\right\}_e$ - potential energy of *e*-th element on its r-th main oscillation mode,

$\left(e_{k,r}\right)_e = \frac{1}{2}\omega_r^2 \left\{q_r^s\right\}_e^T [m]_e \left\{q_r^s\right\}_e$ - kinetic energy of *e*-th element on r-th main oscillation mode,

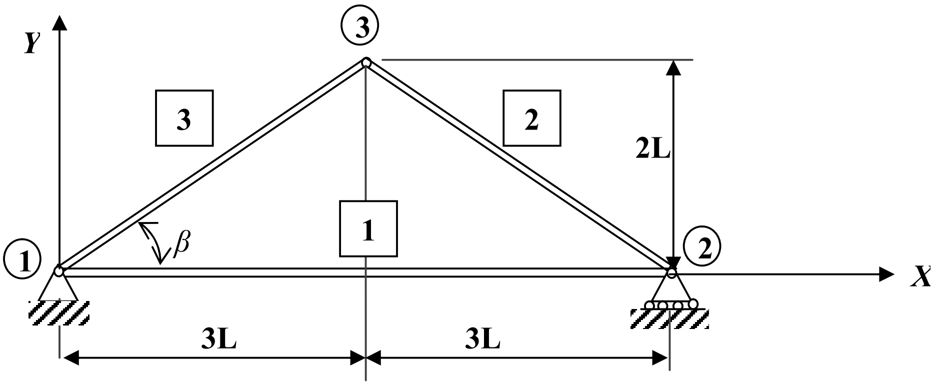
$\left\{q_r^s\right\}_e$ - is the corresponding r-th eigenvector, of *e*-th element with s degrees of freedom.

Consequently, the dynamic analysis can be done according to the difference between potential and kinetic energy distribution (e_p - e_k) through all structure's elements.

4.4 Analysis of the energy distribution for a simple structure

Truss composed of three connected rods

The following simple example is presented to illustrate the procedures of determine the distributions of potential and kinetic energies through all elements of the whole structure.



For plane truss, the transformed stiffness matrix and the transformed mass matrix are as following [120]:

$$[k_e] = \frac{AE}{L_e}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \tag{4.13}$$

$$[m_e] = \frac{\rho A L_e}{6}\begin{bmatrix} 2C^2 & 2CS & C^2 & CS \\ 2CS & 2S^2 & CS & S^2 \\ C^2 & CS & 2C^2 & 2CS \\ CS & S^2 & 2CS & 2S^2 \end{bmatrix} \tag{4.14}$$

For a uniform member, *A* and *E* are the area of the cross-section and the elastic modulus, respectively. In addition, L_e is a length of member and ρ is a density.

Element 1;

The orientated angle with respect to the horizontal axis X, θ =0 so that C=cos θ =1 and S=sin θ=0. The element stiffness matrix is:

$$[k_1] = \frac{A_1E_1}{6L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[k_1] = \frac{A_1E_1}{L}\begin{bmatrix} 0.1667 & 0 & -0.1667 & 0 \\ 0 & 0 & 0 & 0 \\ -0.1667 & 0 & 0.1667 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Let:

$$[k_{11}]_1 = [k_{22}]_1 = \frac{A_1E_1}{L}\begin{bmatrix} 0.1667 & 0 \\ 0 & 0 \end{bmatrix} , [k_{12}]_1 = [k_{21}]_1 = \frac{A_1E_1}{L}\begin{bmatrix} -0.1667 & 0 \\ 0 & 0 \end{bmatrix}$$

Element 2; θ =180- β. The element stiffness matrix is:

$$[k_2] = \frac{A_2E_2}{\sqrt{13}L}\begin{bmatrix} 0.6922 & -0.4615 & -0.6922 & 0.4615 \\ -0.4615 & 0.3076 & 0.4615 & -0.3076 \\ -0.6922 & 0.4615 & 0.6922 & -0.4615 \\ 0.4615 & -0.3076 & -0.4615 & 0.3076 \end{bmatrix}$$

$$[k_2] = \frac{A_2E_2}{L}\begin{bmatrix} 0.1920 & -0.1280 & -0.1920 & 0.1280 \\ -0.1280 & 0.0853 & 0.1280 & -0.0853 \\ -0.1920 & 0.1280 & 0.1920 & -0.1280 \\ 0.1280 & -0.0853 & -0.1280 & 0.0853 \end{bmatrix}$$

Let:

$$[k_{11}]_2 = [k_{22}]_2 = \frac{A_2E_2}{L}\begin{bmatrix} 0.1920 & -0.1280 \\ -0.1280 & 0.0853 \end{bmatrix},$$

$$[k_{12}]_2 = [k_{21}]_2 = \frac{A_2E_2}{L}\begin{bmatrix} -0.1920 & 0.1280 \\ 0.1280 & -0.0853 \end{bmatrix}$$

Element 3; *θ* = *β*. The element stiffness matrix is:

$$[k_3] = \frac{A_3E_3}{\sqrt{13}L}\begin{bmatrix} 0.6922 & 0.4615 & -0.6922 & -0.4615 \\ 0.4615 & 0.3076 & -0.4615 & -0.3076 \\ -0.6922 & -0.4615 & 0.6922 & 0.4615 \\ -0.4615 & -0.3076 & 0.4615 & 0.3076 \end{bmatrix}$$

$$[k_3] = \frac{A_3E_3}{L}\begin{bmatrix} 0.1920 & 0.1280 & -0.1920 & -0.1280 \\ 0.1280 & 0.0853 & -0.1280 & -0.0853 \\ -0.1920 & -0.1280 & 0.1920 & 0.1280 \\ -0.1280 & -0.0853 & 0.1280 & 0.0853 \end{bmatrix}$$

Let:

$$[k_{11}]_3 = [k_{22}]_3 = \frac{A_2E_2}{L}\begin{bmatrix} 0.1920 & 0.1280 \\ 0.1280 & 0.0853 \end{bmatrix},$$

$$[k_{12}]_3 = [k_{21}]_3 = \frac{A_2E_2}{L}\begin{bmatrix} -0.1920 & -0.1280 \\ -0.1280 & -0.0853 \end{bmatrix}$$

The next step after getting the element matrices will be to assemble the element matrices into a global finite element matrix. So, the global stiffness matrix for the truss is:

$$[K]_{glob} = \begin{bmatrix} [k_{11}]_1 + [k_{22}]_3 & [k_{12}]_1 & [k_{21}]_3 \\ [k_{21}]_1 & [k_{22}]_1 + [k_{11}]_2 & [k_{12}]_2 \\ [k_{12}]_3 & [k_{21}]_2 & [k_{22}]_2 + [k_{11}]_3 \end{bmatrix} \tag{4.15}$$

For the same material and Area of rods (E_1=E_2=E_3=E, $A1$=A_2=A_3 =A), the global stiffness matrix is:

$$[K]_{glob} = \frac{AE}{L}\left[\begin{array}{cccccc} 0.3587 & 0.1280 & -0.1667 & 0 & -0.1920 & -0.1280 \\ 0.1280 & 0.0853 & 0 & 0 & -0.1280 & -0.0853 \\ -0.1667 & 0 & 0.3587 & -0.1280 & -0.1920 & 0.1280 \\ 0 & 0 & -0.1280 & 0.0853 & 0.1280 & -0.0853 \\ -0.1920 & -0.1280 & -0.1920 & 0.1280 & 0.3840 & 0 \\ -0.1280 & -0.0853 & 0.1280 & -0.0853 & 0 & 0.1707 \end{array}\right]\begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{array}$$

By applying the boundary condition ($u_1 = v_1 = v_2 = 0$), the global stiffness matrix becomes:

$$[K]_{glob} = \frac{AE}{L}\left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3587 & 0 & -0.1920 & 0.1280 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1920 & 0 & 0.3840 & 0 \\ 0 & 0 & 0.1280 & 0 & 0 & 0.1707 \end{array}\right]\begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{array}$$

The reduced global stiffness matrix for (u_2, u_3 and v_3) is:

$$[K]_{glob,red} = \frac{AE}{L}\left[\begin{array}{ccc} 0.3587 & -0.1920 & 0.1280 \\ -0.1920 & 0.3840 & 0 \\ 0.1280 & 0 & 0.1707 \end{array}\right]\begin{array}{c} u_2 \\ u_3 \\ v_3 \end{array}$$

The mass matrix for each element can be obtained as following:

Element 1;

The orientated angle with respect to the horizontal axis X, $\theta = 0$ so that C=cos $\theta = 1$ and S=sin θ=0. Substituting in eq.(4.9), the element mass matrix is:

$$[m_1] = \frac{\rho A.6L}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For $m_1 = m_2 = m_3 = m = \rho\, A\, L$,

$$[m_1] = m\begin{bmatrix}2 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\\ 1 & 0 & 2 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Element 2; θ =180- β and $Le = \sqrt{13}\ L$

$$[m_2] = m\begin{bmatrix}0.8321 & -0.5547 & 0.4160 & -0.2774\\ -0.5547 & 0.3698 & -0.2774 & 0.1849\\ 0.4160 & -0.2774 & 0.8321 & -0.5547\\ -0.2774 & 0.1849 & -0.5547 & 0.3698\end{bmatrix}$$

Element 3; θ =β and $Le = \sqrt{13}\ L$

$$[m_3] = m\begin{bmatrix}0.8321 & 0.5547 & 0.4160 & 0.2774\\ 0.5547 & 0.3698 & 0.2774 & 0.1849\\ 0.4160 & 0.2774 & 0.8321 & 0.5547\\ 0.2774 & 0.1849 & 0.5547 & 0.3698\end{bmatrix}$$

The global mass matrix can be written as:

$$[M]_{glob} = m\begin{bmatrix}2.8321 & 0.5547 & 1 & 0 & 0.4160 & 0.2774\\ 0.5547 & 0.3698 & 0 & 0 & 0.2774 & 0.1849\\ 1 & 0 & 2.8321 & -0.5547 & 0.4160 & -0.2774\\ 0 & 0 & -0.5547 & 0.3698 & -0.2774 & 0.1849\\ 0.4160 & 0.2774 & 0.4160 & -0.2774 & 1.6641 & 0\\ 0.2774 & 0.1849 & -0.2774 & 0.1849 & 0 & 0.7396\end{bmatrix}$$

By applying boundary condition (u_1= v_1= v_2=0), the global mass matrix becomes:

$$[M]_{glob} = m \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.8321 & 0 & 0.4160 & -0.2774 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.4160 & 0 & 1.6641 & 0 \\ 0 & 0 & -0.2774 & 0 & 0 & 0.7396 \end{bmatrix}$$

The reduced mass matrix for (u_2, u_3 and v_3) is:

$$[M]_{glob} = m \begin{bmatrix} 2.8321 & 0.4160 & -0.2774 \\ 0.4160 & 1.6641 & 0 \\ -0.2774 & 0 & 0.7396 \end{bmatrix}$$

Now, the next step is to determine Eigen value and Eigen vector:

$$\left[[K]_{glob,red} - \omega^2 [M]_{glob,red}\right] . \{Q\} = \{0\} \tag{4.16}$$

$$\left[\frac{EA}{L} \begin{bmatrix} 0.3587 & -0.1920 & 0.1280 \\ -0.1920 & 0.3840 & 0 \\ 0.1280 & 0 & 0.1707 \end{bmatrix} - m\,\omega^2 \begin{bmatrix} 2.8321 & 0.4160 & -0.2774 \\ 0.4160 & 1.6641 & 0 \\ -0.2774 & 0 & 0.7396 \end{bmatrix} \right] . \begin{Bmatrix} a_{3i} \\ a_{5i} \\ a_{6i} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

To obtain the Eigen value, the characteristic equation is:

$$\left\| \begin{bmatrix} 0.3587 & -0.1920 & 0.1280 \\ -0.1920 & 0.3840 & 0 \\ 0.1280 & 0 & 0.1707 \end{bmatrix} - \frac{m\,L\omega^2}{AE} \begin{bmatrix} 2.8321 & 0.4160 & -0.2774 \\ 0.4160 & 1.6641 & 0 \\ -0.2774 & 0 & 0.7396 \end{bmatrix} \right\| = 0$$

Solving the previous equation gives the three natural frequencies as:

$$\omega_1 = 0.1858 \sqrt{\frac{EA}{mL}}$$

$$\omega_2 = 0.4804\sqrt{\frac{EA}{mL}}$$

$$\omega_3 = 0.6515\sqrt{\frac{EA}{mL}}$$

The corresponding Eigen vector can be determine by solving the next equation

$$\left[\frac{EA}{L}\begin{bmatrix} 0.3587 & -0.1920 & 0.1280 \\ -0.1920 & 0.3840 & 0 \\ 0.1280 & 0 & 0.1707 \end{bmatrix} - m\omega_i^2\begin{bmatrix} 2.8321 & 0.4160 & -0.2774 \\ 0.4160 & 1.6641 & 0 \\ -0.2774 & 0 & 0.7396 \end{bmatrix}\right].\begin{Bmatrix} a_{3i} \\ a_{5i} \\ a_{6i} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

The Eigen vector matrix is:

$$\begin{bmatrix} 0.438 & 0 & 0.4351 \\ .2768 & 0.5481 & -0.4977 \\ -04152 & 0.8222 & 0.7465 \end{bmatrix}$$

For ω_1;

a_{51}/a_{51}=1, a_{31}/a_{51}= 1.5823 , and a_{61}/a_{51}= -1.5

The first mode shape vector is:

$$\{Q_1\} = \begin{Bmatrix} a_{31}/a_{51} \\ a_{51}/a_{51} \\ a_{61}/a_{51} \end{Bmatrix} = \begin{Bmatrix} 1.5823 \\ 1 \\ -1.5 \end{Bmatrix} ;or\ \{Q_1\}^T = \{1.5823 \quad 1 \quad -1.5\}$$

For ω_2;

a_{52}/a_{52}=1, a_{32}/a_{52}= 0 , and a_{62}/a_{52}= 1.5

The second mode shape vector is:

$$\{Q_2\} = \begin{Bmatrix} a_{322}/a_{52} \\ a_{52}/a_{52} \\ a_{62}/a_{52} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \\ 1.5 \end{Bmatrix} ;or\ \{Q_2\}^T = \{0 \quad 1 \quad 1.5\}$$

For ω_3;

a_{53}/a_{53}=1, a_{33}/a_{53}= -0.8742 , and a_{63}/a_{53}= -1.5

The third mode shape vector is:

$$\{Q_3\} = \begin{Bmatrix} a_{33}/a_{53} \\ a_{53}/a_{53} \\ a_{63}/a_{53} \end{Bmatrix} = \begin{Bmatrix} -0.8742 \\ 1 \\ -1.5 \end{Bmatrix} ;or\ \{Q_3\}^T = \{-0.8742 \quad 1 \quad -1.5\}$$

4.4.1 Kinetic and Potential Energy of the Whole Structure:

The kinetic and potential energies of the whole structure can be determined as following:

$$E^r = E_k^r = E_p^r = \omega_r^2\{Q_r\}^T[M]\{Q_r\} = \{Q_r\}^T[K]\{Q_r\} \tag{4.17}$$

The Kinetic and potential energy of the whole structure of the first main form of oscillation are:

$$E_k^1 = \omega_1^2\{Q_1\}^T[M]\{Q_1\}$$

$$E_k^1 = 0.1858^2.\{1.5823 \quad 1 \quad -1.5\}.\frac{EA}{mL}.m\begin{bmatrix} 2.8321 & 0.4160 & -0.2774 \\ 0.4160 & 1.6641 & 0 \\ -0.2774 & 0 & 0.7396 \end{bmatrix}.\begin{Bmatrix} 1.5823 \\ 1 \\ -1.5 \end{Bmatrix}$$

$$E_k^1 = 0.4509\ \frac{EA}{L}$$

$$E_p^1 = \{Q_1\}^T[K]\{Q_1\}$$

$$E_p^1 = \{1.5823 \quad 1 \quad -1.5\}.\frac{EA}{L}.\begin{bmatrix} 0.3587 & -0.1920 & 0.1280 \\ -0.1920 & 0.3840 & 0 \\ 0.1280 & 0 & 0.1707 \end{bmatrix}.\begin{Bmatrix} 1.5823 \\ 1 \\ -1.5 \end{Bmatrix}$$

$$E_p^1 = 0.4509\ \frac{EA}{L}$$

Similarly, the Kinetic and potential energy of the whole structure of the second main form of oscillation are:

$$E_k^2 = \omega_2^2\{Q_2\}^T[M]\{Q_2\}$$

$$E_k^2 = 0.4804^2.\{0 \quad 1 \quad 1.5\}.\frac{EA}{mL}.m\begin{bmatrix} 2.8321 & 0.4160 & -0.2774 \\ 0.4160 & 1.6641 & 0 \\ -0.2774 & 0 & 0.7396 \end{bmatrix}.\begin{Bmatrix} 0 \\ 1 \\ 1.5 \end{Bmatrix}$$

$$E_k^2 = 0.7681\ \frac{EA}{L}$$

$$E_p^2 = \{Q_2\}^T[K]\{Q_2\} = \{0 \quad 1 \quad 1.5\}.\frac{EA}{L}.\begin{bmatrix} 0.3587 & -0.1920 & 0.1280 \\ -0.1920 & 0.3840 & 0 \\ 0.1280 & 0 & 0.1707 \end{bmatrix}.\begin{Bmatrix} 0 \\ 1 \\ 1.5 \end{Bmatrix}$$

$$E_p^2 = 0.7681\ \frac{EA}{L}$$

The kinetic and potential energy of the whole structure of the third main form of oscillation are:

$$E_k^3 = \omega_3^2\{Q_3\}^T[M]\{Q_3\}$$

$$= 0.1858^2.\{-0.8742 \quad 1 \quad -1.5\}.\frac{EA}{mL}.m\begin{bmatrix} 2.8321 & 0.4160 & -0.2774 \\ 0.4160 & 1.6641 & 0 \\ -0.2774 & 0 & 0.7396 \end{bmatrix}.\begin{Bmatrix} -0.8742 \\ 1 \\ -1.5 \end{Bmatrix}$$

$$E_k^3 = 1.7136\,\frac{EA}{L}$$

$$E_p^3 = \{Q_3\}^T[K]\{Q_3\}$$

$$= \{-0.8742 \quad 1 \quad -1.5\}.\frac{EA}{L}.\begin{bmatrix} 0.3587 & -0.1920 & 0.1280 \\ -0.1920 & 0.3840 & 0 \\ 0.1280 & 0 & 0.1707 \end{bmatrix}.\begin{Bmatrix} -0.8742 \\ 1 \\ -1.5 \end{Bmatrix}$$

$$E_p^3 = 1.7136\,\frac{EA}{L}$$

4.4.2 Kinetic and potential energy of individual elements

Kinetic e_k^r and potential e_p^r energy for each element can be individually determined as following:

$$E_{k(e)r} = \omega_r^2\{q_{e,red,r}\}^T\left[M_{glob,rod,e,red}\right]\{q_{e,red,r}\} \tag{4.18}$$

$$E_{p(e)r} = \{q_{e,red,r}\}^T\left[K_{glob,rod,e,red}\right]\{q_{e,red,r}\} \tag{4.19}$$

For the first natural frequency:

Rod 1

$$E_{k(1)1} = \omega_1^2 \{q_{1,red,1}\}^T [M_{glob,rod1,red}] \{q_{1,red,1}\}$$

$$E_{k(1)1} = 0.1858^2 . \frac{EA}{L} . \{0 \quad 0 \quad 1.5823 \quad 0\} . \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \begin{Bmatrix} 0 \\ 0 \\ 1.5823 \\ 0 \end{Bmatrix}$$

$$E_{k(1)1} = 0.1730 \ \frac{EA}{L}$$

$$E_{p(1)1} = \{q_{1,red,1}\}^T [K_{glob,rod1,red}] \{q_{1,red,1}\}$$

$$E_{p(1)1} = \{0 \quad 0 \quad 1.5823 \quad 0\} . \frac{EA}{L} . \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1667 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \begin{Bmatrix} 0 \\ 0 \\ 1.5823 \\ 0 \end{Bmatrix}$$

$$E_{p(1)1} = 0.4174 \ \frac{EA}{L}$$

Rod 2

$$E_{k(2)1} = \omega_1^2 \{q_{2,red,1}\}^T [M_{glob,rod2,red}] \{q_{2,red,1}\}$$

$$E_{k(2)1}$$

$$= 0.1858^2 . \frac{EA}{L} . \{1.5823 \quad 0 \quad 1 \ -1.5\} . \begin{bmatrix} 0.8321 & 0 & 0.416 & -0.2774 \\ 0 & 0 & 0 & 0 \\ 0.416 & 0 & 0.8321 & -0.5547 \\ -0.2774 & 0 & -0.5547 & 0.3698 \end{bmatrix} . \begin{Bmatrix} 1.5823 \\ 0 \\ 1 \\ -1.5 \end{Bmatrix}$$

$$E_{k(2)1} = 0.2779 \ \frac{EA}{L}$$

$$E_{p(2)1} = \{q_{2,red,1}\}^T [K_{glob,rod2,red}] \{q_{2,red,1}\}$$

$$E_{p(1)1} = \{1.5823 \quad 0 \quad 1 \; -1.5\}.\frac{EA}{L}.\begin{bmatrix} 0.192 & 0 & -0.192 & 0.128 \\ 0 & 0 & 0 & 0 \\ -0.192 & 0 & 0.192 & -0.128 \\ 0.128 & 0 & -0.128 & 0.0853 \end{bmatrix}.\begin{Bmatrix} 1.5823 \\ 0 \\ 1 \\ -1.5 \end{Bmatrix}$$

$$E_{p(1)1} = 0.0335\,\frac{EA}{L}$$

Rod 3

$$E_{k(3)1} = \omega_1^2\{q_{3,red,1}\}^T[M_{glob,rod3,red}]\{q_{3,red,1}\}$$

$$E_{k(3)1} = 0.1858^2.\frac{EA}{L}.\{1 \quad -1.5 \quad 0 \;\; 0\}.\begin{bmatrix} 0.8321 & 0.5547 & 0 & 0 \\ 0.5547 & 0.3698 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.\begin{Bmatrix} 1 \\ -1.5 \\ 0 \\ 0 \end{Bmatrix}$$

$$E_{k(3)1} \approx 0$$

$$E_{p(3)1} = \{q_{3,red,1}\}^T[K_{glob,rod3,red}]\{q_{3,red,1}\}$$

$$E_{p(3)1} = \{1 \quad -1.5 \quad 0 \;\; 0\}.\frac{EA}{L}.\begin{bmatrix} 0.192 & 0.128 & 0 & 0 \\ 0.128 & 0.0853 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.\begin{Bmatrix} 1 \\ -1.5 \\ 0 \\ 0 \end{Bmatrix}$$

$$E_{p(3)1} \approx 0$$

The sum of kinetic and potential energies of all elements of the first form of oscillation is:

$$\sum_{i=1}^{3} E_{k(i)1} = E_{k(1)1} + E_{k(2)1} + E_{k(3)1} = (0.1730 + 0.2779 + 0)\frac{EA}{L} = 0.4509\frac{EA}{L}$$

$$\sum_{i=1}^{3} E_{p(i)1} = E_{p(1)1} + E_{p(2)1} + E_{p(3)1} = (0.4174 + 0.0335 + 0)\frac{EA}{L} = 0.4509\frac{EA}{L}$$

This implies the equality of kinetic and potential energy of the whole structure of the first form oscillation.

$$\sum_{i=1}^{3} E_{k(i)1} = \sum_{i=1}^{3} E_{p(i)1} \quad (4.20)$$

The same procedure can be repeated for the second and third form of oscillation, and the next table shows the obtained results:

Table (4.1): energy distribution on the truss rods for all forms of oscillation

Rod number	For the first mode		For the second mode		For the third mode	
	E_k [J] *	E_p [J]	E_k [J]	E_p [J]	E_k [J]	E_p [J]
1	0.1730	0.4174	0	0	0.6486	0.1275
2	0.2779	0.0335	0	0	1.0650	1.5861
3	0	0	0.7681	0.7681	0	0
$\sum_{i=1}^{3} E_{k(i)} = \sum_{i=1}^{3} E_{p(i)}$	0.4509	0.4509	0.7681	0.7681	1.7136	1.7136

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* All members of the energy are multiplied by a factor of $\frac{EA}{L}$. The numbers in the table have the dimension [m^2].

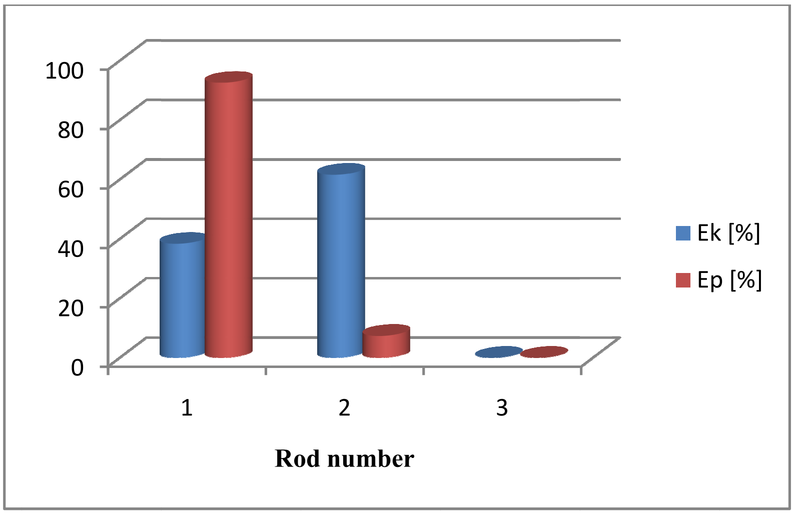


Figure (3.2) Percentages of kinetic and potential energy distribution for the first mode

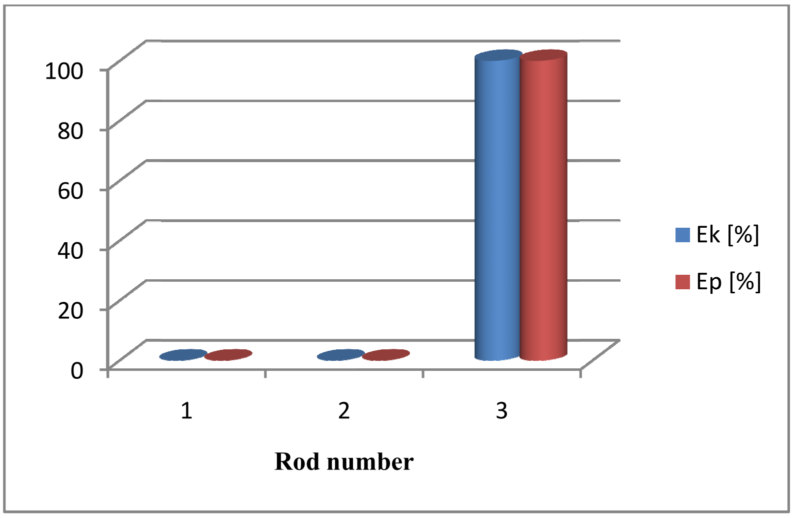


Figure (3.3) Percentages of kinetic and potential energy distribution for the second mode

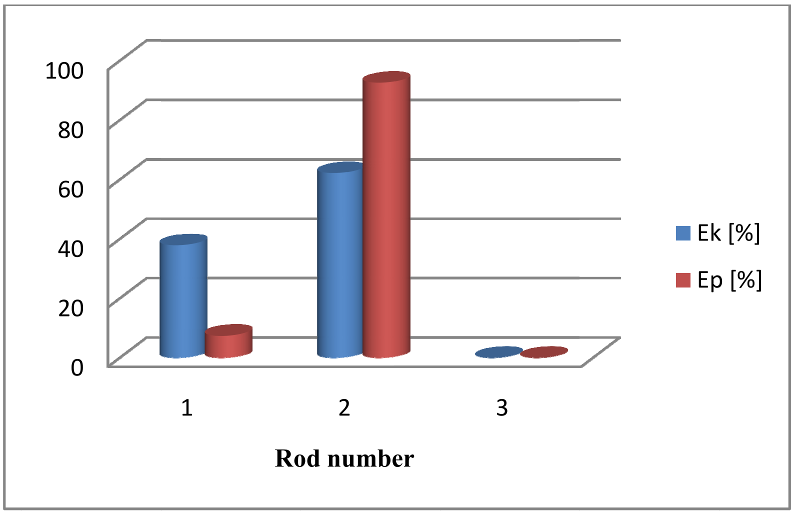


Figure (3.3) Percentages of kinetic and potential energy distribution for the third mode

4.5. Modification of dynamic parameters

For free vibration case the modified system can be describe by a modified equation (perturbation equation) as:

$$[K]'\{Q_r\}' = \lambda_i'[M]' \cdot \{Q_r\}' \qquad (4.21)$$

Introducing [ΔK] and [ΔM] are the corresponding changes in stiffness and mass matrices respectively. Then,

$$[K]' = [K] + [\Delta K], [M]' = [M] + [\Delta M]$$

$$\{Q_r\}' = \{Q_r\} + \{\Delta Q_r\}, \quad \lambda_i' = \lambda_i + \Delta\lambda_i \qquad (4.22)$$

Where: $\Delta\lambda$ and $\{\Delta Q_r\}$ are changes of eigenvalues and eigenvectors, respectively.

For a modified system equation (4.7) can be rewritten as:

$$\begin{aligned}&\left(\left[K\right]+\left[\Delta K\right]\right)\left(\{Q_r\}+\{\Delta Q_r\}\right)=\\&\left(\lambda_r+\Delta\lambda_r\right)\left(\left[M\right]+\left[\Delta M\right]\right)\left(\{Q_r\}+\{\Delta Q_r\}\right)\end{aligned}\tag{4.23}$$

In the same manner, the balanced equation of potential and kinetic energy (3) can be rewritten in its perturbed form as:

$$\begin{aligned}&\left(\{Q_r\}+\{\Delta Q_r\}\right)^T\left(\left[K\right]+\left[\Delta K\right]\right)\left(\{Q_r\}+\{\Delta Q_r\}\right)=\\&\left(\lambda_r+\Delta\lambda_r\right)\left(\{Q_r\}+\{\Delta Q_r\}\right)^T\left(\left[M\right]+\left[\Delta M\right]\right)\left(\{Q_r\}+\{\Delta Q_r\}\right)\end{aligned}\tag{4.24}$$

After some manipulations and neglecting the higher order terms [121], the change of i-th eigenvalue under system modification can be expressed as:

$$\frac{\Delta\lambda_r}{\lambda_r'}=\frac{\frac{1}{2}\{Q_r\}^{\prime T}[\Delta K]\{Q_r\}'-\frac{1}{2}\lambda_r'\{Q_r\}^{\prime T}[\Delta M]\{Q_r\}'}{\frac{1}{2}\lambda_r'\{Q_r\}^{\prime T}[M]'\{Q_r\}'}\tag{4.25}$$

Equation (4.25) can be considered as a basic formula for reanalysis procedure to improve structure dynamic characteristics.

Furthermore, the next formula can be used for the unmodified system:

$$\frac{\Delta\lambda_r}{\lambda_r}=\frac{\frac{1}{2}\{Q_r\}^T[\Delta K]\{Q_r\}-\frac{1}{2}\lambda_r\{Q_r\}^T[\Delta M]\{Q_r\}}{\frac{1}{2}\lambda_r\{Q_r\}^T[M]\{Q_r\}}\tag{4.26}$$

The denominator in equation (4.26) represents the kinetic energy of a certain oscillation mode and having in mind equation (4.8), it also represents the potential energy, for reasons of energy balance in the main oscillation modes.

The stiffness and mass matrices after the modification is done in e-th finite element can be expressed as:

$$\begin{aligned}
&[k]_e' = [k]_e + [\Delta k]_e = [k]_e + \alpha_e [k]_e, \\
&[m]_e' = [m]_e + [\Delta m]_e = [m]_e + \beta_e [m]_e
\end{aligned} \tag{4.27}$$

Where α_e and β_e are values that define the modification of *e*-th element, and are called *modification coefficients*. In this case, the members of stiffness matrices and mass matrices within the matrices of construction parameters are all equal to zero except for those corresponding to *e*-th finite element, so that the nominator in equation (4.26) for *r*-th oscillation mode becomes

$$\begin{aligned}
&\frac{1}{2}\{Q_r\}^T[\Delta K]\{Q_r\} - \frac{1}{2}\lambda_r\{Q_r\}^T[\Delta M]\{Q_r\} = \\
&\frac{1}{2}\alpha_e \{q_r^s\}_e^T [k]_e \{q_r^s\}_e - \frac{1}{2}\beta_e \lambda_r \{q_r^s\}_e^T [m]_e \{q_r^s\}_e \\
&= \frac{1}{2}\left(\alpha_e e_{p,r} - \beta_e e_{k,r}\right)
\end{aligned} \tag{4.28}$$

Where: $\{q_r^s\}_e$ - is the corresponding r-th eigenvector of *e*-th element with s degrees of freedom,

$e_{p,r} = \frac{1}{2}\{q_r^s\}_e^T [k]_e \{q_r^s\}_e$ - is the potential energy of *e*-th element in r-th main oscillation mode without constructional modification, and

$e_{k,r} = \frac{1}{2}\omega_r^2 \{q_r^s\}_e^T [m]_e \{q_r^s\}_e$ is the kinetic energy of e-th element in r-th main oscillation mode without constructional modification. Consequently, equation (4.26) can be written as:

$$\frac{\Delta\lambda_r}{\lambda_r}=\frac{\frac{1}{2}\{Q_r\}^T[\Delta K]\{Q_r\}-\frac{1}{2}\lambda_r\{Q_r\}^T[\Delta M]\{Q_r\}}{\frac{1}{2}\lambda_r\{Q_r\}^T[M]\{Q_r\}}= \qquad (4.29)$$

$$=\frac{\alpha_e e_{p,r}-\beta_e e_{k,r}}{E_{k,r}}$$

The previous equation has an important definition to understand the procedures of reanalysis and to define the position of elements that require modifications to improve the dynamic behavior of the structure. Because the denominator has the same value, the numerator is the main interest of analysis. The natural frequency of the structure increased or decreased according to the values of α_e and β_e. When α_e has a positive value, hence increased rigidity, the natural frequency is increased. When α_e has negative values, hence decreased rigidity, the natural frequency is decreased. On the other hand, when β_e has a positive value, hence increased mass, the natural frequency is decreased. When β_e has negative values, hence decreased mass, the natural frequency is increased. Consequently, the modification (increase/decrease structure rigidity or mass) which will be done for the structure depends on the sign value of numerator in equation (4.29). The main point of improving dynamic behavior of the structure is increasing its natural frequencies and maximizing the interval between adjacent natural frequencies. Hence, study of energy distribution will be done for each element in the structure to determine places of modification.

4.6 Calculation of the Supporting Structure

The basic idea of the finite element method is to find approximate solutions (numerically) of complicated problems. Modeling is the most important step in engineering practice. Mathematical modeling, or idealization, is a process by which an engineer passes from the actual physical system under study, to a mathematical model of the system. Modeling is achieved through the selection of the type, number and size of the finite element discretization, the degree of freedom of nodes and boundary conditions, as well as the introduction of some idealization, approximations and appropriate alternatives. Types of finite elements depend on the geometry of the problem. The main classification is based on the types of supporting structures that are modeled. Accordingly, the element can be classified as a line (1 D - rods, beams), surface (2 D - membranes, plates, shells) and volumetric (3 D - tetrahedron, hexahedron ...). Also, the choice of finite element modeling and idealized models depends on the dimensions (length, width, height), the degree of freedom in the nodes (translation, rotation), and the type of loading (longitudinal strain, twisting, bending, complex, planar...), etc. The size and number of elements directly affect the accuracy and convergence solutions.

4.6.1 Dynamic Analysis and Diagnostics of Model and its Groups

The procedures which are used in this thesis are concerned with distribution of potential and kinetic energy in all elements of the structure which gives predictions for reanalysis. Calculations of main modes of oscillation were performed using Abaqus software [115], while the energy distributions using KOMIPS software [104].

These procedures were developed by Natasa [105] and the following cases should be considered for reanalysis of similar constructions:

a) Elements in which the kinetic and potential energies (and the difference in their increase) are negligible with respect to other elements.
b) Elements in which the kinetic energy is dominant compared to potential energy
c) Elements in which the potential energy is dominant compared to kinetic energy
d) Elements in which the potential and kinetic energy exist and are not negligible in comparison with other elements.

4.6.2 Concepts of the KOMIPS program

The basis for structural performance diagnostics is the computer modeling and structural analysis calculation software (KOMIPS) [104] with the application of finite element numerical method throughout static, dynamic, and thermal calculation of consisting structural elements. The main parts of the program are as follows:

- Preprocessor for interactive computer model generation,

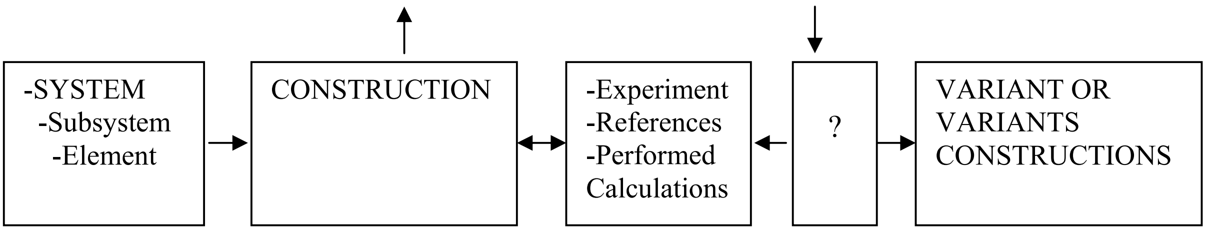
- Processor for static, dynamic and thermal calculation
- Postprocessor for analysis and diagnostic of structure behavior,
- Users functions,
- Interactive computer graphics and
- Model conversion.

KOMIPS

computer modeling and structural calculations
Author: Prof.dr Taško Maneski, Faculty of Mechanical Engineering, Beograd

MODELING	PREPROCESSOR	PROCESSOR	POSTPROCESSOR
-Mapping -Defining the problem -The choice of the finite-element -discretization -Boundary conditions, - load	-Local-generating -Global-generating -Graphics -Optimize. connecting nodes -Conversions: Acad and HPGL	-Static -Dynamic -Thermo -Linear and non-linear -Stationary and non-stationary	-Analysis of results -Specific calculations - Graphics -Elements of optimization -Conversions: Acad and HPGL

1 D - 2 D - 3D PROBLEMS



KOMIPS allows modeling and the calculation of complex structures and problems, determination of real displacements and stresses, and real structural behavior including the consisting elements, it gives a reliable forecast of structural behavior in service and depicts the parameters for decision making (operating regime, repairs, reconstructions, revitalizations, optimizations, confirmations of selected solution variants), poor performance sample identification or structural deterioration, service life estimation and time of reliable operation efficiency. Every improvement of structural performance that can be reached by this approach allows service life extension and increase of reliability.

4.6.3 Reanalysis Algorithm

The following algorithm is established based on the previous analysis as illustrated in the following steps [105]:

Step 1: The observed structure is divided into appropriate number of finite elements for which kinetic energy $e_{k,r} = \frac{1}{2}\omega_r^2 \{q_r^s\}_e^T [m]_e \{q_r^s\}_e$, and potential energy $e_{p,r} = \frac{1}{2}\{q_r^s\}_e^T [k]_e \{q_r^s\}_e$, are calculated separately, on those main modes which are interest in the analysis.

Step 2: Comparing the values of potential and kinetic energy over zones or elements, as well as corresponding energy differences, based on which the following courses of analysis are formed:

Step 3: In elements for which is valid:

$e_{pr} \to 0,\ e_{kr} \to 0$, there are no possibilities for successful modifications with respect to increasing eigenfrequencies. These elements do not have significant effect on dynamic behavior of structure, but they might be suitable for other types of optimizations. In general, reducing the mass of those elements lightens the weight of whole structure without endangering its dynamical behavior.

Step 4: For those elements where $e_{pr} >> e_{kr}$, eigenvalues can be increased by increasing the stiffness of structure. The modifications to increase these values are not arbitrary, but they are done according to the principle of energy distributions through the elements of structure.

Step 5: For those elements where $e_{kr} >> e_{pr}$, eigenvalues can be increased by decreasing the mass of structure. Also, this operation can be done based on distribution of energy through the elements of structure. According to many criteria, decreasing of mass is a generally desired type of modification.

Step 6: Most often, elements appear in structure for which the values of e_{kr}, e_{pr} are not negligible. Therefore, the situation is more complex and those elements are suitable for reanalysis. In this case, the reanalysis of structure is done based on the differences in increases of potential and kinetic energy $\Delta e_{pr} - \Delta e_{kr}$ between modified and original system. The modification parameters α and β are independently calculated for each element. It has been shown that modification parameters depend on type of cross sectional area, type of material used, and boundary conditions. Reanalysis formula can be applied to achieve the purpose of increase eigenvalues.

$$\Delta\lambda_1 = \frac{\sum_{j=1}^{N}\left(\alpha_j e_{pj}^{(1)} - \beta_j e_{kr}^{(1)}\right)}{\{Q_1\}^T[M]\{Q_1\}} \tag{4.30}$$

Step 7: When the desired value of increase is achieved, it is possible to conduct the check of modified structure by running the software based on the finite element analysis, with modified parameters. Then, the evaluation of modified structure can be obtained based on new energy distribution schemes. If the difference of energy increase on the redesigned places is less than the previous that means that the procedure converges, and vice versa. Convergence is the goal of every optimization procedure.

Chapter 5

Implementation of reanalysis technique on 1D and 2D structures

In this chapter, numerical examples of some different 1D and 2D structures subjected to structural modifications are presented in order to assess numerically the effectiveness of the method proposed in this thesis.

As mentioned previously, the main idea of improving dynamic behavior of the structure is increasing its natural frequencies and maximizing the interval between adjacent natural frequencies. Hence, the distribution of kinetic and potential energy in the structural elements of the main forms of oscillations will be deemed as an indicator of the direction of dynamic modification.

5.1 Calculation of the Supporting Structure

The basic idea of the finite element method is to find approximate solutions (numerically) of complicated problems. Modeling is the most important step in engineering practice. Mathematical modeling, or idealization, is a process by which an engineer passes from the actual physical system under study, to a mathematical model of the system. Modeling is achieved through the selection of the type, number and size of the finite element discretization, the degree of freedom of nodes and boundary conditions, as well as the introduction of some idealization, approximations and appropriate alternatives.

Types of finite elements depend on the geometry of the problem. The main classification is based on the types of supporting structures that are modeled. Accordingly, the element can be classified as a line (1 D - rods, beams), surface (2 D - membranes, plates, shells) and volumetric (3 D - tetrahedron, hexahedron ...). Also, the choice of finite element modeling and idealized models depends on the dimensions (length, width, height), the degree of freedom in the nodes (translation, rotation), and the type of loading (longitudinal strain, twisting, bending, complex, planar...), etc. The size and number of elements directly affect the accuracy and convergence solutions.

5.2 Structural modification on 1D structure

In this section, structural modification is applied to trusses and beams as one-dimensional (1D) structures.

5.2.1 Simply- supported Truss

The first example is a simply-supported truss as shown in figure 5.1 which is subjected to structural modification in order to improve its dynamic behavior. The geometry and material properties of the original proposed truss are listed in Table 5.1.

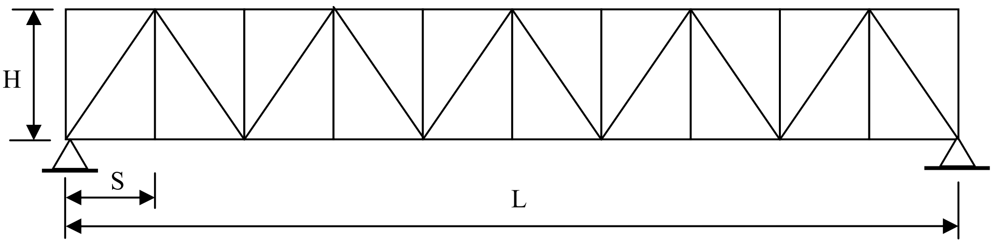


Figure 5.1 FE model of the original truss

Table 5.1 Geometry and material parameters of the original simply-supported truss

Length (L)	10 m
Height (H)	1.5 m
Distance (S)	1.0 m
Cross-sectional Area of the horizontal Rods	0.0024 m^2
Cross-sectional Area of the vertical and inclined Rods	0.0009 m^2
Young's modulus	200 GPa
Poisson's Ratio	0.3
Density	7800 Kg/m^3
Total Mass	616 Kg

Figure 5.2 shows the obtained results for the first mode of oscillation of this model (bending). Potential and kinetic energies have been calculated using Equations (4.9) and (4.10) and the differences in increment were determined.

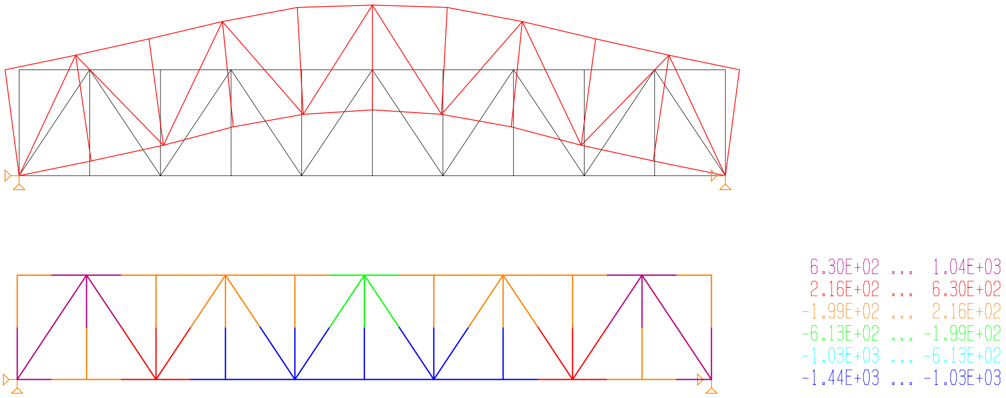


Figure 5.2 FEA of original model. The first frequency is f_{01}= 39.7 Hz. Difference between potential and kinetic energy [Nm]

Based on the distribution of energy through the structure, it can be seen that the zones which have positive values in the difference between potential and kinetic energy (red and purple colors) require increasing in the stiffness. In this example, the first natural frequency of the truss can be increased 12.59% (f_{01}= 44.7 Hz) by increasing the area of both two inclined rods in the corners to become 0.0024 m^2. This is accompanied by an increase in the total mass to become a 701 kg. Although the target is increasing the first natural frequency, the total mass of the structure is important factor which should be taken in account in structural design. Accordingly, after many reanalysis process without increasing the area of any rod , figure 5.3 shows the best modified structure with approximately the same value of the first natural frequency (f_{01}= 44.1 Hz), but the total mass is 463 Kg (33.95% decreases).

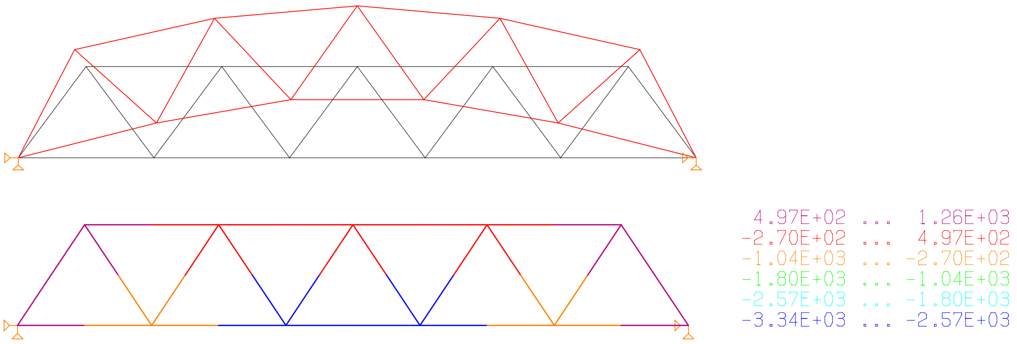


Figure 5.3 FEA of model A. The first frequency is f_{01}= 44.1 Hz. Difference between potential and kinetic energy [Nm]

In order to investigate the effect of truss height (H) on the natural frequency, some models are presented. Figures (5.3, 5.4) show the distribution of energy through the structure and its first mode of oscillation for models B and C respectively. The obtained results are listed on table 5.2, and the comparison between all models is shown in figure 5.6. Although model C has a highest value of the first natural frequency, model A can be considered as the best model because it has a natural frequency value close to model C with lower weight.

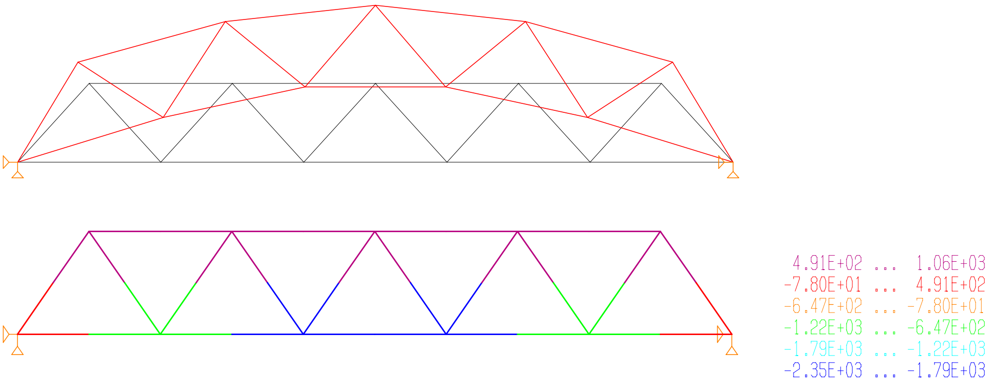


Figure 5.4 FEA of model B. The first frequency is f_{01}= 36. Hz. Difference between potential and kinetic energy [Nm]

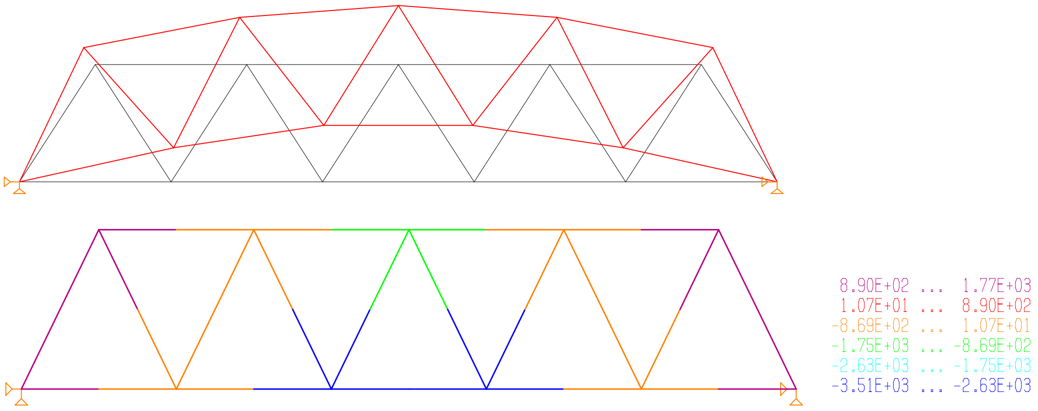


Figure 5.5 FEA of model C. The first frequency is f_{01}= 46.4 Hz. Difference between potential and kinetic energy [Nm]

Table 5.2 Comparison of natural frequencies for different models of truss

Model	Height (H)	Length (L)	Total mass	First Natural Frequency f_{01}
A	1.5 m	10 m	463.51 Kg	44.1 Hz
B	1 m	10 m	436.23 Kg	36.0 Hz
C	2 m	10 m	493.93 Kg	46.4 Hz

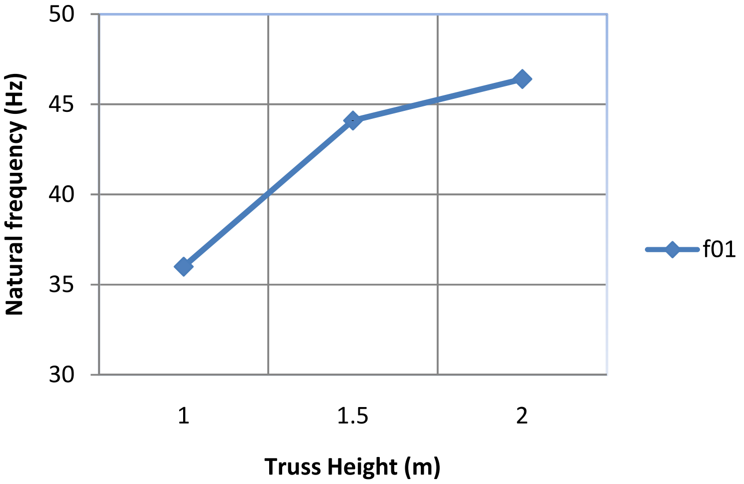


Figure 5.6 Comparison between models by considering the effect of Truss height on its natural frequency

5.2.2 Frame structure

A planer frame with a uniform cross sectional area as shown in figure 5.7 is presented to demonstrate the effectiveness of the proposed method to improve the performance of the structure. The frame consists of three beams and the geometry and material properties of the frame are listed in Table 5.3.

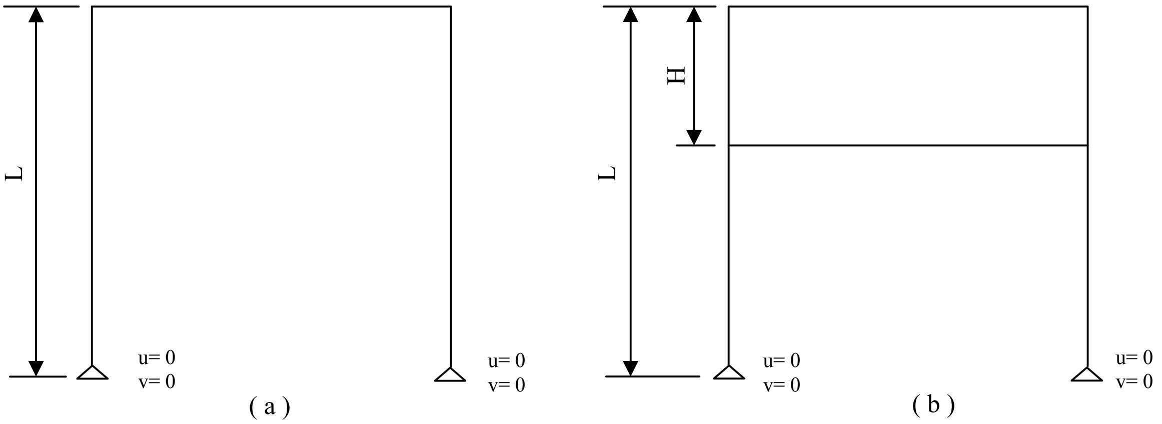


Figure 5.7 Frame structure. (a) Without a horizontal Beam link. (b) With a horizontal Beam link

Table 5.3 Geometry and material parameters of the Frame

Length (L)	10 m
Cross-sectional Area	1.121 m^2
Second Moment of Inertia	0.1 m^4
Young's modulus	210 GPa
Poisson's Ratio	0.3
Density	7800 Kg/m^3

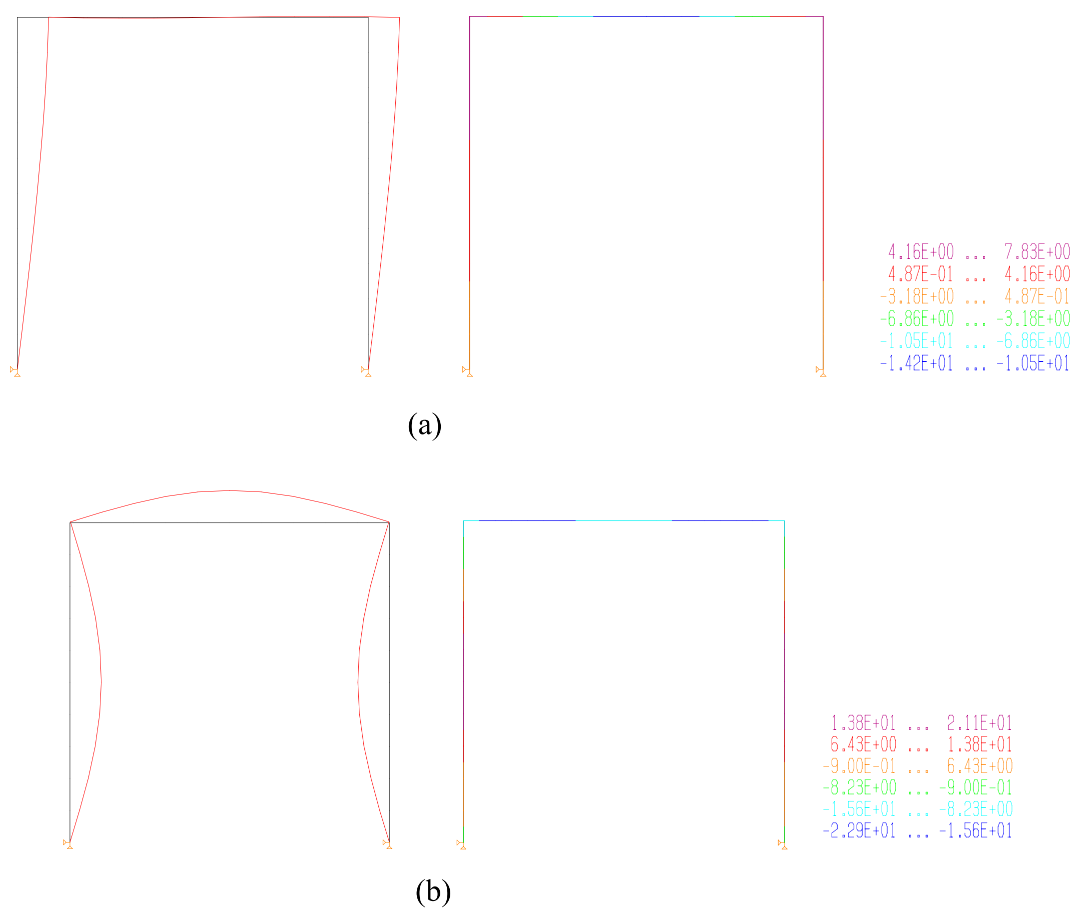
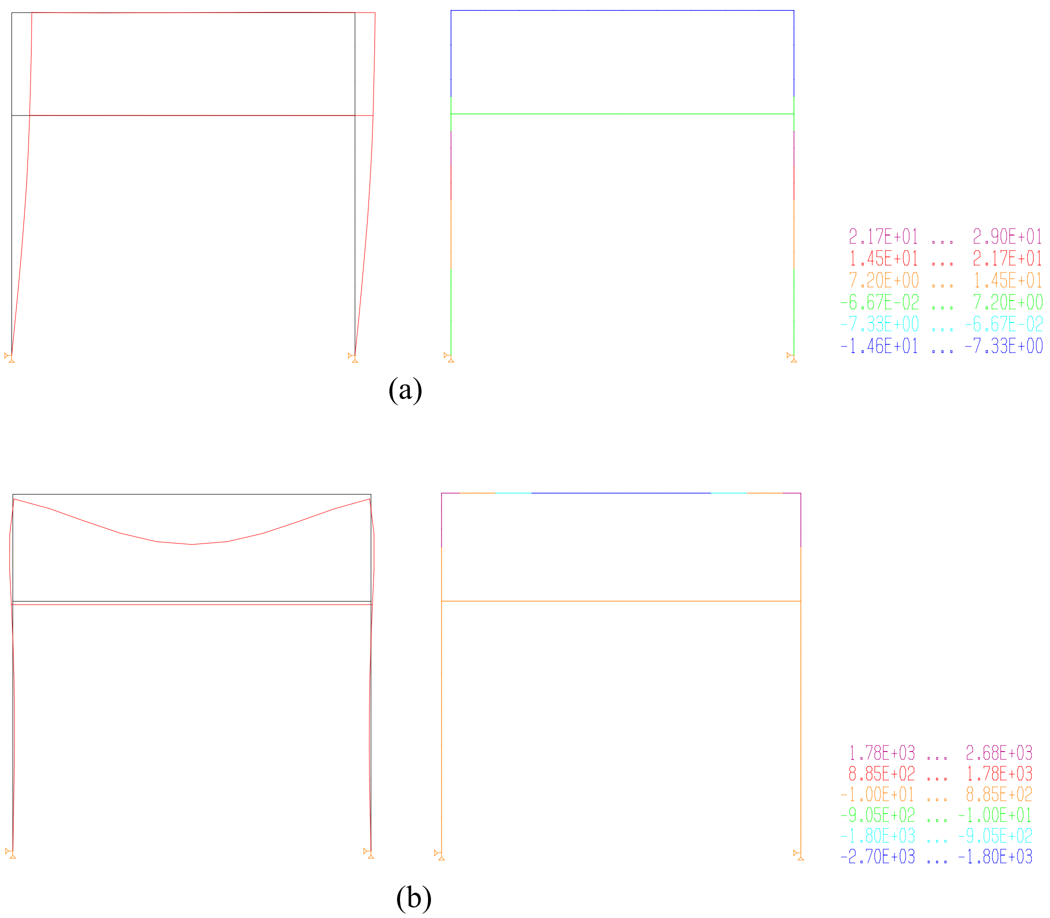


Figure 5.8 FEA of Frame - model A. (a) First mode f_{01}= 3.7 Hz. (b) Second mode f_{02}= 24.9 Hz. Difference between potential and kinetic energy [Nm]

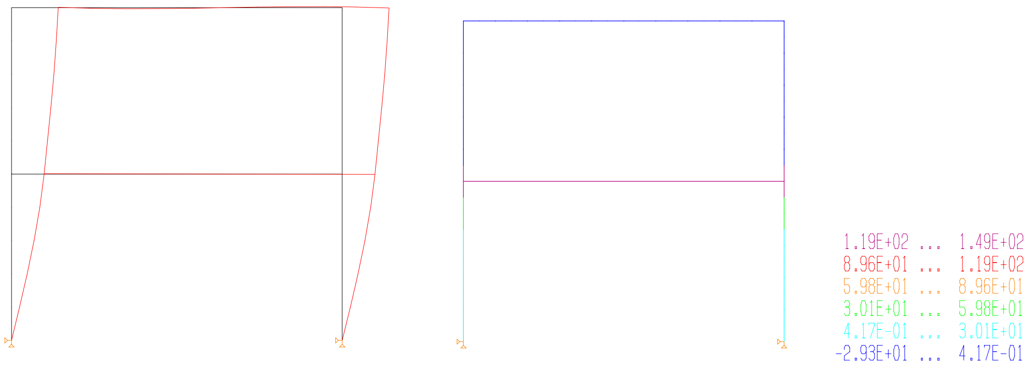
Based on the energy distribution through the frame elements, one can figure out from figure 5.8 that the performance of the frame can be improved by linking the two vertical beams with a horizontal beam as shown in figure 5.7 (b). To this end, two additional models, model A and Model B respectively, are proposed. Figures (5.9, 5.10) show the effect of a horizontal beam position (H/L) on the behavior of the frame. The first two natural frequencies of the frame in different cases of (H/L) are shown in figure 5.11. Accordingly, it is clear that model B has the best structure behavior.



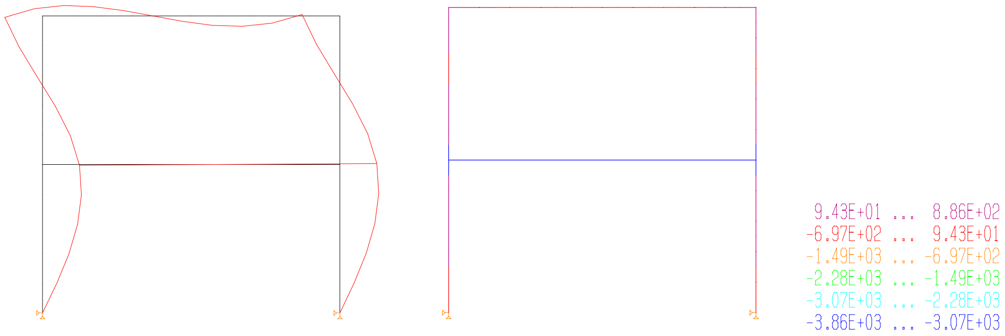
(a)

(b)

Figure 5.9 FEA of Frame - model B. (a) First mode f_{01}= 4.6 Hz. (b) Second mode f_{02}= 40.8 Hz. Difference between potential and kinetic energy [Nm]



(a)



(b)

Figure 5.10 FEA of Frame - model C. (a) First mode f_{01}= 6.0 Hz. (b) Second mode f_{02}= 30.1 Hz. Difference between potential and kinetic energy [Nm]

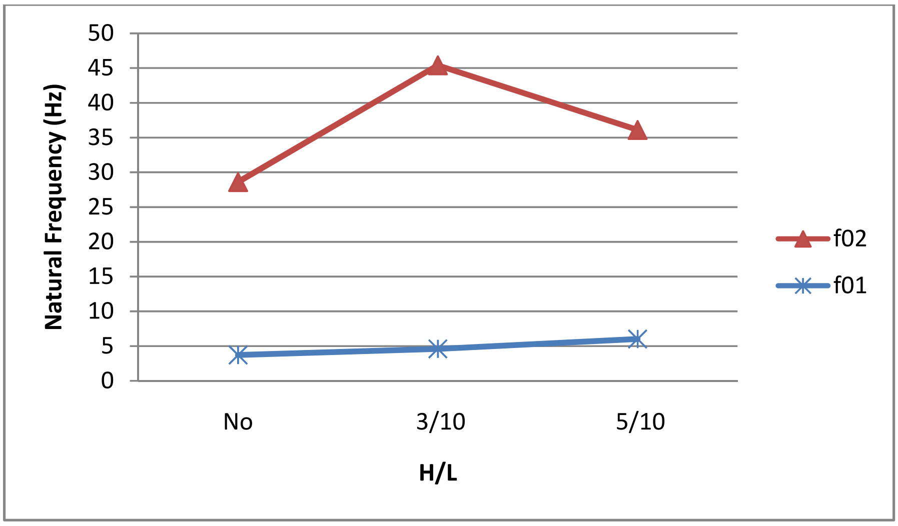


Figure 5.11 Comparison between models by considering the effect of a horizontal beam position (H/L) on the behavior of frame.

5.2.3 Excavator Boom

The boom of a mining excavator can be modeled as a simple 1D structure as shown in figure 5.12.The structure consists of two beams elements and two rods (truss) elements. The dimensions of the horizontal cantilever beam are L=10 m, L_1= L/3, L_2= L/4.

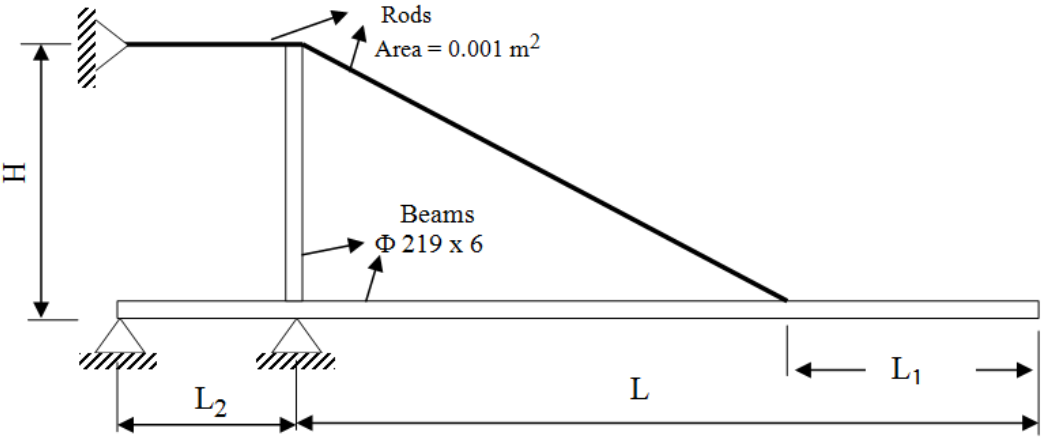


Figure 5.12 Planer 1D model of the Excavator Boom

ρ = 7800 Kg/m^3 **E**= 2.1 Gpa

Three models A, B, C, which have (H = L/4, L/3, L/2) respectively are presented in order to study the effect of the height (H) on the behavior of structure. Figures (5.13-5.21) respectively show the results of the three first natural frequencies and its mode shapes of the three models.

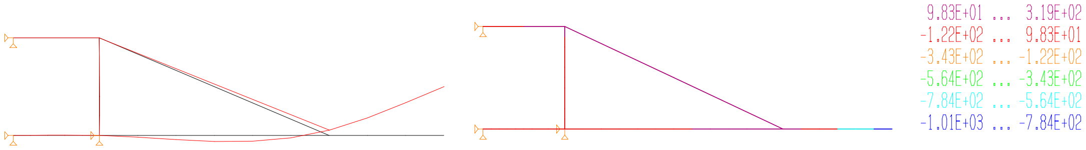


Figure 5.13 FEA of Boom - model A. First mode f_{01}= 9.25Hz. Difference between potential and kinetic energy [Nm]

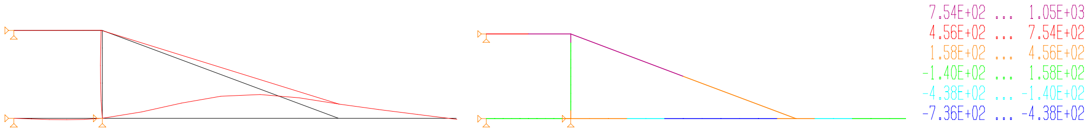


Figure 5.14 FEA of Boom - model A. Second mode f_{02}= 17.37 Hz. Difference between potential and kinetic energy [Nm]

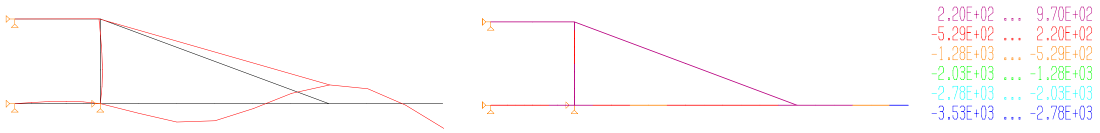


Figure 5.15 FEA of Boom - model A. Third mode f_{03}= 36.93 Hz. Difference between potential and kinetic energy [Nm]

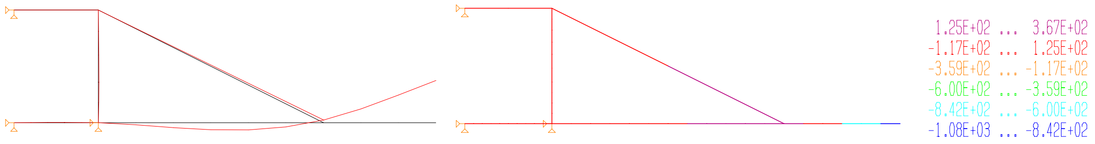


Figure 5.16 FEA of Boom - model B. First mode f_{01}= 9.43Hz. Difference between potential and kinetic energy [Nm]

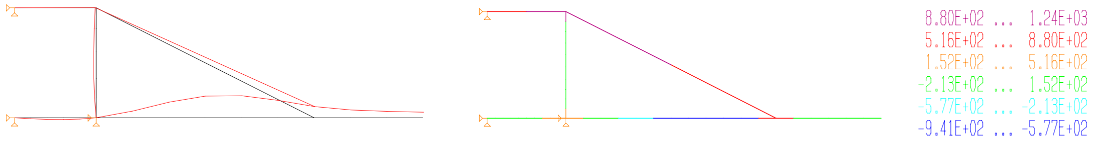


Figure 5.17 FEA of Boom - model B. Second mode f_{02}=19.33 Hz. Difference between potential and kinetic energy [Nm]

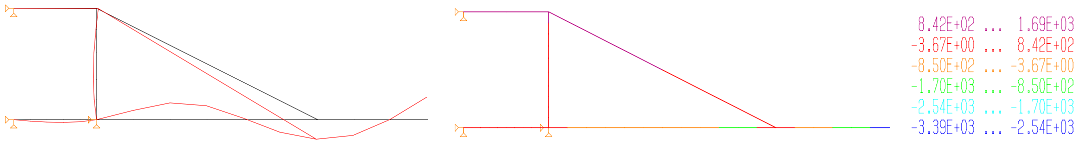


Figure 5.18 FEA of Boom - model B. Third mode f_{03}=38.45 Hz. Difference between potential and kinetic energy [Nm]

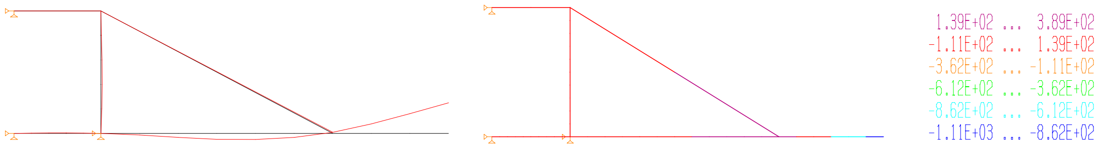


Figure 5.19 FEA of Boom - model C. First mode f_{01}= 9.62Hz. Difference between potential and kinetic energy [Nm]

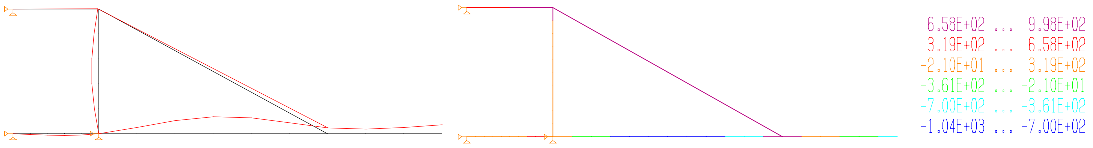


Figure 5.20 FEA of Boom - model C. Second mode f_{02}=20.97 Hz. Difference between potential and kinetic energy [Nm]

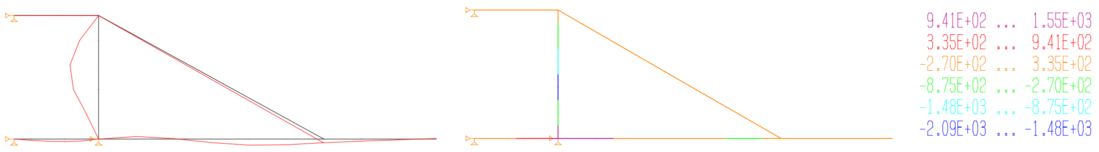


Figure 5.21 FEA of Boom - model C. Third mode f_{03}=33.10 Hz. Difference between potential and kinetic energy [Nm]

By reference to figures (5.13-5.21) one can figure out that model B has a best behavior compared with other two models A and C. Also, based on the energy distribution through the whole structure, it can be seen that the positive values in difference between potential and kinetic energies (purple color) are located on the rods, which means that the structure performance can be improved by increasing the stiffness of rods. Table 5.4 shows the obtained results of the first three natural frequencies when the rod area is 0.002 m^2 instead of 0.001 m^2.

Table 5.4 Comparison of natural frequencies for different models of Excavator Boom

H/L	Rod Area = 0.001 m^2			Rod Area = 0.002 cm^2		
	f_{01} Hz	f_{02} Hz	f_{03} Hz	f_{01} Hz	f_{02} Hz	f_{03} Hz
1/4	9	17.1	36	9.4	19.2	36.3
1/3	9.3	19.1	38.1	9.6	21.1	39.6
1/2	9.5	20.8	33.1	9.6	22.2	33.5

5.3 Structural modification on 2D structure

In this section, the structural modification procedures which are mentioned in Chapter 4 are applied to a rectangular plate as an example of a 2-D structure. In order to investigate the effect of the boundary condition on the dynamic behavior of plate, three cases are presented which are: clamped rectangular plate (membrane structure), simply supported and cantilever plate (shell structure) respectively. In addition, and in order to highlight the effectiveness and the ease of implementation of the proposed approach, a rectangular clamped plate along four edges as the same as that in [38] is presented to investigate the frequency response functions FRFs under structural dynamic modifications.

5.3.1 Clamped rectangular Plate (membrane structure)

A clamped plate is chosen as an example of a 2-D structure. The dimensions of the original plate are 1m X 1m, 10 mm thickness. The material properties are: Young's modulus=210 GPa; Poisson's ratio=0.3; and density=7800 kg/m^3. The purpose is to investigate the effect of boundary condition on the dynamic behavior of plate. Figures (5.22-5.30) show the dynamic response of the plate for five different cases of clamping. It can be seen that the dynamic behavior of plate can be improved by increasing the fixation sides. Furthermore, in each case of clamping, one can clearly decide the interest area of modification according to the distribution in difference between potential and kinetic energy.

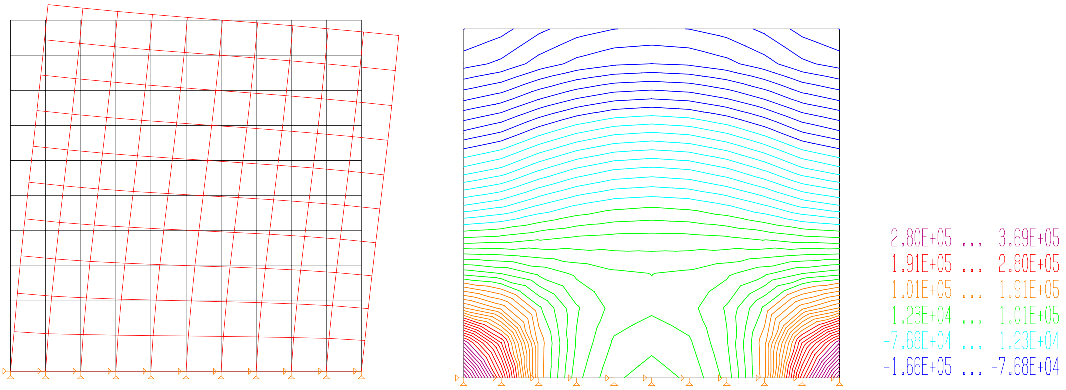


Figure 5.22 FEA of a plate clamped along one side, First mode f_{01}= 532.7 Hz. Difference between potential and kinetic energy [Nm]

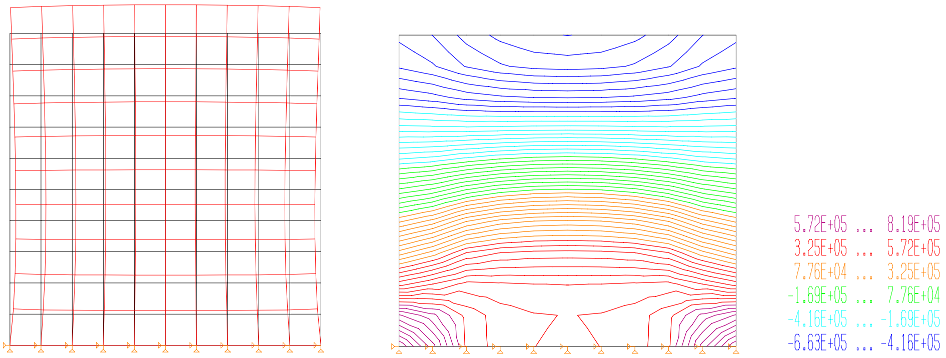


Figure 5.23 FEA of a plate clamped along one side, Second mode f_{02}= 1273. Hz. Difference between potential and kinetic energy [Nm]

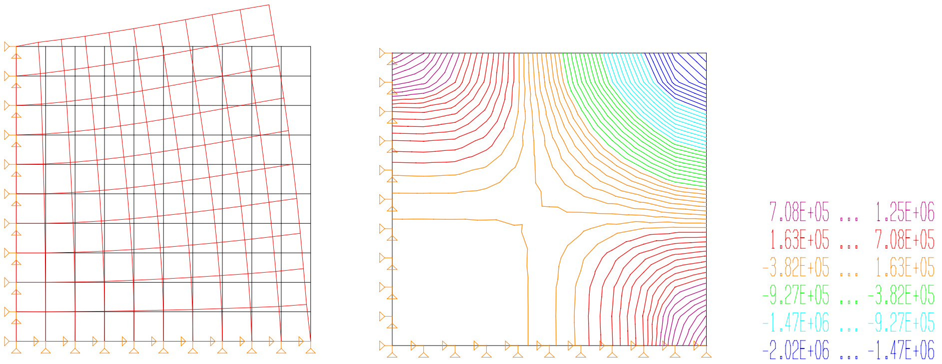


Figure 5.24 FEA of a plate clamped along two adjacent sides, First mode f_{01}= 1238.5 Hz. Difference between potential and kinetic energy [Nm]

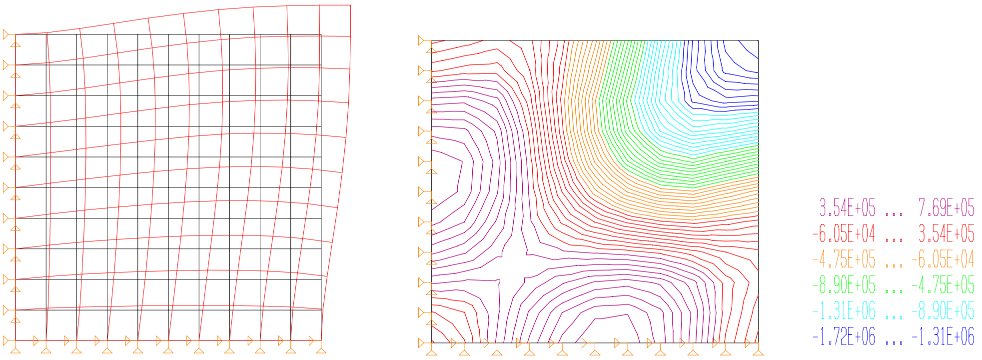


Figure 5.25 FEA of a plate clamped along two adjacent sides, Second mode f_{02}= 1590. Hz. Difference between potential and kinetic energy [Nm]

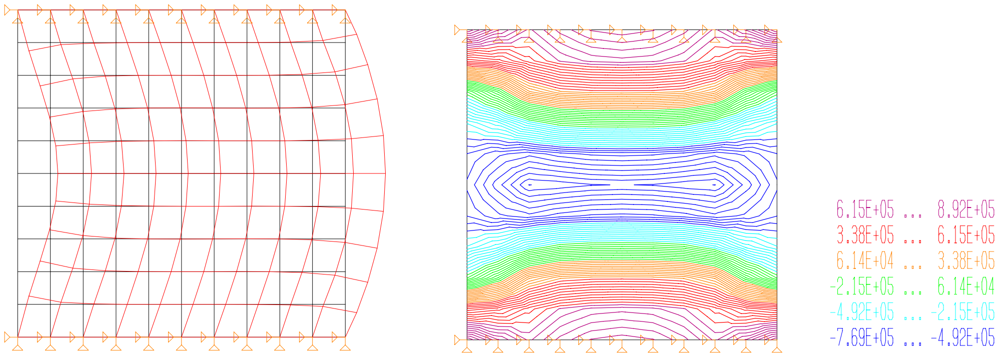


Figure 5.26 FEA of a plate clamped long two opposite sides, First mode f_{01}= 1441. Hz. Difference between potential and kinetic energy [Nm]

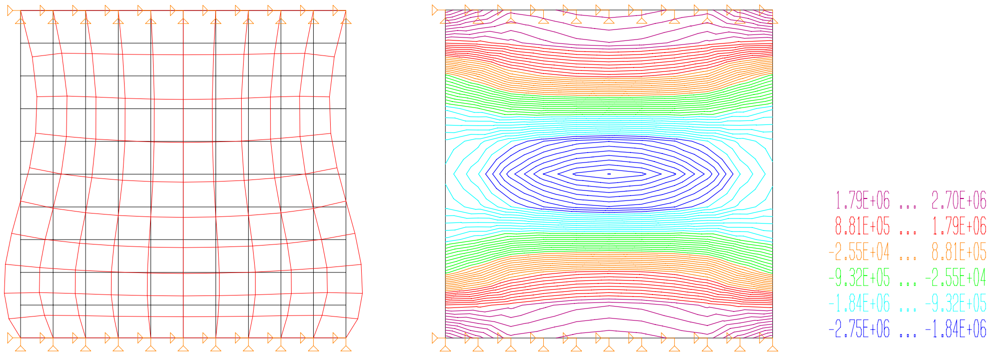


Figure 5.27 FEA of a plate clamped long two opposite sides, Second mode f_{02}= 2546. Hz. Difference between potential and kinetic energy [Nm]

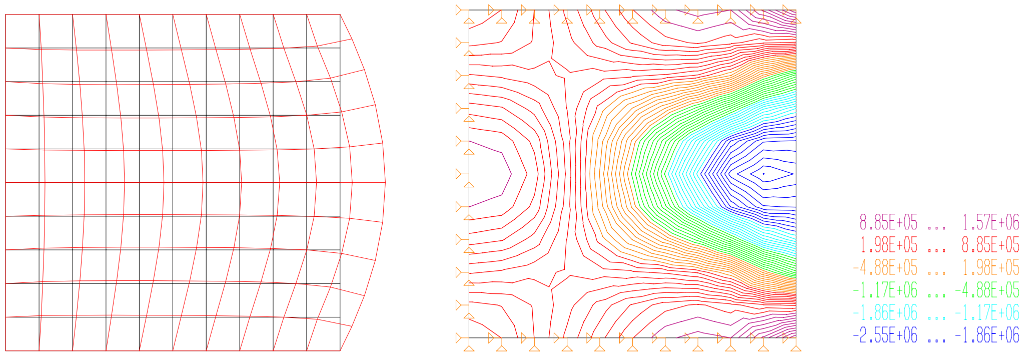


Figure 5.28 FEA of a plate clamped along three sides, First mode f_{01}= 1917. Hz. Difference between potential and kinetic energy [Nm]

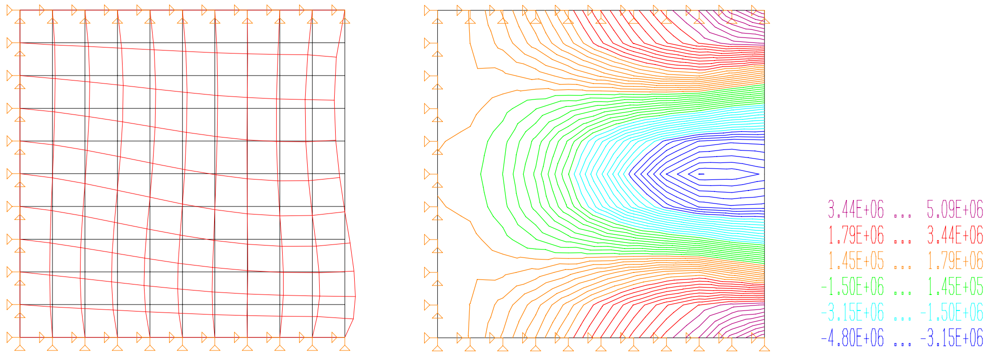


Figure 5.29 FEA of a plate clamped along three sides, Second mode f_{02}= 2661. Hz. Difference between potential and kinetic energy [Nm]

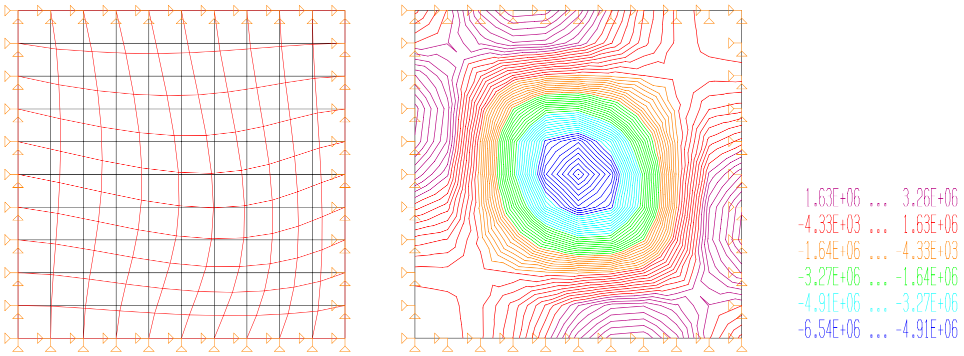


Figure 5.30 FEA of a plate clamped along four sides, First mode f_{01}= 2974. Hz Second mode f_{02}= 2974. Hz. Difference between potential and kinetic energy [Nm]

5.3.2 Simply Supported rectangular Plate (shell structure)

A simply supported plate, which has the same dimensions and properties of the plate described in previous section, is presented to examine the effect of boundary condition on the dynamic behavior of plate. Similar to the previous example, four cases of different boundary conditions are considered and figures (5.31-5.38) show the results.

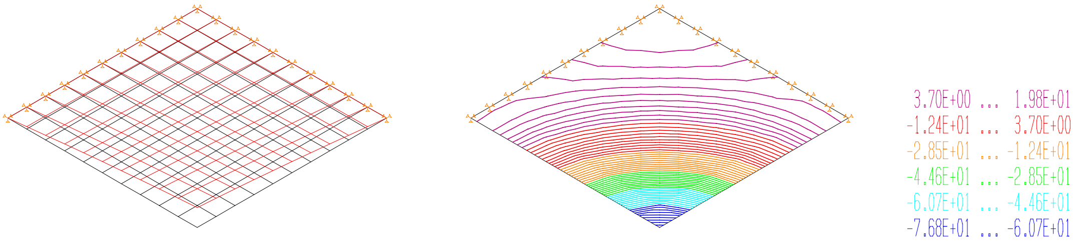


Figure 5.31 FEA of a simply supported plate along two adjacent sides, First mode f_{01}= 8.2 Hz. Difference between potential and kinetic energy [Nm]

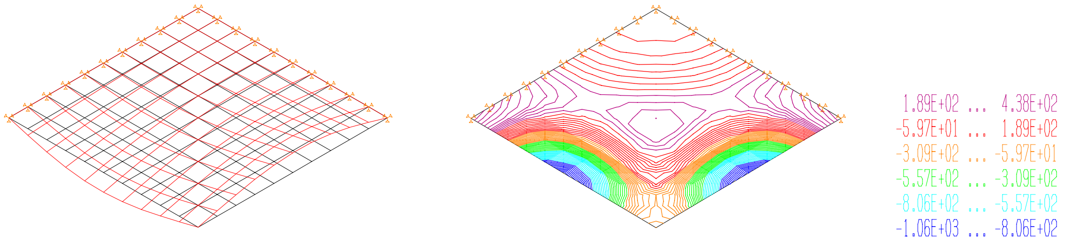


Figure 5.32 FEA of a simply supported plate along two adjacent sides, Second mode f_{02}= 41.6 Hz. Difference between potential and kinetic energy [Nm]

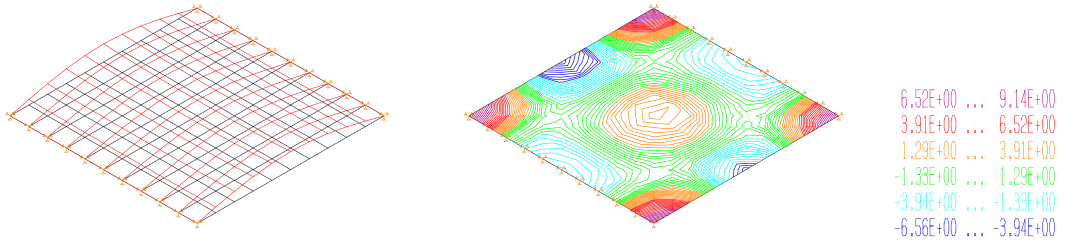


Figure 5.33 FEA of a simply supported plate along two opposite sides, First mode f_{01}= 23.5 Hz. Difference between potential and kinetic energy [Nm]

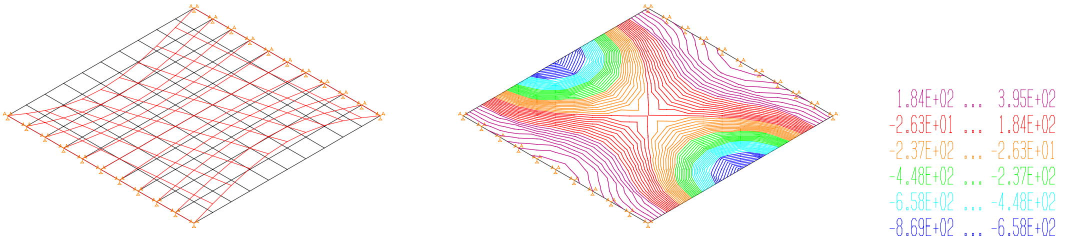


Figure 5.34 FEA of a simply supported plate along two opposite sides, Second mode f_{02}= 39.1 Hz. Difference between potential and kinetic energy [Nm]

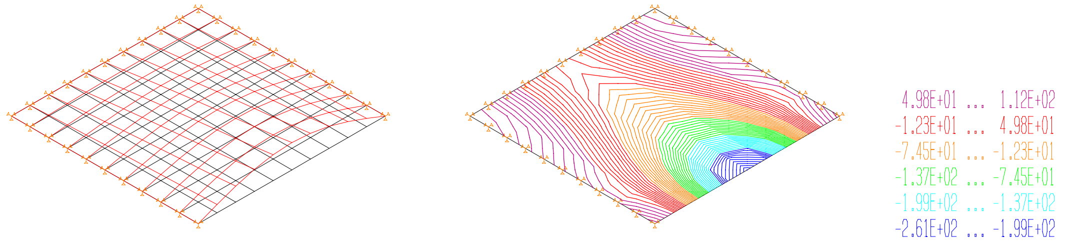


Figure 5.35 FEA of a simply supported plate along three sides, First mode f_{01}= 28.5 Hz. Difference between potential and kinetic energy [Nm]

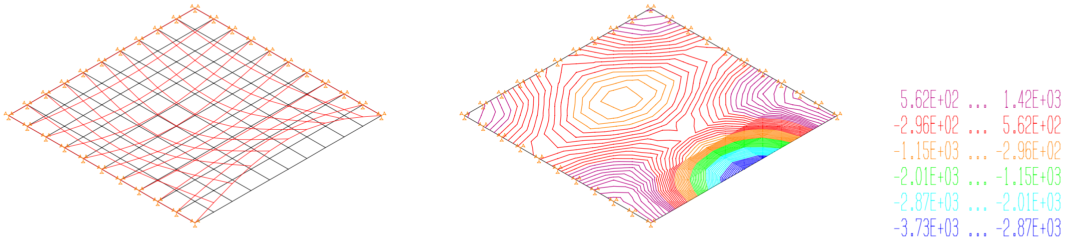


Figure 5.36 FEA of a simply supported plate along three sides, Second mode f_{02}= 67.2 Hz. Difference between potential and kinetic energy [Nm]

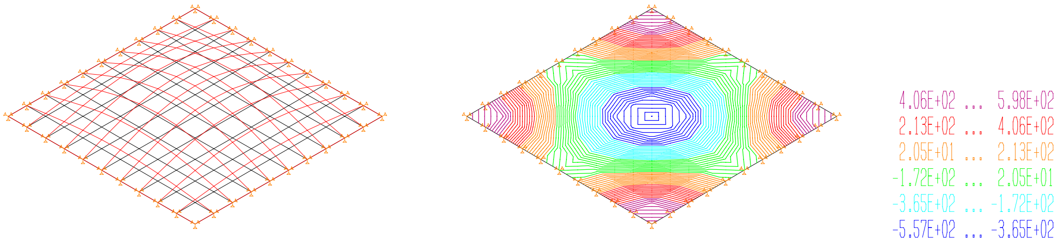


Figure 5.37 FEA of a simply supported plate along four sides, First mode f_{01}= 47.9 Hz. Difference between potential and kinetic energy [Nm]

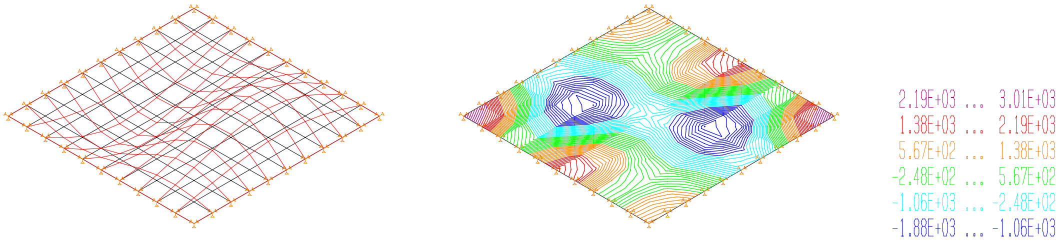
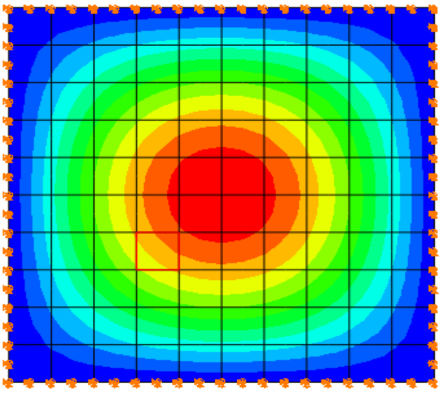


Figure 5.38 FEA of a simply supported plate along four sides, Second mode f_{02}= 119.9 Hz. Difference between potential and kinetic energy [Nm]

In each case, it is clear through the energy distribution that the dynamic behavior of the plate can be improved by increasing the rigidity of the elements which close to the boundary conditions, where the positive values in difference between potential and kinetic energy (purple and red colors) are located at those zones. The sensitivity analysis can be done for the case of the simply supported plate along four sides by change the thickness of some certain elements in the plate. Table (5.5) shows the sensitivity analysis of first and second natural frequencies to the change in the thickness of some element. The analysis has been carried out by using Abaqus software [115].



1	1	2	2	3	3	2	2	1	1
1	1	2	2	3	3	2	2	1	1
2	2	4	4	4	4	4	4	2	2
2	2	4	4	4	4	4	4	2	2
3	3	4	4	5	5	4	4	3	3
3	3	4	4	5	5	4	4	3	3
2	2	4	4	4	4	4	4	2	2
2	2	4	4	4	4	4	4	2	2
1	1	2	2	3	3	2	2	1	1
1	1	2	2	3	3	2	2	1	1

Figure 5.39 Meshing and section numbering of a simply supported plate along four sides.

Table 5.5 first and second natural frequencies for a simply supported plate depending on the change in the thickness of some element

section	Modified models					
	O	A	B	C	D	E
	Thickness (mm)					
1	10	11	12	12	15	15
2	10	10	10	11	13	13
3	10	10	10	10	10	12
4	10	10	10	10	10	11
5	10	10	10	10	10	10
	f_{01}=47.9 Hz	f_{01}=49.21 Hz	f_{01}=50.64 Hz	f_{01}=51.86 Hz	f_{01}=59.32 Hz	f_{01}=57.18 Hz
	f_{02}=119.9 Hz	f_{02}=121.57 Hz	f_{02}=123.3 Hz	f_{02}=125.85 Hz	f_{02}=136.04 Hz	f_{02}=121.2 Hz

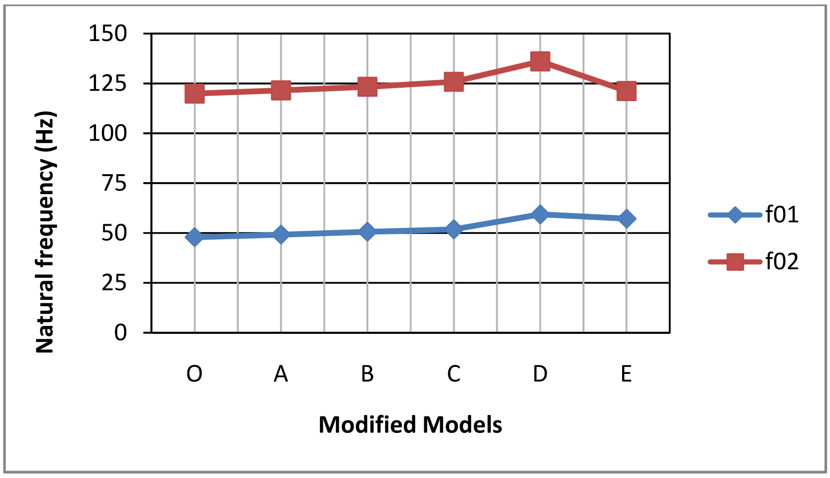
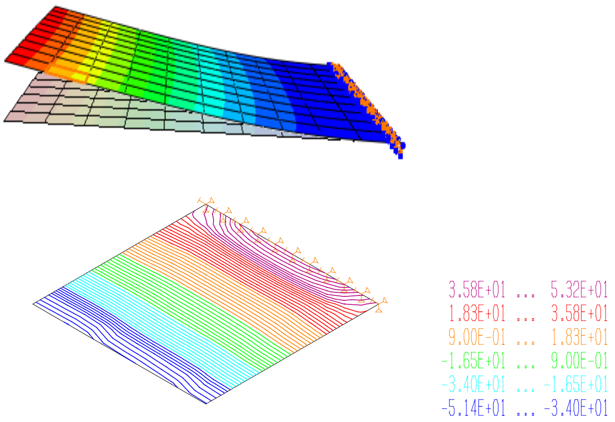


Figure 5.40 studying of the effect of elements thickness on the dynamic behavior of the plate

From table 5.5 and figure 5.40 one can figure out that model D has the best dynamic characteristics. Because the modification for this model was done based on the energy distribution to the elements near to the boundary conditions, the best results were obtained. Hence, that is evidence that the energy distribution gives a clear view to the problem, which helps to make appropriate decision for structure modifications.

5.3.3 Cantilever plate (shell structure)

Similar to the previous example the diagnostic and sensitivity analysis was done to the clamped plate which has the same dimensions and property of the plate described in section 5.3.1. Figure (5.41) shows the first mode of the plate. Table (5.6) shows the sensitivity analysis of first and second natural frequencies to the change in the thickness of some element. The modifications have been done based on the distribution of energy and the results show that model E is the best modified model.



5	5	4	4	3	3	2	2	1	1
5	5	4	4	3	3	2	2	1	1
5	5	4	4	3	3	2	2	1	1
5	5	4	4	3	3	2	2	1	1
5	5	4	4	3	3	2	2	1	1
5	5	4	4	3	3	2	2	1	1
5	5	4	4	3	3	2	2	1	1
5	5	4	4	3	3	2	2	1	1
5	5	4	4	3	3	2	2	1	1
5	5	4	4	3	3	2	2	1	1

Figure 5.41 FEA of a cantilever plate, First mode f_{01}= 8.4 Hz. Difference between potential and kinetic energy [Nm].

Table 5.6 first and second natural frequencies for a clamped plate depending on the change in the thickness of some element

section	Modified models					
	O	A	B	C	D	E
	Thickness (mm)					
1	10	11	12	12	15	15
2	10	10	10	11	12	13
3	10	10	10	10	11	12
4	10	10	10	10	10	11
5	10	10	10	10	10	10
	f_{01}=8.4 Hz	f_{01}=9.17 Hz	f_{01}=9.77 Hz	f_{01}=10.27 Hz	f_{01}=12.74 Hz	f_{01}=13.42 Hz
	f_{02}=20.5 Hz	f_{02}=21.52 Hz	f_{02}=22.28 Hz	f_{02}=22.97 Hz	f_{02}=26.30 Hz	f_{02}=28.00 Hz

5.3.4 Clamped rectangular plate (shell structure)

An aluminum plate clamped along four sides (Figure 5.42) is presented. The focus of this example is to calculate and improve the frequency response functions FRFs of the plate based on the procedure of reanalysis. The plate is divided into 20 shell elements per side and the geometry and material properties of the original plate is the same as that in [38] and are listed in Table 5.7.

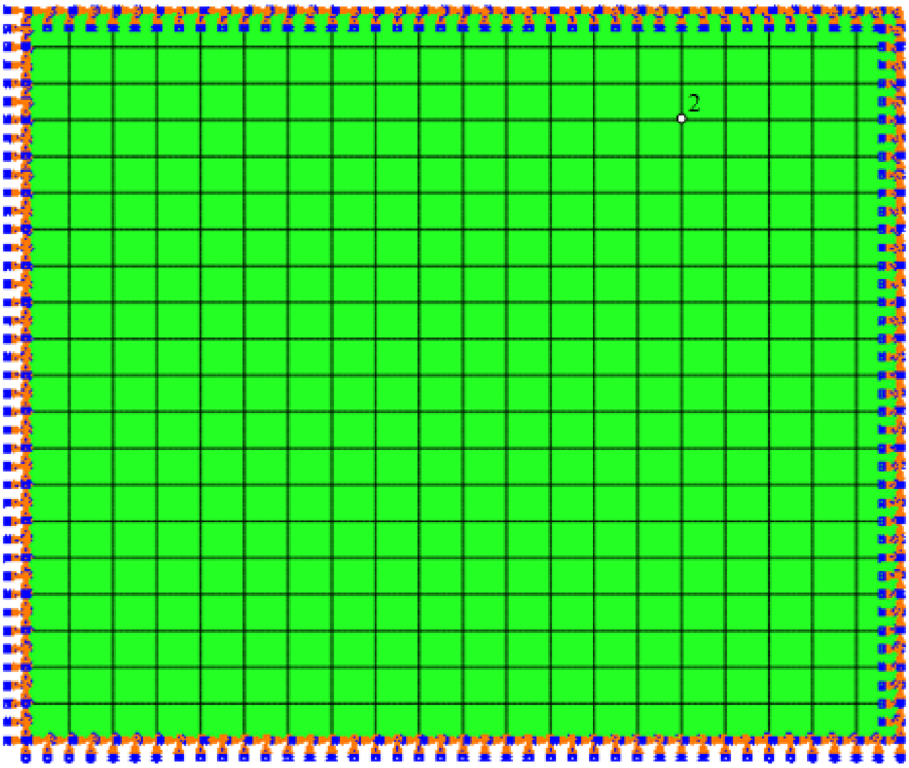


Figure 5.42 FE model of the original clamped rectangular plate.

Table 5.7 Geometry and material parameters of original plate

Length	490 mm
Width	410 mm
Young's modulus	70 GPa
Poisson's Ratio	0.33
Density	2800 Kg/m^3

Similar to the analysis done by Hang [38] (Figure 5.43), and by using the harmonic analysis in ABAQUS V6.7, the FRF of the original plate is obtained within the frequency range of 0.25 Hz to 800 Hz and a 0.25 Hz frequency resolution. Figure 5.44 shows the calculated FRFs of the original plate at location (2). This location was selected because it has reasonable responses for all the modes within the frequency range of interest [38].

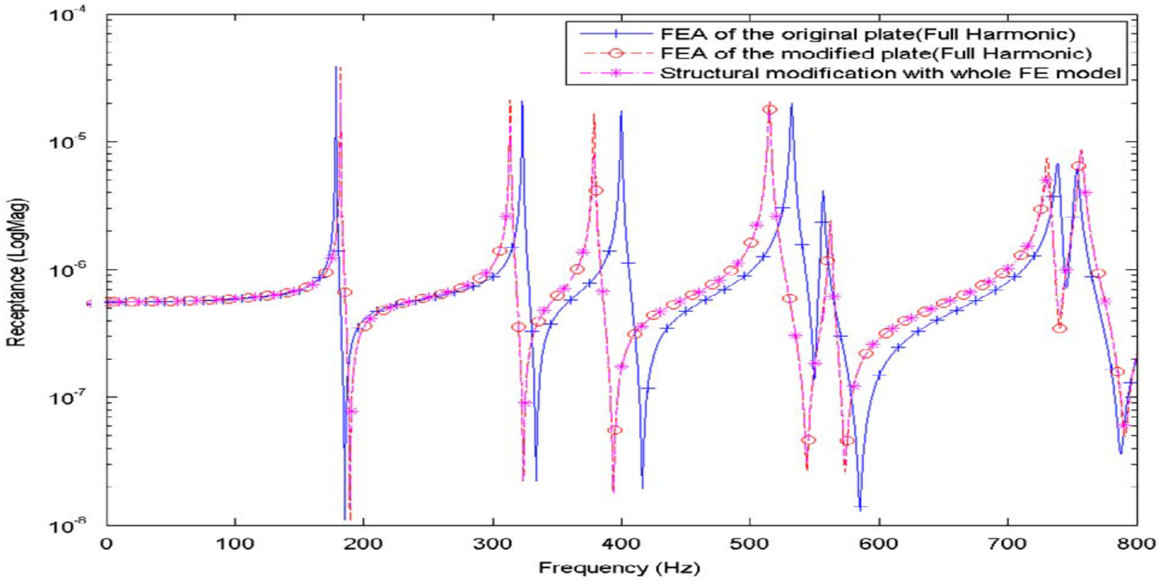


Figure 5.43 Calculated FRF of original plate at location 2. [38].

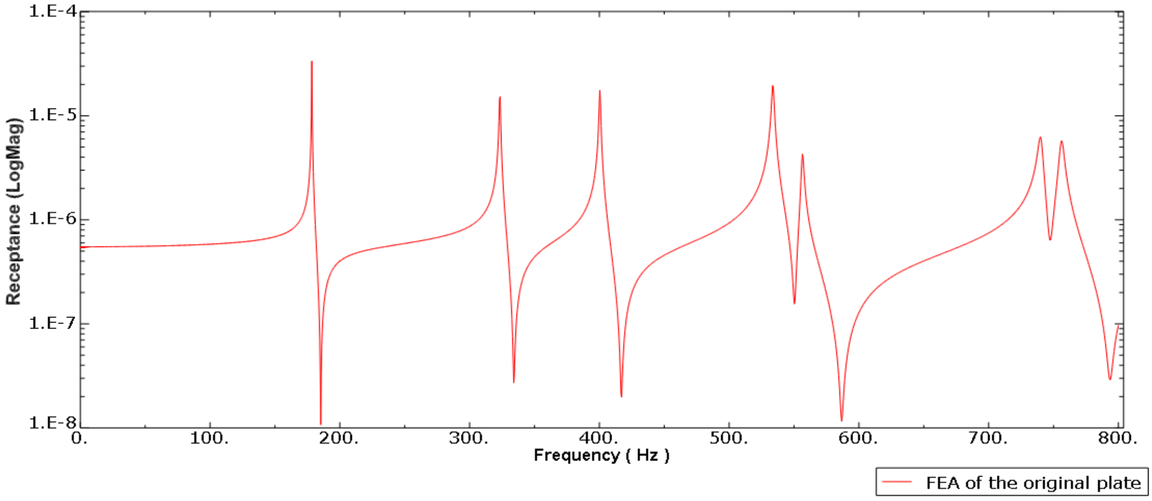


Figure 5.44 Calculated FRF of original plate at location 2.

Figure 5.45 shows the first mode of oscillation of the original plate. Based on the reanalysis procedure described previously in this thesis, the dynamic behavior of the plate can be improved by increasing the rigidity of the elements located near to the boundary conditions, where the positive values in difference between potential and kinetic energy (purple and red colors) are located at those zones. Three models, model 1, model 2 and model 3 respectively, are proposed and the thickness of those elements (shaded elements in figure 5.46 a) have been modified to become 4.5 mm instead of 4 mm for model 1, and 5 mm instead of 4 mm for model 2. While for model 3, the shaded elements in figure 5.46 (b) have been modified to become 5 mm instead of 4 mm.

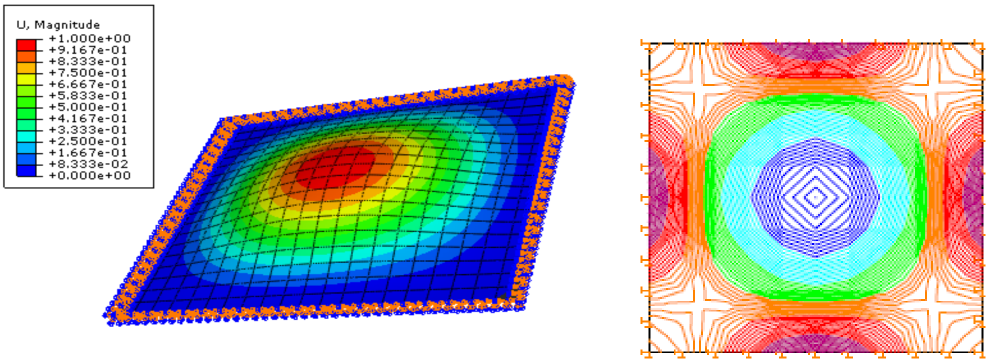
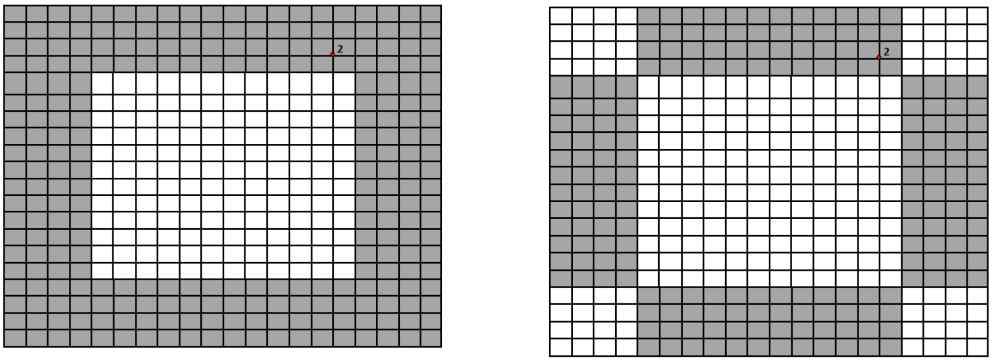


Figure 5.45 FEA of a clamped plate along four sides, First mode f_{01}= 178.4 Hz Second mode f_{02}=323.3 Hz. Difference between potential and kinetic energy [Nm].



(a) (b)

Figure 5.46 FE model of the modified clamped rectangular plate. (a): Model 1 & 2, (b): Model 3.

Figure 5.47 shows the comparison of FRFs of original and modified plates at location (2). In addition, Table 5.8 shows the first seven natural frequencies for the original and modified models, which is located within the frequency range of 0.25 Hz to 800 Hz based on the original plate model. As a consequence, it can be seen that modified models have got a best dynamic behavior compared with the original one. As mentioned before, the highest value of the first frequency of the structure the best structure performance. Moreover, it is apparent that the proposed method improves the plate response, where the natural frequencies of model 2 and model 3 have been reduced in to five frequencies instead of seven frequencies within the frequency range.

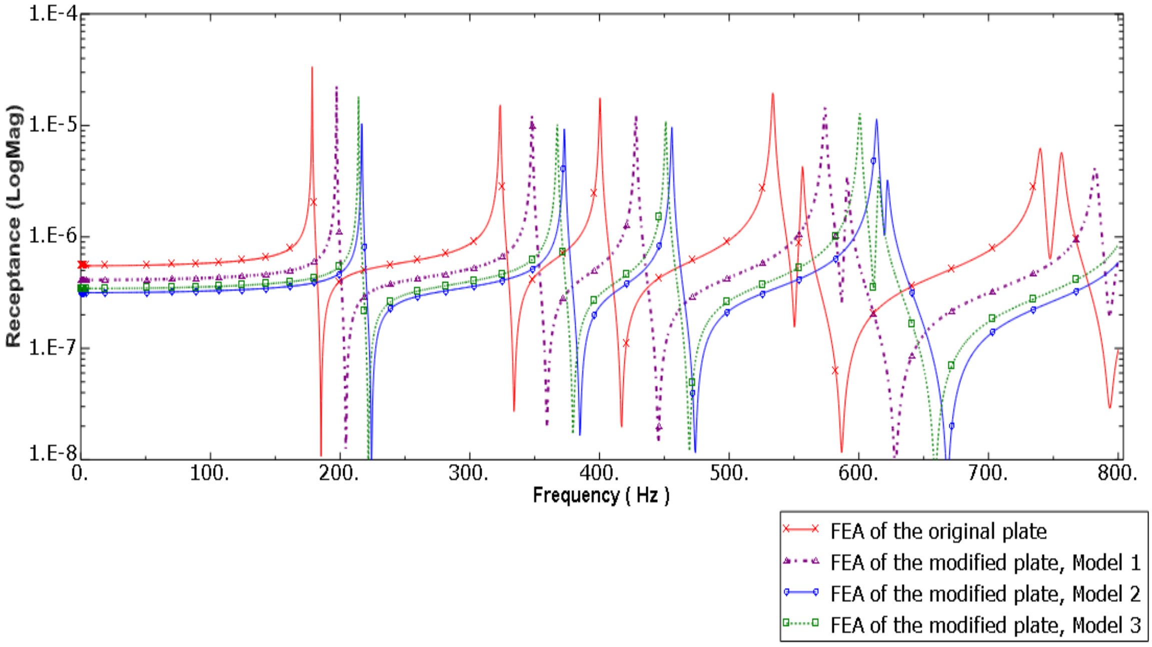


Figure 5.47 Comparison of FRFs of original and modified plate at location 2.

Table 5.8 Comparison of calculated natural frequencies of the original plate and modified models

Mode	**Calculated natural frequency (Hz)**			
	Original plate	Model 1	Model 2	Model 3
1	178.43	197.22	216.76	214.23
2	323.37	347.99	372.93	367.41
3	400.42	428.05	455.78	451.08
4	533.75	573.89	613.76	600.73
5	556.51	590.91	621.81	615.20
6	740.16	782.61	819.48	814.12
7	756.18	810.83	862.87	843.39

Model 3 is the closest modified model which has been proposed based on the distribution of energy as shown in figure 5.45. Although model 2 has a highest value of the first natural frequency, however, model 3 can be considered as the best model because it has natural frequencies values close to model 2 with a less weight, which emphasis that the proposed method provides effective results. Thus, if more precise modifications according to the energy distribution will be done to the plate, one can predict that the plate performance will get more improvement.

Chapter 6

Implementation of reanalysis technique on real complex structures

The aim of developed the proposed method of reanalysis and diagnostic of structure behavior is to determine real behavior of the construction in exploitation. Because most of the dynamic problems that occur during the operation of machinery mainly come from insufficiently geometrical designs, redesign of these structures, therefore, is required in order to overcome the operation problems. In addition, most of complex structures are obtained by assembling components. Thus, it is desirable to simplify a complex problem to an easier problem. The problem of modeling a complicated structure could be greatly simplified by first dividing the structure into components, each of which could be represented as a substructure model. In substructure analysis, it is common to break down the whole structure into a number of components, or substructures, each of which is analyzed individually using whichever method is the most convenient [110].

Modeling of complex structures using finite elements method is a helpful approach in solving problems in short time with reliable results, and it has been considered in many papers and PhD theses [104,107,111,112,113,114].

In order to evaluate structures precisely and obtained accurate results, structures shall be modeled as close to the actual structural conditions in reality as possible. Furthermore, to get a compatible model for real complex structure, numerical analysis and experiments should be done on a prototype model. Accordingly, the best model is achieved when good agreement of results is obtained from the numerical analysis and the experiments.

6.1 Diagnosis of Dynamic Behavior of Real Complex Structures

Unlike the previous chapter where aforementioned structures could be deemed somewhat simple problems, the reanalysis procedures are applied in this chapter on real complex structures. It has already been pointed out that the distributions of potential and kinetic energies of elements of the whole structure give a clear view to the problem, which helps to make appropriate decision for structure modifications. Consequently, proceeding from this principle, energy analysis has been conducted on some real complex structures to locate the position of dynamic modifications if needed.

6.1.1 Diagnosis of Dynamic Behavior of Portal Cranes

Cranes are transport machines, which generally used in heavy machinery industry, shipyards, seaports, warehouses and construction sector. There are several factors that have to be taken into consideration when a crane being designed. Most important factors are; own weight of the crane, the weight of the bulk which has to be transported and the dynamic loads which occur during the movements. Moreover, for the cranes which operate in open-air, the external loads caused by wind and the other climate conditions have to be considered. In order to prevent possible accidents which can cause enormous losses after manufacturing, all these factors have to be taken into account during the design process [116]. In this section, three portal cranes have been investigated in order to diagnosis its behavior without the completeness of reanalysis procedures, and then, one can decides which modification should be done to avoid problems resulting of a heavy duty operating. Calculations of main modes of oscillation were performed using Abaqus [115] while the energy distributions using KOMIPS [104].

Model 1

Electric gantry cranes, span 15m between the legs (Figure 6.1), are designed to work outdoors, on a dam ĐERDAP Kladovo. Gantry cranes are designed for servicing the hydro-mechanical equipment at the dam and the entrance building, mounting block, and for unloading equipment from the locks.

In order to investigate the dynamic behavior of the crane, a Finite Element Model has been proposed. For the finite element analysis, the total node number used is 8666 and the element number is 9702 for the gantry crane. The element type used in the model is four node linear shell element (S4R) for 9686 elements, and 16 elements are triangular elements of type (S3). The material properties are: Young’s modulus=200 GPa; Poisson’s ratio=0.3; and density=7800 kg/m^3.

Figures (6.2-6.4) show the results of the first three natural frequencies and its mode shapes. As previously mentioned in this thesis, one can decide the required modifications for the crane, if needed, based on the energy distribution through the model.

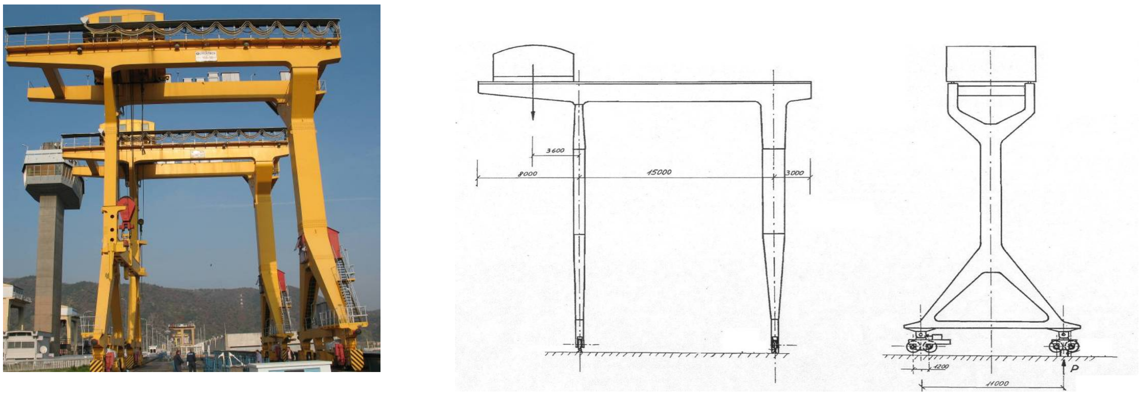


Figure 6.1: Electric gantry cranes in- situ, Model 1.

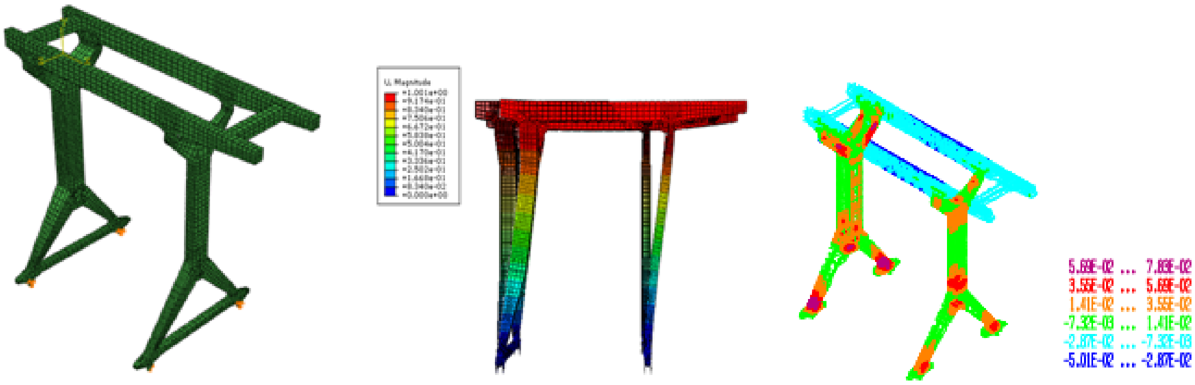


Figure 6.2: FEA of model 1. First mode, f_{01}= 1.327 Hz. Difference between potential and kinetic energy [Nm]

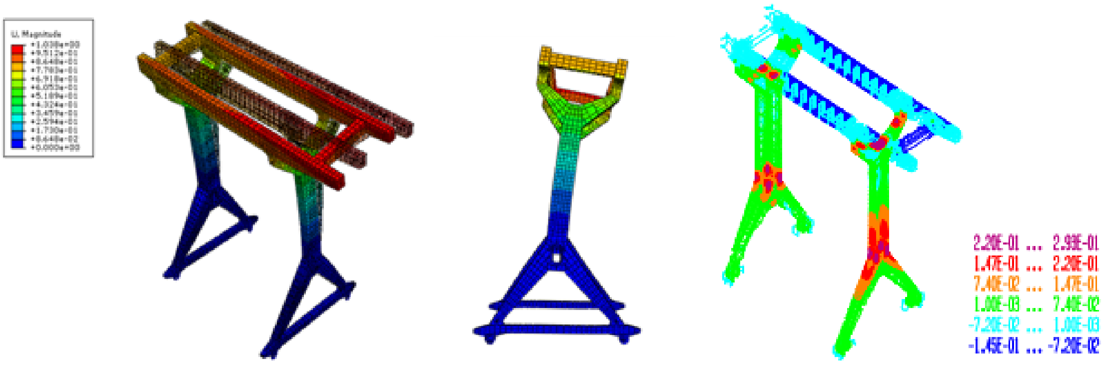


Figure 6.3: FEA of model 1. Second mode, f_{02}= 1.9 Hz. Difference between potential and kinetic energy [Nm]

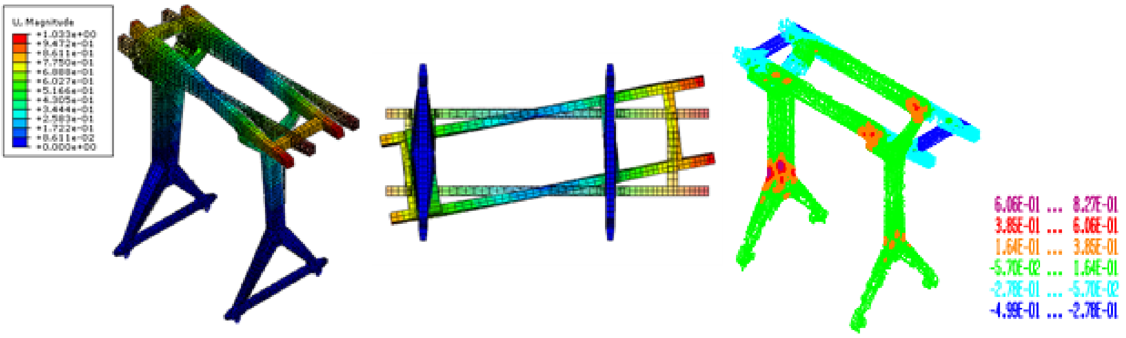


Figure 6.4 FEA of model 1. Third mode, f_{03}= 3.12 Hz. Difference between potential and kinetic energy [Nm].

Model 2

Similar to the previous example, a gantry cranes, as shown in figure 6.5, is modeled in order to diagnosis its dynamic behavior. For the finite element analysis, the total node number used is 6442 and the element number is 7108 for the gantry crane. The element type used in the model is four node linear shell element (S4R). The material properties are same as that used for the previous crane. Figures (6.2-6.4) show the results of the first three natural frequencies and its mode shapes.

Figure 6.5: Gantry cranes in- situ, Model 2.

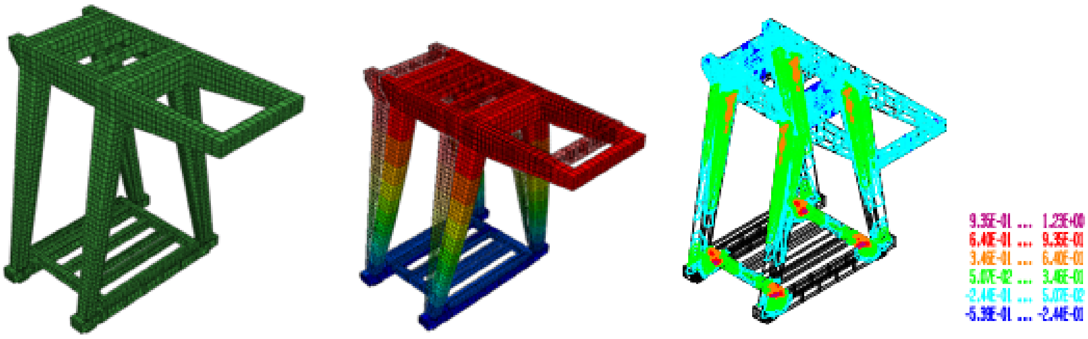


Figure 6.6: FEA of model 2. First mode, f_{01}= 4.46 Hz. Difference between potential and kinetic energy [Nm]

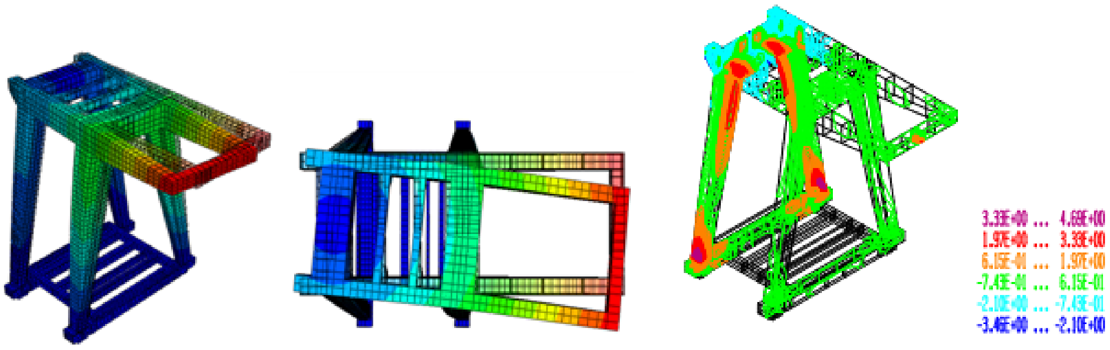


Figure 6.7: FEA of model 2. Second mode, f_{02}= 7.4 Hz. Difference between potential and kinetic energy [Nm]

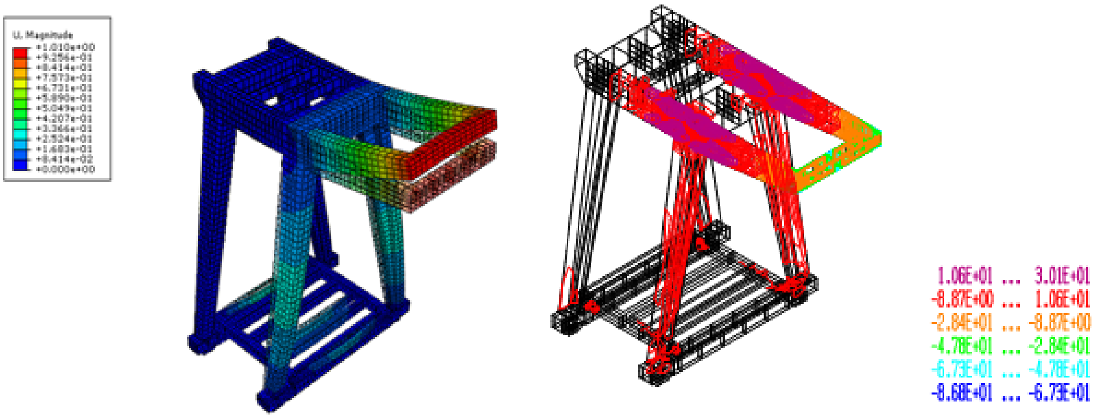


Figure 6.8 FEA of model 2. Third mode, f_{03}= 23.2 Hz. Difference between potential and kinetic energy [Nm].

Model 3

The third model for this investigation is a portal crane as shown in figure 6.9. The investigation of this model had been done as same as model 1 and model 2. For the finite element analysis, the total node number used for this model is 7126 and the element number is 6780. The element type used in the model is four node linear shell element

(S4R). The material properties are same as that used for the previous models. Figures (6.2-6.4) show the results of the first three natural frequencies and its mode shapes.

Figure 6.9: Portal gantry crane in- situ, Model 3.

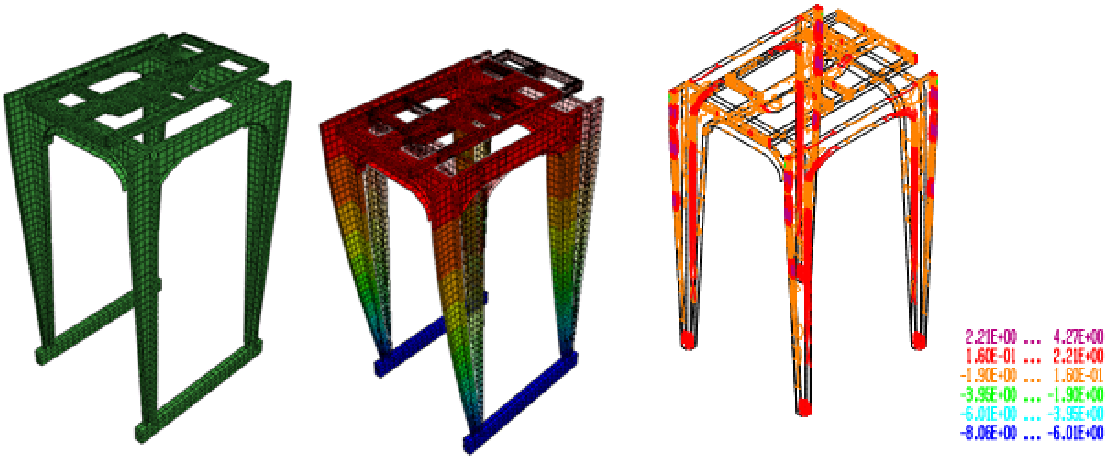


Figure 6.10: FEA of model 3. First mode, f_{01}= 3.04 Hz. Difference between potential and kinetic energy [Nm]

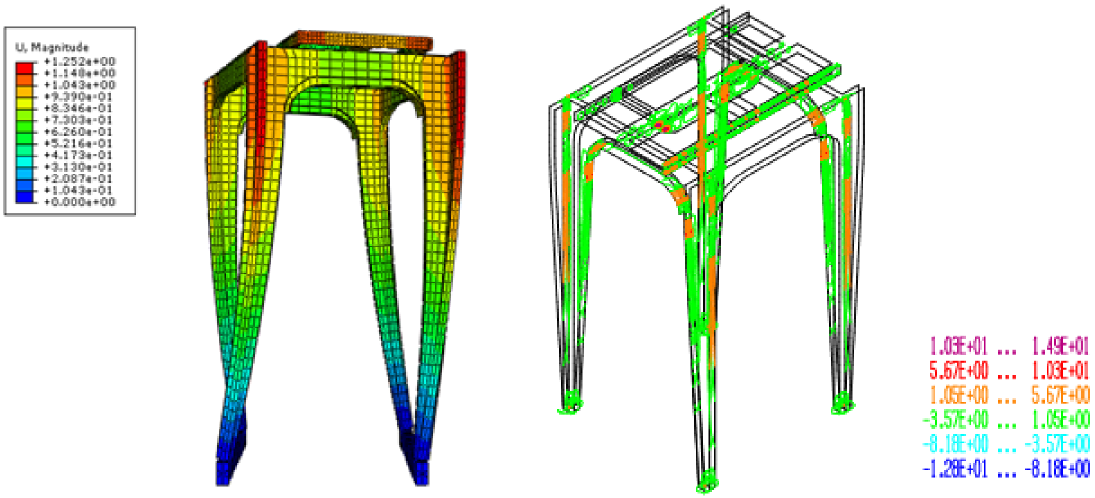


Figure 6.11: FEA of model 3. Second mode, f_{02}= 3.64 Hz. Difference between potential and kinetic energy [Nm]

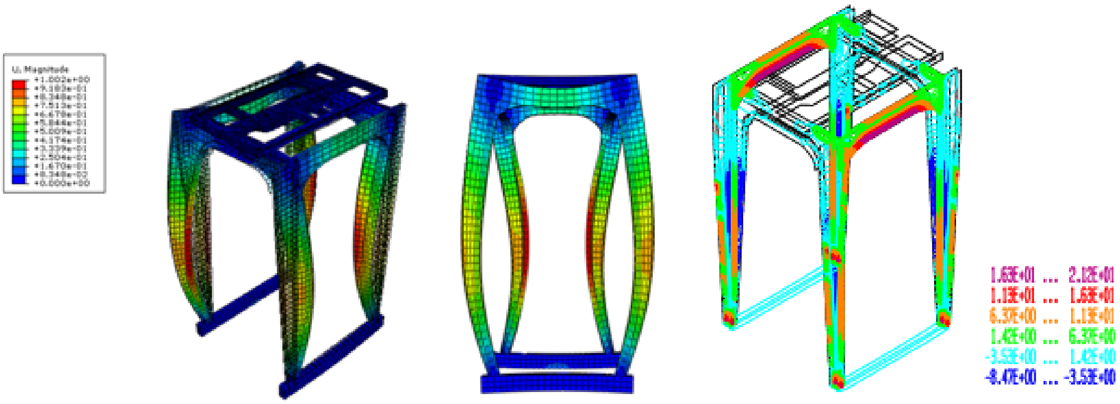


Figure 6.12: FEA of model 3. Third mode, f_{03}= 20.26 Hz. Difference between potential and kinetic energy [Nm].

6.1.2 Case Study 1

Bucket wheel excavators are complex systems, with numerous functionally important components. This wheel excavator is working in cement factory BFC Lafarge Beocin. In this thesis the diagnostic of dynamic behavior of the bogie rotary excavator has been done in order to achieve the appropriate reconstruction. The study consists of two parts. The first is a numerical and experimental study to the prototype which simulates the real structure, and the second is the implementation of the proposed method to improve its dynamic characteristics. Calculations of main modes of oscillation were performed using Abaqus [115] while the energy distributions using KOMIPS [104]. In this analysis plate finite elements are used.

6.1.2.1 Numerical and Experimental study

In order to validate finite element analysis, numerical and experimental analysis has been done on the prototype model which simulates the bogie rotary excavator (Figure 6.13). The prototype was manufactured with dimensions less than the original 5 times. The parts that have been used to manufacture the prototype model are shown in Figure 6.14. The source of excitation is an unbalanced disk which is connected with a rotating motor as shown in figure 6.13. The speed ratio between the motor and the unbalanced disk is 1.5, and the motor operates at a frequency up to 50 Hz. Unlike the original model where the fixation is on all four sides, the prototype has been fastened at four points, because it is stiffer. The experiments have been done at different rotational speeds of the motor as listed in table 6.1. Figures 6.15 show the experimental results of the prototype model at several values of motor rotational speed.

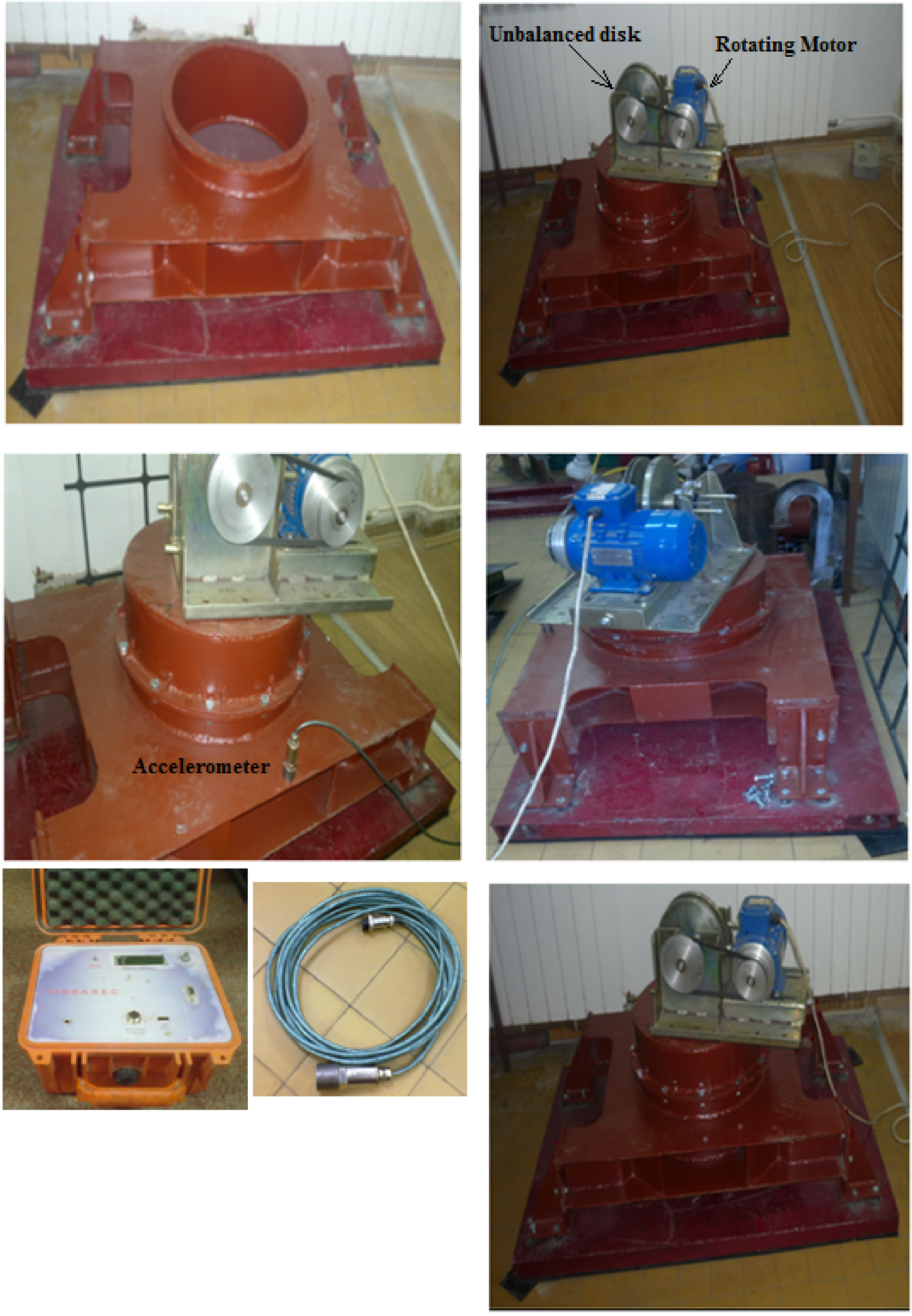


Figure 6.13: Photographs of Prototype modal under modal testing

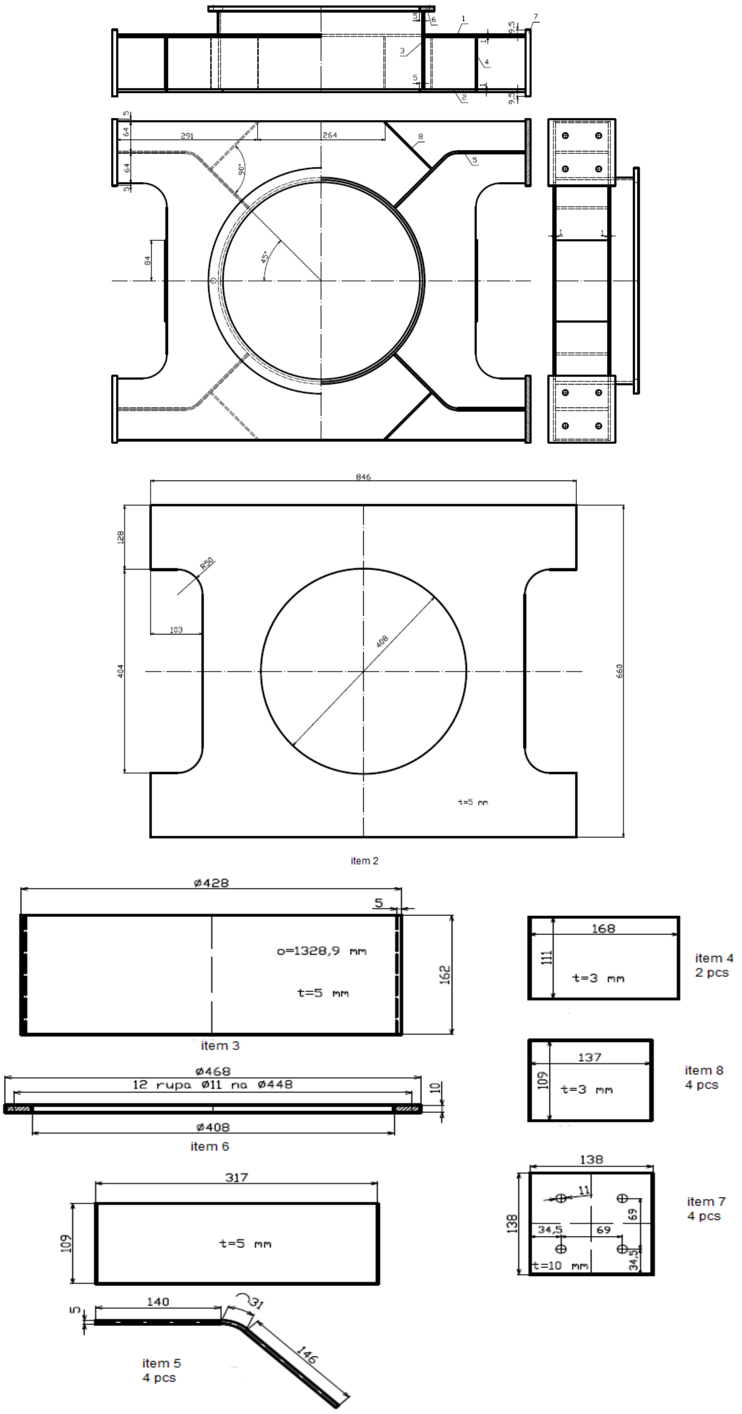
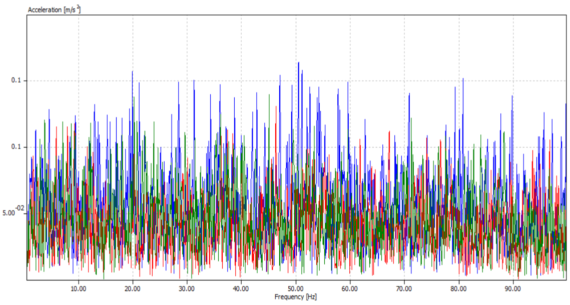


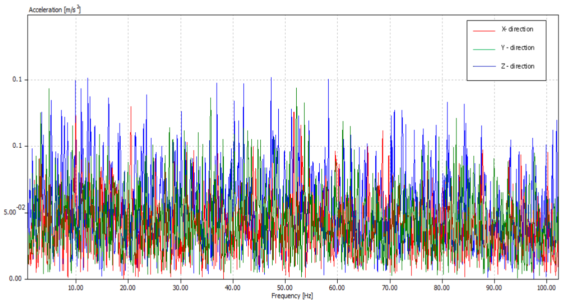
Figure 6.14: drawing for parts used to manufacturing prototype model

Table 6.1: Number of tests corresponding to different rotational speeds of the motor.

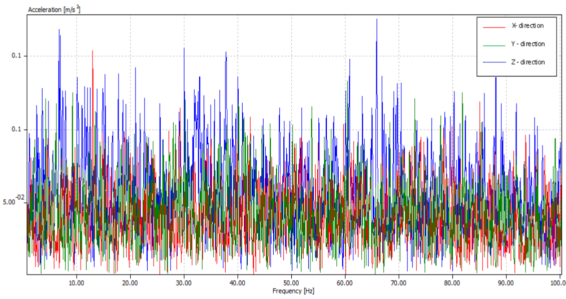
No .of test	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Motor frequency Hz	2	4	6	8	10	12	14	16	18	20	22
frequency /1.5	1.33	2.7	4.0	5.3	6.7	8.0	9.3	10.7	12.0	13.3	14.7
No .of test	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
Motor frequency Hz	24	26	27	28	29	30	31	32	33	34	35
frequency /1.5	16.0	17.3	18.0	18.7	19.3	20.0	20.7	21.3	22.0	22.7	23.3
No .of test	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	
Motor frequency Hz	36	37	38	39	40	41	42	43	45	46	
frequency /1.5	24.0	24.7	25.3	26.0	26.7	27.3	28.0	28.7	30.0	30.6	



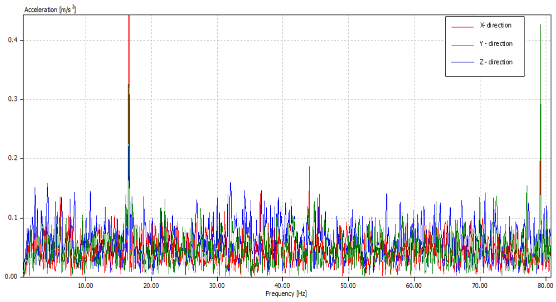
Results of test no. 1 in x-y-z directions



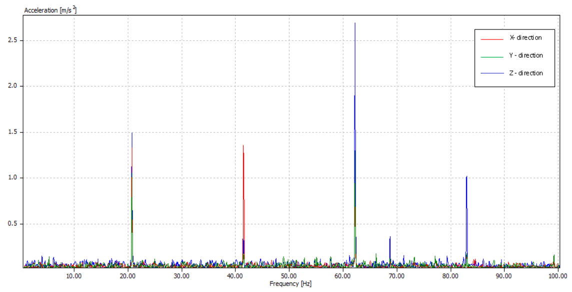
Results of test no. 5 in x-y-z directions



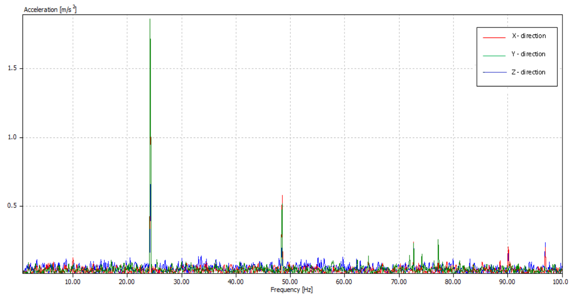
Results of test no. 8 in x-y-z directions



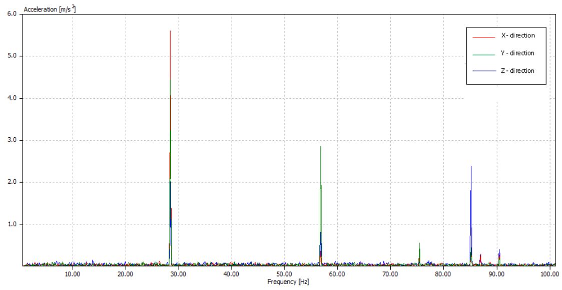
Results of test no. 12 in x-y-z directions



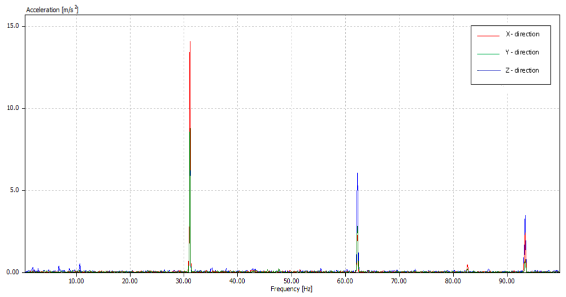
Results of test no. 17 in x-y-z directions



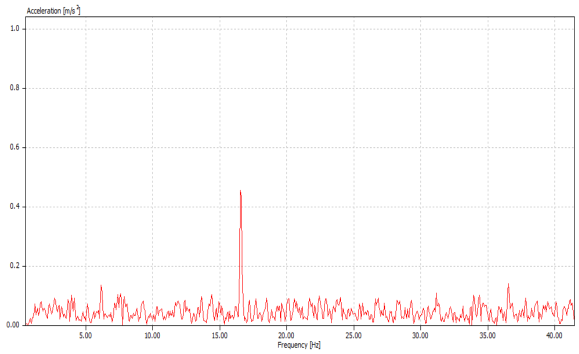
Results of test no. 22 in x-y-z directions



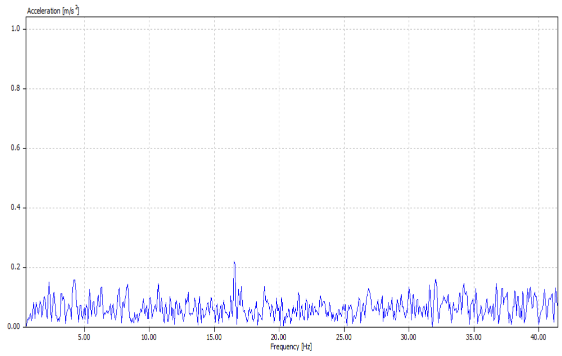
Results of test no. 27 in x-y-z directions



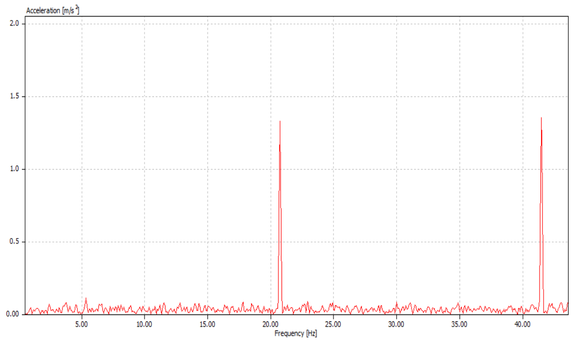
Results of test no. 31 in x-y-z directions



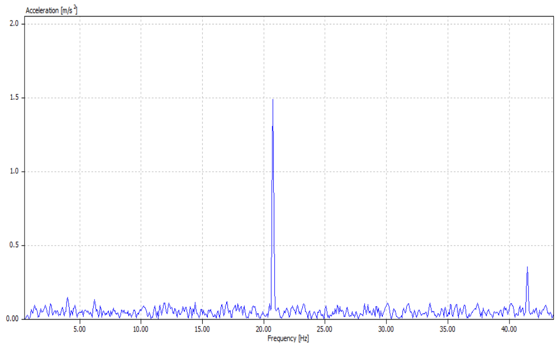
Results of test no. 12 in x - direction



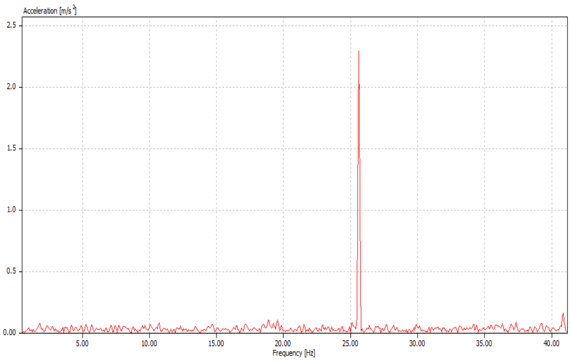
Results of test no. 12 in z - direction



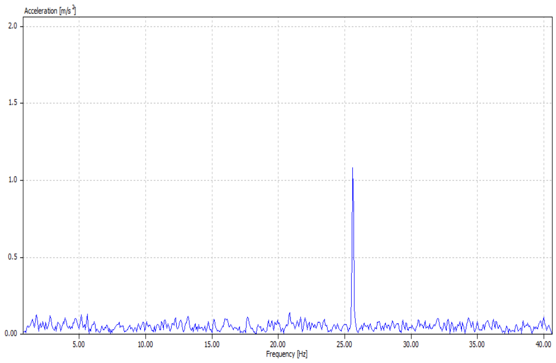
Results of test no. 17 in x - direction



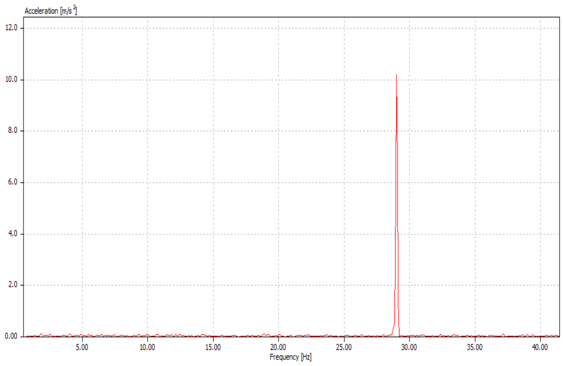
Results of test no. 17 in z - direction



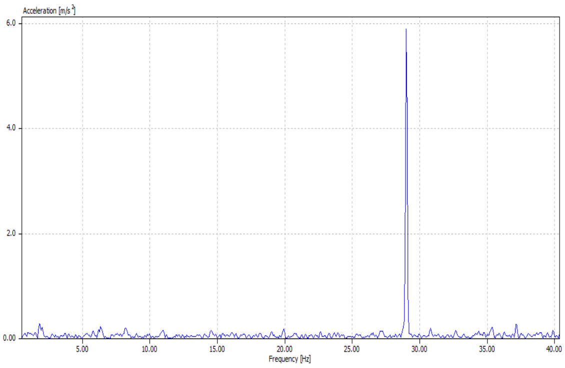
Results of test no. 24 in x - direction



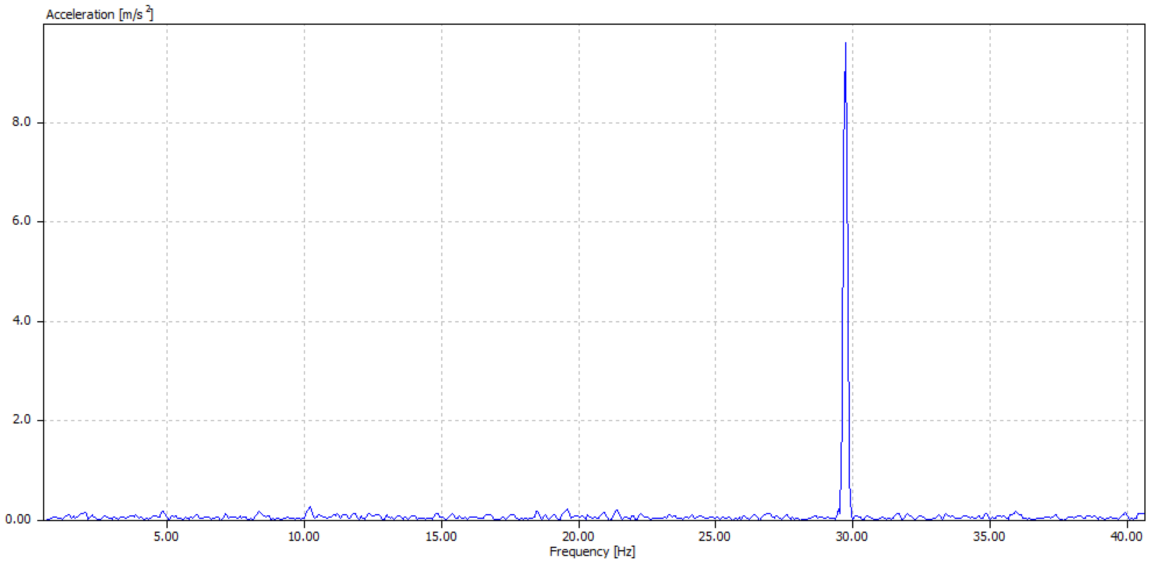
Results of test no. 24 in z - direction



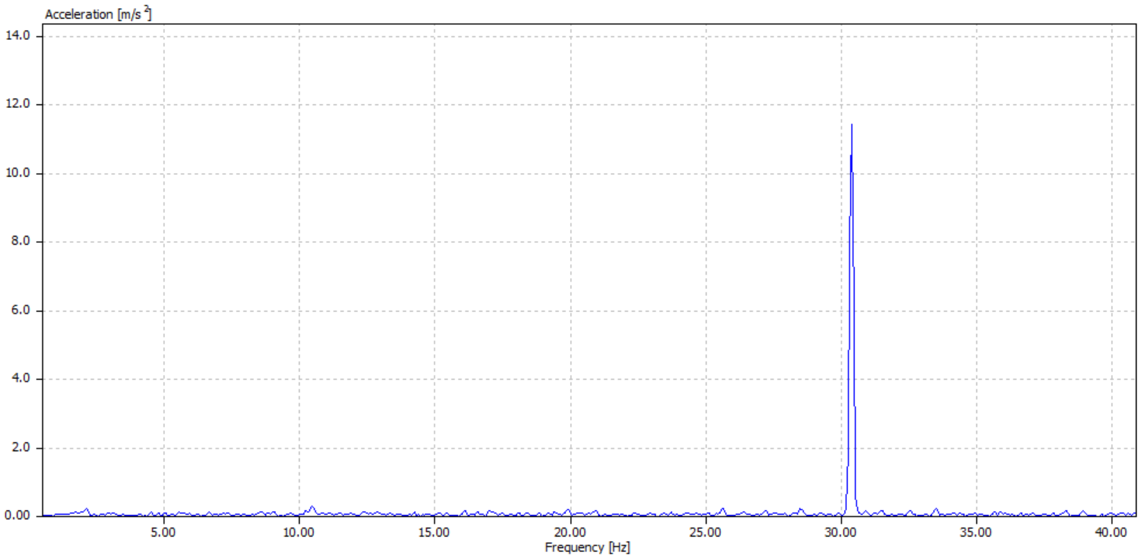
Results of test no. 28 in x - direction



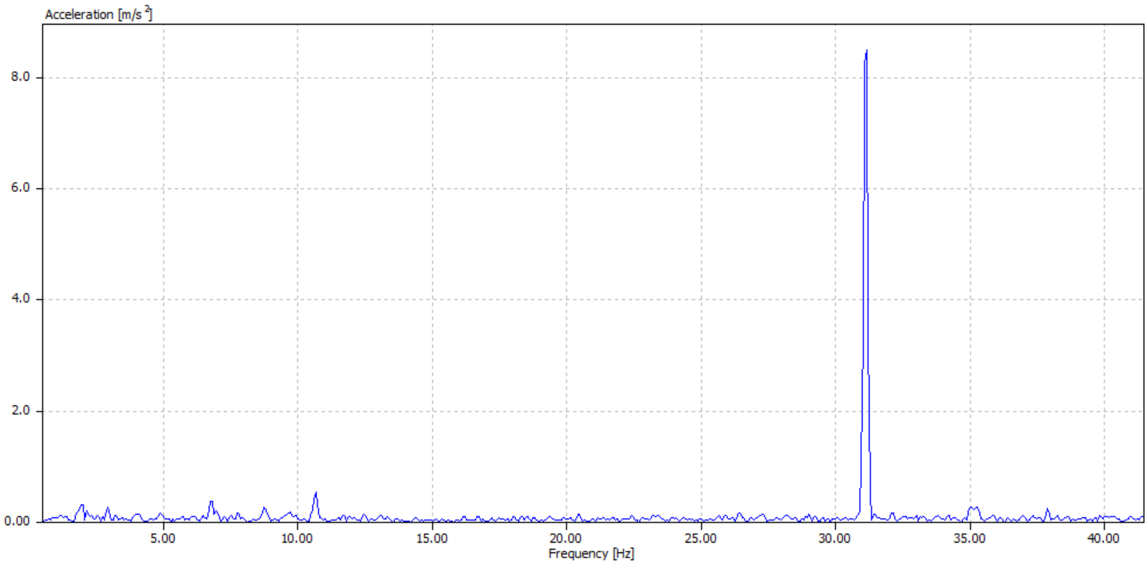
Results of test no. 28 in z - direction



Results of test no. 29 in z - direction



Results of test no. 30 in z - direction



Results of test no. 31 in z - direction

Figure 6.15: measurement results in frequency domain at different values of rotational speed.

For the finite element analysis (Figure 6.16), the total node number used for the prototype model is 2895 and the element number is 2960. The element type used in the model is linear shell element (S4R S3R). The material properties are: Young's modulus = 200 GPa; Poisson's ratio=0.3; and density=7850 kg/m^3.

The FE results for several significant values of the natural frequency are listed in Table 6.2. Also, Figure 6.17 shows the corresponding mode shapes of these natural frequencies.

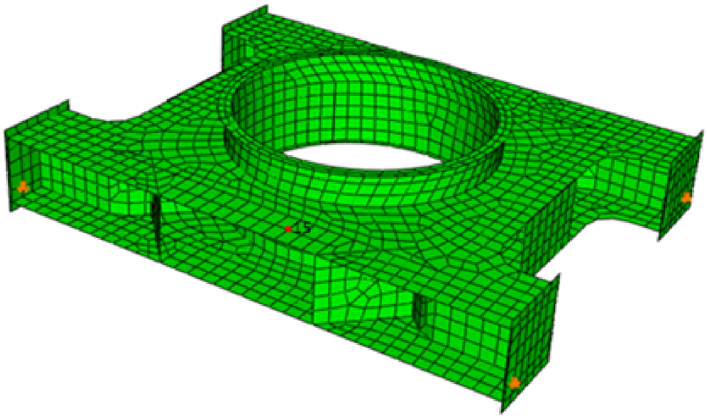
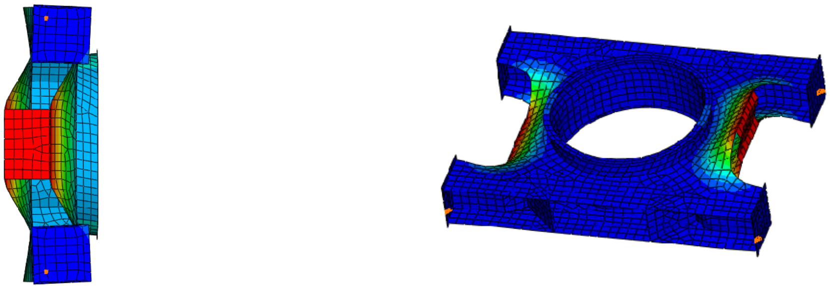


Figure 6.16: FEM of Prototype modal.

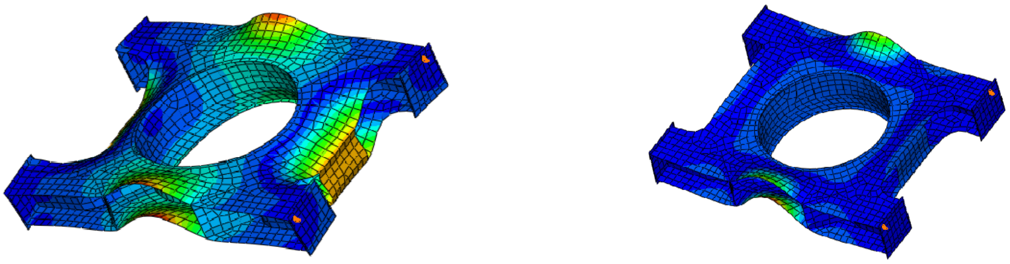
Table 6.2 Some of FE results for natural frequencies of the prototype model

Mode number	Frequency (Hz)
1	8.663
2	9.698
3	10.455
8	16.443
14	24.406
26	28.166
29	30.589
34	31.632



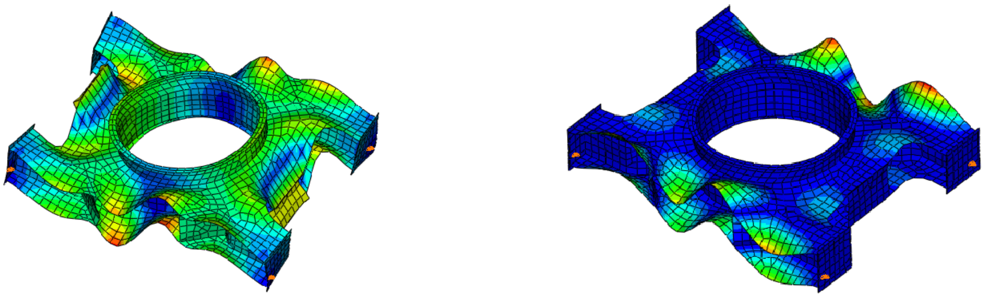
Mode shape 1

Mode shape 2



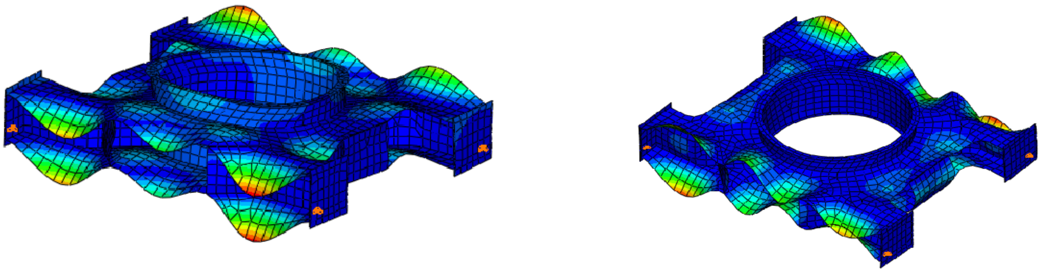
Mode shape 3

Mode shape 8



Mode shape 14

Mode shape 26



Mode shape 29

Mode shape 34

Figure 6.17: Mode shapes of prototype model corresponding to several significant natural frequencies

Moreover, By using the harmonic analysis in ABAQUS V6.7, the FRF of the prototype model is obtained within the frequency range of 0.25 Hz to 35 Hz. Figure 6.18 shows the calculated FRFs of the prototype model at node 15 (Figure 6.16).

According to the obtained results, it can be seen that the results of the FE model are in reasonably good agreement with the measured of the prototype model.

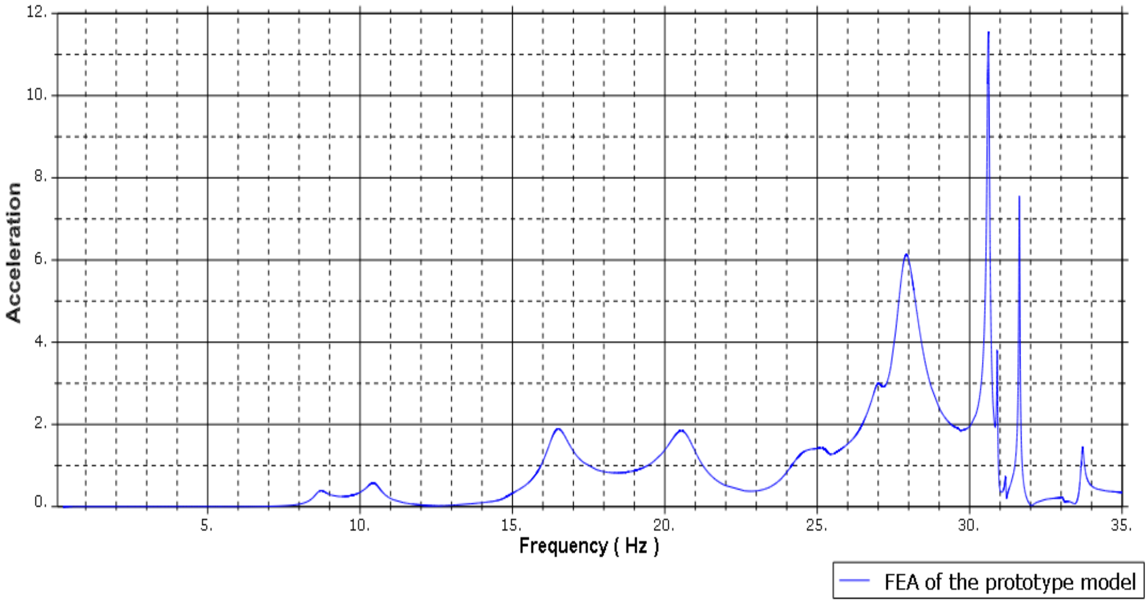


Figure 6.18: Calculated FRF of prototype model at node 15.

6.1.2.2 Improving dynamic behavior of bogie rotary excavator

After the validation of the finite element model had been done, and acceptable results were obtained, the next step of this study is to investigate and improve the dynamic performance of the real structure based on the method proposed in this thesis. This study consists of seven models for structure reanalysis. Figure 6.19 shows the first model which is the model of the original structure.

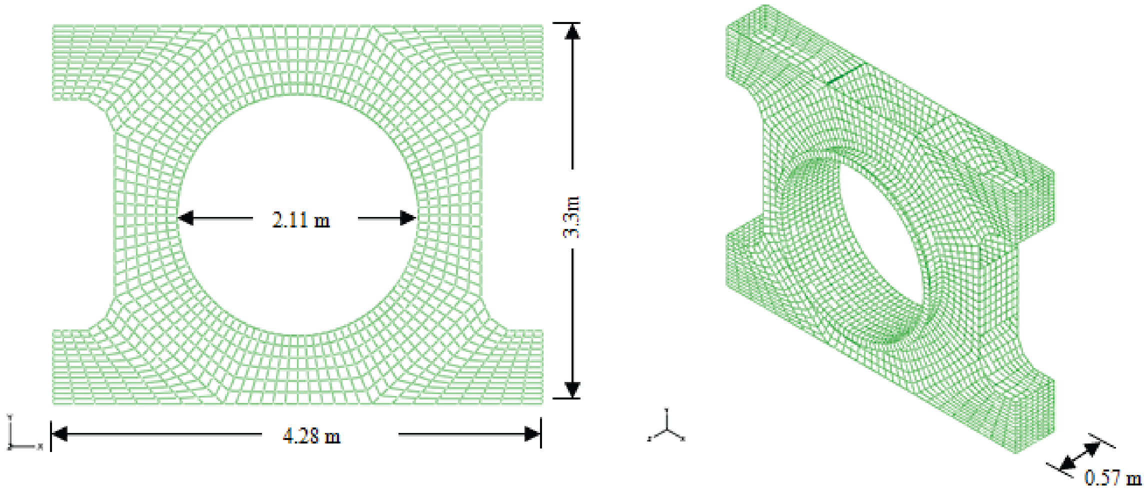


Figure 6.19: The existing structure of bogie rotary excavator

Figure 6.20 shows the obtained results for the first mode of oscillation of this model (bending). Based on the procedures described in 4.6.3, potential and kinetic energies have been calculated using Equations (4.9, 4.10, and 4.12) and the differences in increment were determined, as presented in Figure 6.20.

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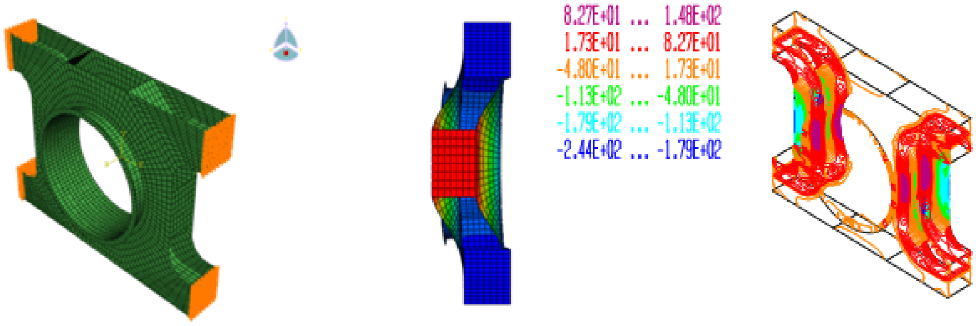

Figure 6.20: FEA of model 1.The first frequency is f_{01}= 63.132 Hz. Difference between potential and kinetic energy [Nm]

Model 2 represents the first proposed modifications for the structure. The additional materials were added around the hole in the center. Figure 6.21 shows the obtained results of this model. Based on the distribution of energy through the structure, it can be seen that the zones which have positive values in the difference between potential and kinetic energy (red and purple colors) require increasing in the rigidity. Therefore, the rigidity of the structure was increased in model 3 (figure 6.22) by increasing the distance between the upper and lower plates. According to the obtained results of model 3, it is clear that the dynamic behavior of the structure has been improved, where the value of the first frequency for this model is 92.993 Hz while the first frequency for model 1 was 63.132 Hz.

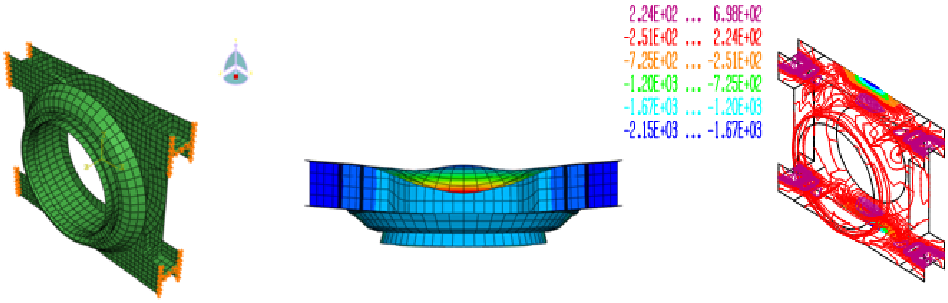

Figure 6.21: FEA of model 2.The first frequency is f_{01}= 88.975 Hz. Difference between potential and kinetic energy [Nm].

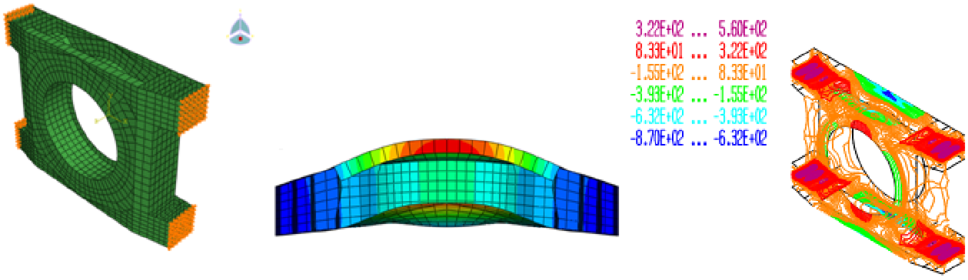

Figure 6.22: FEA of model 3.The first frequency is f_{01}= 92.993 Hz. Difference between potential and kinetic energy [Nm].

To get better results, some modifications have been done to the structure. where both sides of structure have been covered by additional plates. Figures 6.23, 6.24 and 6.25 show the effect of these modifications on models 4, 5 and 6.

Model 7 is the final proposed modification model for the structure. The additional stiffeners have been added to the both sides of the Bucket wheel excavator as shown in figure 6.26. This model has the best results compared with other previous models. Figure 6.26 shows the obtained results of this model. The first frequency of this model is f_{01} = 143.72 Hz which is considered a higher value in all models.

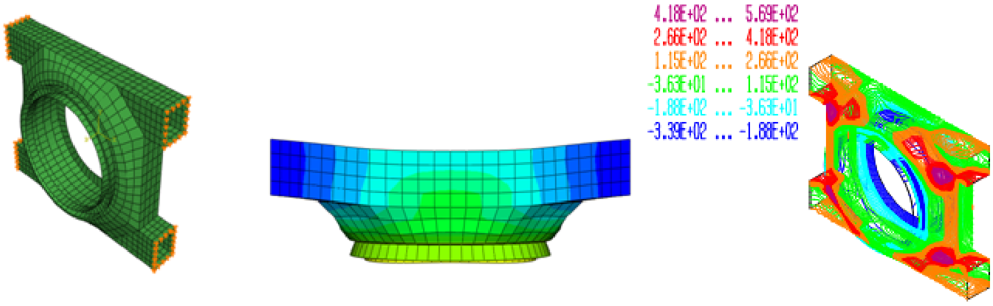

Figure 6.23: FEA of model 4.The first frequency is f_{01}= 101.88 Hz. Difference between potential and kinetic energy [Nm].

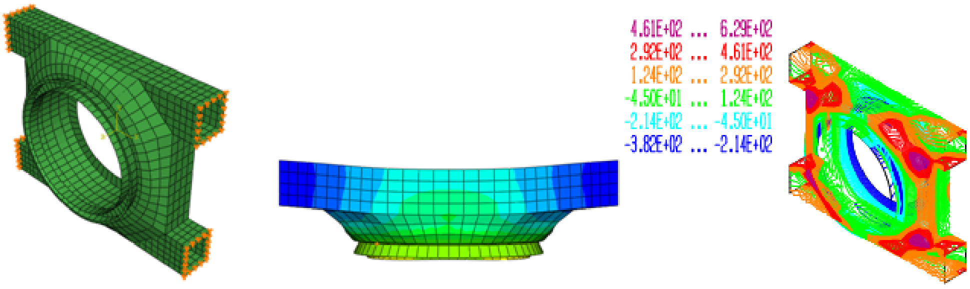

Figure 6.24: FEA of model 5.The first frequency is f_{01}= 107.4 Hz. Difference between potential and kinetic energy [Nm].

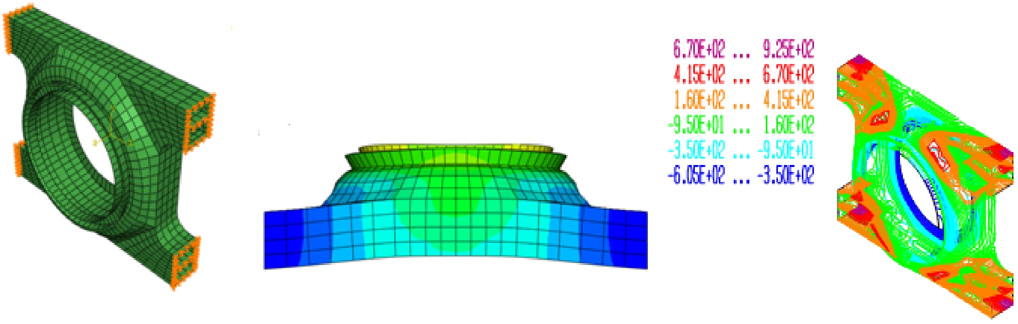

Figure 6.25: FEA of model 6. The first frequency is f_{01}= 129.42 Hz. Difference between potential and kinetic energy [Nm].

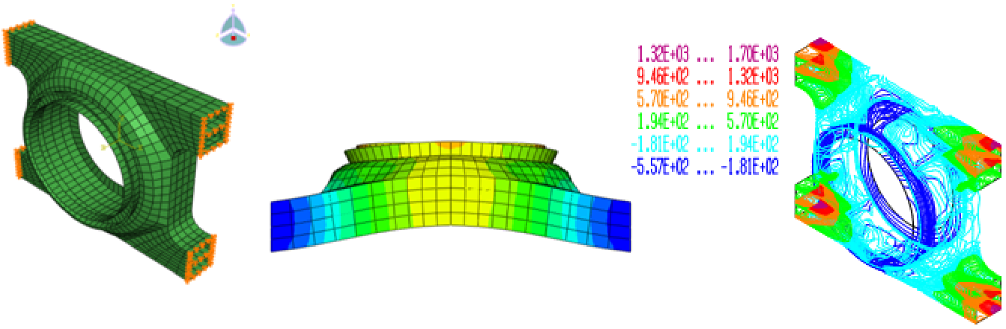

Figure 6.26: FEM of model 7.The first frequency is f_{01}= 143.72 Hz. Difference between potential and kinetic energy [Nm].

Although the height of first frequency is a good criterion for improving the structure's behavior, the difference between frequencies is also a very important factor as mentioned before. Therefore, in order to observe the difference between adjacent frequencies, the first three frequencies have been determined for all models. The comparison between all models is shown in Figure 6.27.

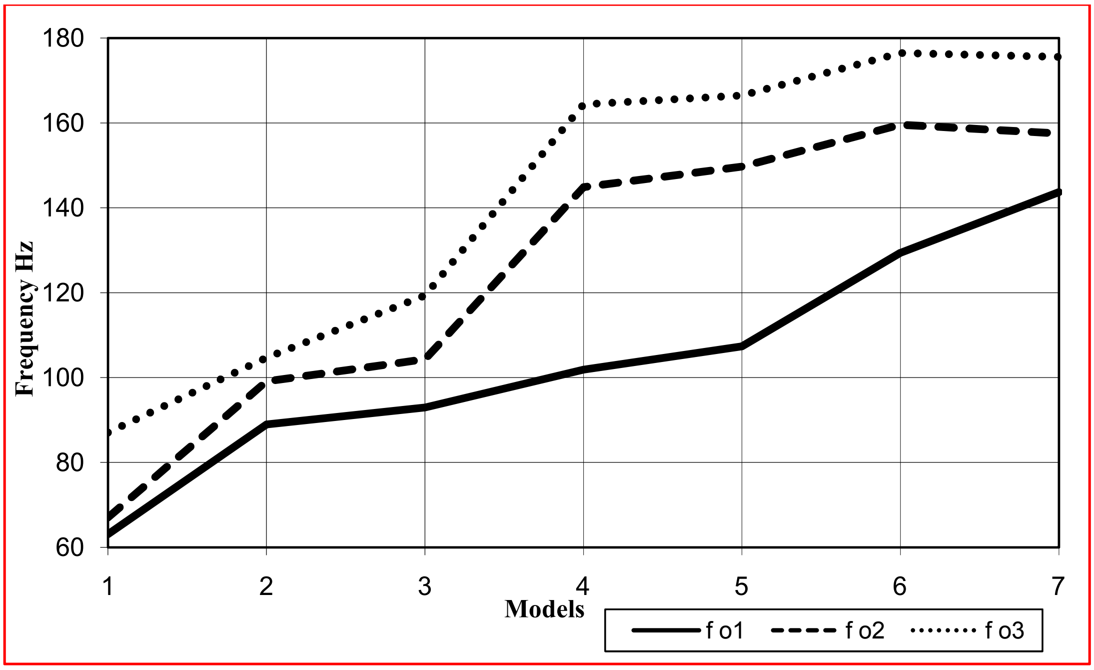

Figure 6.27: Comparison between models considering the differences between adjacent frequencies

According to the results obtained from the dynamic behavior of the bogie rotary excavator after the modifications have been done on the base structure, it can be clearly concluded that the study of distribution of potential and kinetic energy gives a clear definition for interest zones and elements for modifications.

The new solution of structure increases the first main mode about 2.2 times of the original structure. As a result, the improving of the structure's dynamic behavior is achieved.

6.1.3 Case study 2

Similar to the previous case study (except the experimental part), the procedures of reanalysis are conducted to the rotary bucket well excavator (BWE C700, Kolubara opencast mine, Lazarevac, Serbia, Figure 6.28). The first FE model, which simulates the original structure and its component parts, is as shown in figure 6.29.

Figure 6.28

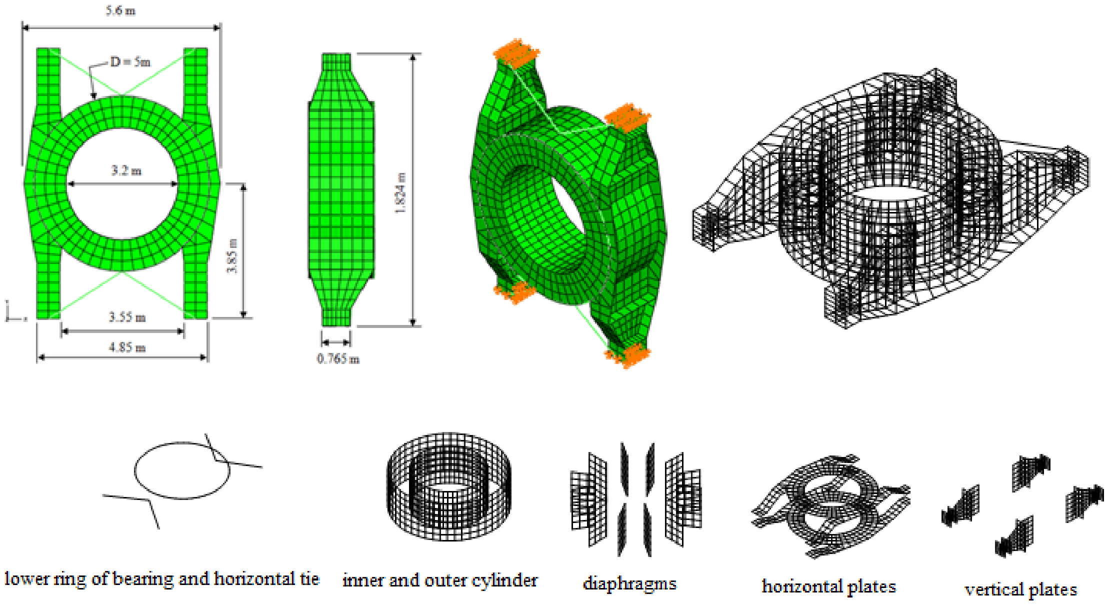

Figure 6.29: The existing structure of BWE C700 rotary excavator

This study consists of five models for structure reanalysis. Model 1 is referred to the original structure. Figure 6.30 (a, b) shows the obtained results for the first and second modes of oscillation of this model. Similar to what done in case study 1, the modified models are proposed according to the energy analysis. As had been mentioned, the zones which have positive values in the difference between potential and kinetic energy (red and purple colors) require increasing in the rigidity.

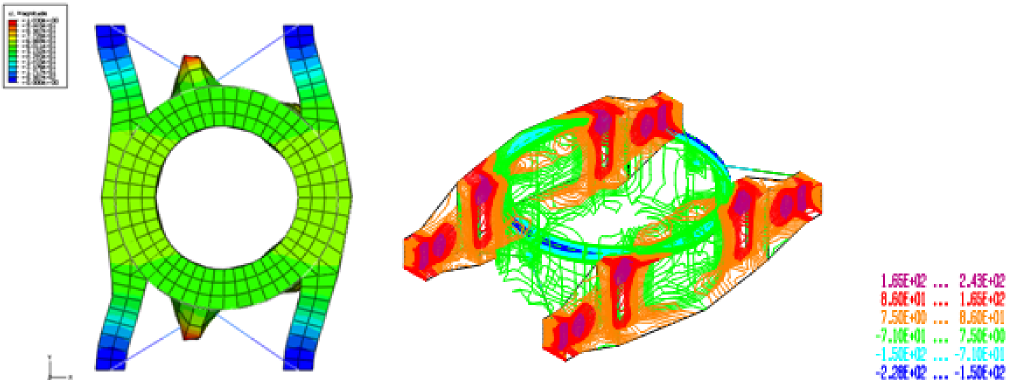

Figure 6.30-a: FEA of model 1.The first frequency is f_{01}= 59.57 Hz. Difference between potential and kinetic energy [Nm]

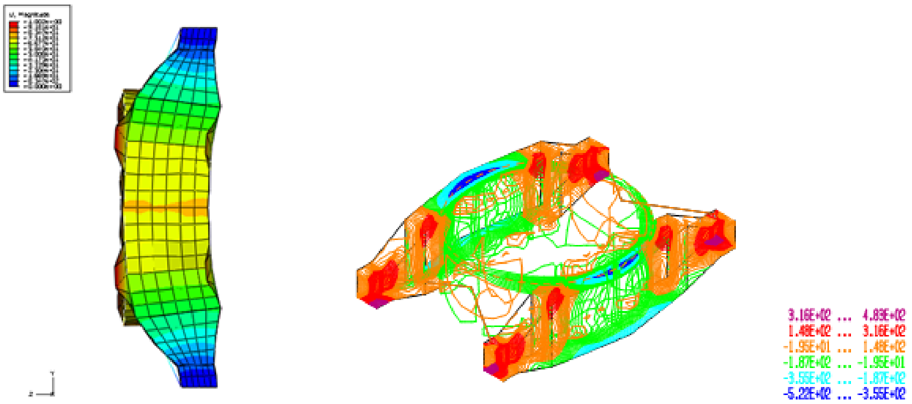

Figure 6.30-b: FEA of model 1.The second frequency is f_{02}= 64.83 Hz. Difference between potential and kinetic energy [Nm]

Model 2 represents the first proposed modifications for the structure. The outer and inner sides have been covered by additional materials. Figure 6.31 shows the first mode of oscillation of this model. In contrast to what is required, the obtained results were unexpected for this model. The first natural frequency is decreased compared with the original model. Accordingly, in model 3, the outer part of the additional material has been removed. It can be seen from figure 6.32 that the results are improved compared with previous and original models. Model 4 and Model 5 have the same geometry, where the additional inner plates that cover the two sides of structure have been removed. The modifications of these two models have been done by increasing the thickness of some plates in the structure. Different results have been obtained as pointed in figures 6.33 and 6.34.

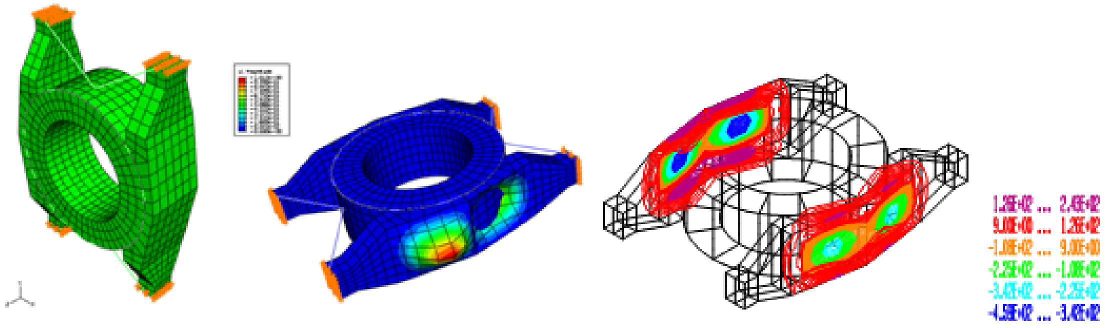

Figure 6.31: FEA of model 2.The first frequency is f_{01}= 42.12 Hz. Difference between potential and kinetic energy [Nm]

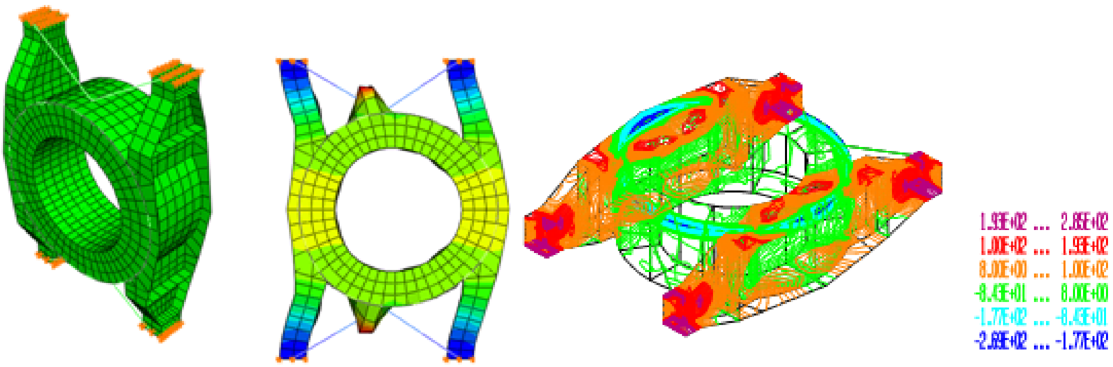

Figure 6.32: FEA of model 3.The first frequency is f_{01}= 62.97 Hz. Difference between potential and kinetic energy [Nm]

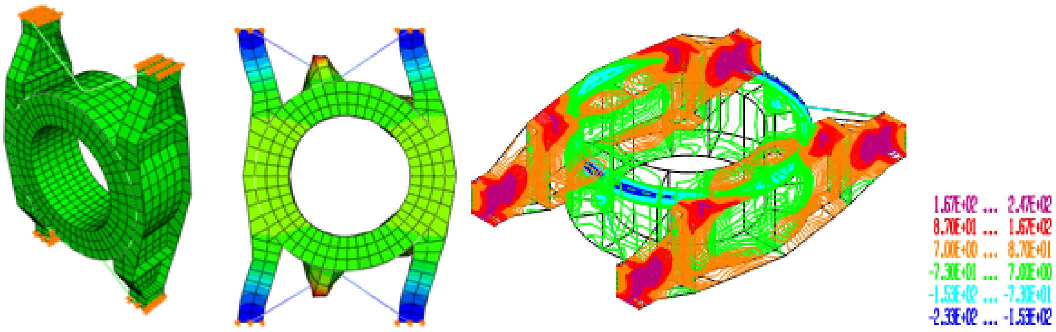

Figure 6.33: FEA of model 4.The first frequency is f_{01}= 58.92 Hz. Difference between potential and kinetic energy [Nm]

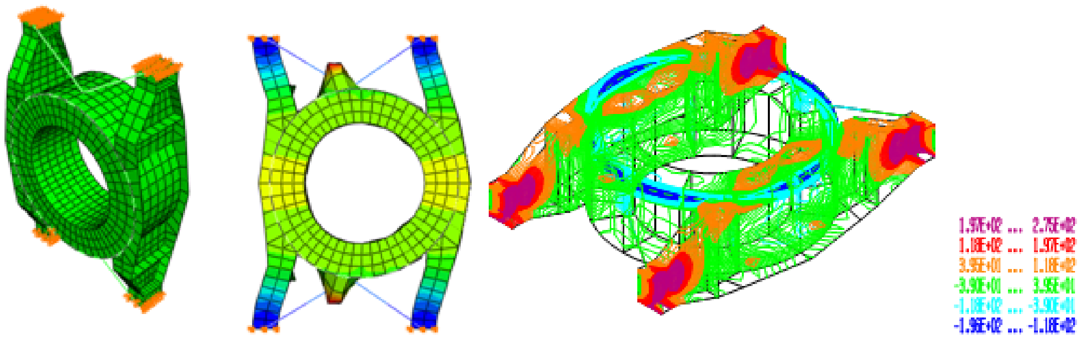

Figure 6.34: FEA of model 5.The first frequency is f_{01}= 60.22 Hz. Difference between potential and kinetic energy [Nm]

The first three frequencies have been determined for all models. The comparison between all models is shown in Figure 6.35. As a result, it can be seen that model 3 has the best dynamic performance where the first natural frequency is increased and the difference between the first and second frequencies are also increased. Also, good results can be achieved by modifying the geometry of the structure especially in zones of intersection of cylindrical part with the outsides plates.

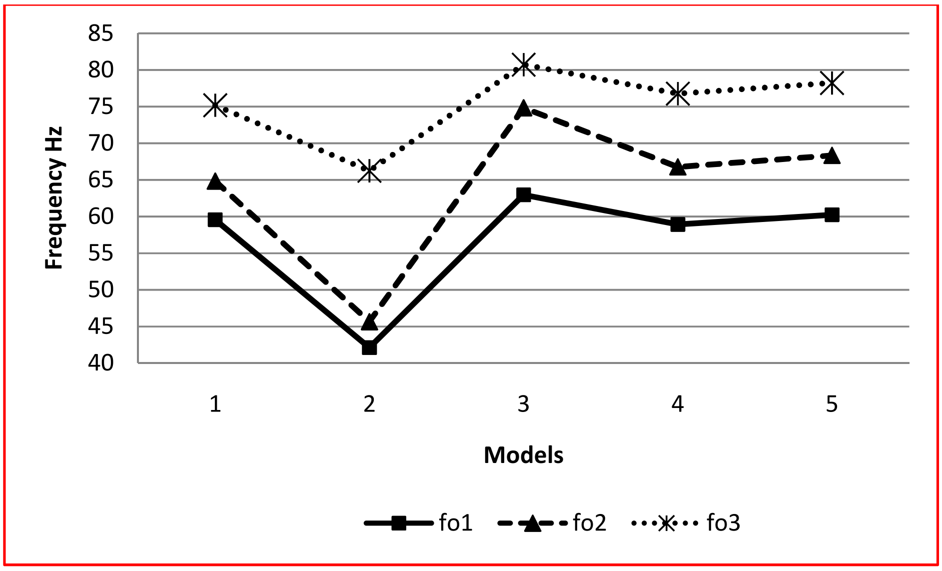

Figure 6.35: Comparison between models considering the differences between adjacent frequencies

Chapter 7

Conclusion

In this Thesis, the methodology of Diagnosis of Dynamic Behavior of Structures using the Distribution of Kinetic and Potential energy is presented. The aim of developed method for analysis and diagnostic of structure behavior is to determine real behavior of the construction in exploitation. The procedures used in this thesis are concerned with the analysis of the distribution of potential and kinetic energy and the differences between them in elements of the structure, which gives prediction for which elements need a modification in order to improve the dynamic characteristic of a structure.

Structural Dynamic Modifications are often undertaken to improve the dynamic behavior of an existing structure. In many cases, the objective is to modify structure eigenvalues or eigenvectors to reduce the vibration responses of the structure. There are two opposite approaches for structural modifications. The first one is direct structural modification, and the second is inverse structural modification. The direct structural modification problem is treated as prediction problem which is concerned with determining the dynamic response of a structure brought about by modification. On the other hand, the inverse structural dynamic modification is treated as an optimization procedure which is used to determine necessary modifications in order to achieve the desired dynamic behavior of structure usually in terms of the desired values for natural frequencies and mode shapes. The researches concerned with these two approaches have been briefly reviewed in the second Chapter of thesis.

Because of enormous development in the industry, where complex shapes of the structures, FE analysis has become the most popular technique in structural dynamic analysis. Modeling of complex structures using finite elements method is a helpful approach in solving problems in short time with reliable results. The procedure involved in deriving the finite element equation of dynamic problems has been described in the third Chapter.

Procedures of Reanalysis Technique based on the energy method, which are used in this thesis, have been presented with an analytical example in the fourth Chapter. Distribution of potential and kinetic energy in main oscillation modes is the base methodology for improving dynamic behavior of structure using reanalysis procedures technique. Study of distribution of potential and kinetic energy of structure gives obvious prediction which elements need some modifications to achieve the best dynamic characteristics. The main point of improving dynamic behavior of a structure is increasing its natural frequencies and maximizing the interval between adjacent natural frequencies.

The algorithm of reanalysis has the following aspects:

I. Elements in which the kinetic and potential energies (and the difference in their increase) are negligible with respect to other elements.
II. Elements in which the kinetic energy is dominant compared to potential energy.
III. Elements in which the potential energy is dominant compared to kinetic energy.
IV. Elements in which the potential and kinetic energy exist and are not negligible in comparison with other elements.

In order to demonstrate the effectiveness of the proposed method, the aforementioned procedures are applied numerically to trusses and beams as 1D structures, as well as to different cases of plates as 2 D structures in Chapter five. In the first section of this Chapter, structural dynamic modification is implemented to a simply supported truss structure and beams as one-dimensional (1D) structures in order to improve its dynamic behavior. Best results were obtained. For instance, in truss example, the value first natural frequency of the original truss was f_{01}= 39.7 Hz. after many reanalysis processes, which have been done based on the distribution of energy through the structure, the best modified structure with a value of the first natural frequency f_{01}= 44.1 Hz has been obtained with 33.95% decreases in its total mass, which emphasis that the proposed method provides effective results. Furthermore, in the second section of Fifth Chapter, structural modification procedures are applied to some different cases of a rectangular plate as an example of a 2-D structure. In order to investigate the effect of the boundary condition on the dynamic behavior of plate, three cases are presented which are: clamped rectangular plate (membrane structure), simply supported and cantilever plate (shell structure) respectively. In addition, and in order to highlight the effectiveness and the ease of implementation of the proposed approach, a rectangular clamped plate along four edges is presented to investigate the frequency response functions FRFs under structural dynamic modifications. Also, the satisfactory results were obtained. For example, in the case of clamped rectangular structure (shell elements), the aim was to calculate and improve the frequency response functions FRFs of the plate based on the procedure of reanalysis. Three modified models, model 1, model 2 and model 3 respectively, are proposed with different thickness of some certain elements. The calculated of FRFs of the original and modified plates are obtained within the frequency range of 0.25 Hz to 800 Hz and a 0.25 Hz frequency resolution by using the harmonic analysis in ABAQUS V6.7. According to the comparison of FRFs between the original plate and the modified plates, it can be seen that modified models have got a best dynamic behavior compared with the original one. Consequently, it is apparent that the proposed method improves the plate response, where the natural frequencies of model 2 and 3 have been reduced to five frequencies instead of seven frequencies within the frequency range.

Moreover, the implementation of reanalysis technique on real complex structures has been presented in the Chapter six of this thesis. First, Finite Element Analysis is made in order to diagnosis of dynamic behavior of some real complex structures without the completeness of reanalysis procedures to improve structures' behavior. In addition, energy analysis has been conducted to locate the position of dynamic modifications if needed. Furthermore, in the second part of this Chapter, complete reanalysis procedures have been conducted to two complexes sub-structures (Bucket wheel excavators). Although modeling of complex structures using finite elements method is a helpful approach in solving problems in short time with reliable results, experiments should be considered in order to validate Finite Element model. Therefore, In order to validate finite element analysis, numerical and experimental analysis has been done, for one case study, on a prototype model. The obtained results of the FE model were in reasonably good agreement with the measured of the prototype model.

After the validation of the finite element model had been done, the Bucket wheel excavator’s models have been investigated based on the method proposed in this thesis in order to improve their dynamic behavior. As mentioned before, the highest value of the first frequency of the structure the best structure performance. Consequently, for instance, the new solution of bogie rotary excavator increases the first main mode about 2.2 times of the original structure. Therefore, the distributions of potential and kinetic energies of elements of the whole structure give a clear view to the problem, which helps to make appropriate decision for structure modifications. As a result, the improving of the structure's dynamic behavior is achieved.

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Изјављујем да је штампана верзија мог докторског рада истоветна електронској верзији коју сам предао/ла за објављивање на порталу **Дигиталног репозиторијума Универзитета у Београду.**

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