STOCHASTIC MODEL FOR THE ASSESSMENT OF ACCEPTABLE TOLL RATES

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СТОХАСТИЧКИ МОДЕЛ ЗА УТВРЂИВАЊЕ ОПТИМАЛНЕ ПУТАРИНЕ

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STOCHASTIC MODEL FOR THE ASSESSMENT OF ACCEPTABLE TOLL RATES

ABSTRACT

Private participation in the delivery of toll road projects has been used worldwide. It is a model which incorporates private sector knowledge and experience in the management of highway projects and mobilizes private capital through Public-Private Partnerships (PPP). One of the most prevailing characteristics of PPP projects is risk sharing between the public and private partners. Assessment of a project’s financial soundness, a crucial factor for private sector involvement, is the basic underlying process throughout the project’s development until the project reaches financial closure.

The traditional cash flow analysis of the financial feasibility of a project has shown weaknesses in many cases. From the pool of delivered projects which have experienced difficulties in their operations, it can be learned that advanced probabilistic models need to be introduced due to their feature of representing uncertainties more realistically. It is important to capture a project’s uncertainties even in early phases of financial analysis since this information helps in the identification of potential financial risks and assists all sides to structure the deal properly. Parameters commonly used for the evaluation of a project’s financial feasibility are the annual debt service cover ratio ($ADSCR$), the internal rate of return ($IRR$), and the return on equity ($ROE$). Although some existing models for analysis of a project may seem difficult for decision makers and stakeholders to interpret and understand, there are prospective ways of describing and representing the problem in more understandable and meaningful ways.

This research presents a methodological framework for an early assessment of acceptable toll rates for PPP toll road projects taking into account multiple uncertainties. A toll rate is considered acceptable if it is acceptable for all stakeholders. This approach takes into account predefined financial constraints $ADSCR$, $IRR$ and $ROE$ on one side, and the project’s uncertainties, such as volatility of traffic volumes, construction costs variation, and operation and maintenance costs variation on the other side. Selected financial parameters represent the preferences or requirements of potential investors that must be fulfilled in order for them to invest in a PPP project. These preferences and
financial requirements are based on investors' assessments of a project's risk profile and also depend on activities on capital markets.

The results of this approach provide the range of toll rates covering possible risks scenarios. These results can serve as a basis for a comparative analysis of the socially acceptable toll rate, assuming it is known, and the financially required toll rate. It is anticipated that the early identification of the possible gap between these two values represents valuable information for all parties involved in the project. This gap helps in the identification of the need for additional financial instruments, such as guarantees or subsidies, in order to implement a project that is acceptable for private and public partners, equity investors, lenders and users.

The mathematical model used in this research has two hierarchical implementation modes - deterministic (which is quicker) and stochastic (which is more extensive). In the deterministic model, the mathematical re-formulation of the World Bank (WB) Toolkit enables the toll rate calculation for the given set of selected financial constraints. This approach is used to test the sensitivity of toll rates to changes in traffic volume, construction costs, and operation and maintenance (O&M) costs as three risks under the consideration in this research. The sensitivity analysis shows the existence of relations between changes in these parameters which were further elaborated in the stochastic mathematical model.

Recognizing the number of risks in PPP toll road projects, the three mentioned risks have been chosen to be included in the stochastic mathematical model as models of random variables. Construction and O&M costs are presented as continuous random variables, while the traffic volume has the additional stochastic component which represents traffic evolution over time as random walk.

The application of this stochastic model is presented through two case studies, chosen to present how a country's macro-economic conditions and traffic forecast may be captured with the model. Results reveal the minor share of risks covered by socially acceptable toll rates. Concluding remarks show the importance of timely and understandable information about potential risk scenarios that are crucial in raising awareness among decision makers and stakeholders about possible costly downturns.
Key words:
Public-Private Partnerships, toll roads, toll rates, risk management, project uncertainties, traffic risks, construction costs, operation and maintenance costs, financial feasibility, stochastic models.

Scientific area:
Civil engineering

Specific scientific area:
Construction and maintenance of roads and airports, Construction management

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Учешће приватног сектора у реализацији путних пројеката са наплатом путарине заступљено је широм света. Овај модел користи знање и искуство приватног сектора у управљању путним пројектима и мобилише приватни капитал кроз јавно-приватно партнерство (ЈПП). Једна од најзначајнијих карактеристика ЈПП пројеката је подела ризика између јавног и приватног партнера. Процена финансијске основаности пројекта, кључног фактора за учешће приватног сектора, основни је процес целокупног развоја пројекта све до закључења финансијског аранжмана за пројекат.

Традиционална анализа новчаних токова финансијске оправданости пројекта је показала своје недостатке у доста случајева. Из узорка реализованих пројеката који су имали потешкоће у оперативној фази, може се закључити да је неопходно увести напредне модели вероватноће због њихове могућности да реалније представе неизвесност. Важно је да се уvide ризици пројекта и у раним fazama финансијске анализе обзиром да ова информација помаже у сагледавању потенцијалних финансијских ризика и омогућава свим заинтересованим странама да правилно склопе споразум. Параметри који се често користе за процену финансијске оправданости пројекта су годишњи рацио покрића дуга, интерна стопа рентабилитета пројекта и интерна стопа повраћаја уложеног капитала. Иако неки од постојећих модела за анализу пројекта могу доносиоцима одлука и кључним интересним групама да делују компликовано за разумевање и интерпретацију, постоје други разумљиви и смислени начини описивања и презентовања проблема.

Ово истраживање представља методолошки оквир за рано утврђивање оптималне висине путарине за ЈПП путне пројекте са наплатом путарине узимајући у обзир различите неизвесности. Путарина се сматра оптималном ако је прихватљива за све кључне учеснике у пројекту. Овај приступ користи унапред дефинисана финансијска ограничења годишњи рацио покрића дуга, интерне стопе
рентабилитета пројекта и интерне стопе повраћаја уложеног капитала са једне стране, и ризике пројекта као што су нестабилност обима саобраћаја, варијације у трошковима изградње, и варијације у трошковима управљања и одржавања, са друге стране. Изабрани финансијски параметри представљају приоритете или захтеве потенцијалних инвеститора који морају бити испуњени како би се инвестирало у ЈПП пројекат. Ови приоритети и финансијски захтеви се базирају на процени профилра ризика пројекта од стране инвеститора, а такође зависе и од активности на тржишту капитала.

Резултат овог приступа је опсег висине путарине који покрива могућа сценарија ризика. Ови резултати могу да послуже као основа за компаративну анализу социјално прихватљиве висине путарине, претпостављајући да је позната, и финансијски захтеване путарине. Очекује се да анализа идентификација разлике између ове две вредности путарине представља значајну информацију за све учеснике у пројекту. Ова разлика омогућава да се сагледају потребе за додатним финансијским инструментима, као што су гаранције или субвенције, како би се реализовао пројекат који је прихватљив и за јавног и за приватног партнера, инвеститоре, банке и кориснике.

Математички модел коришћен у овом истраживању има два хиерарсхијска модуса имплементације - детерминистички (који је бржи) и стохастички (који је опширнији). У детерминистичком моделу, математичка преформулација апликације Светске банке омогућава прорачун путарине за задати сет финансијских ограничења. Овај приступ је коришћен за тестирање осетљивости путарине на промене у обиму саобраћаја, трошкова изградње и трошкова управљања и одржавања као три изабрана ризика у овом истраживању. Анализа осетљивости покажује постојање везе између промена ових параметара која је даље разматрана у стохастичком математичком моделу.

Препознајући значајан број ризика у ЈПП путним пројектима са наплатом путарине, три споменута ризика су изабрана за стохастички математички модел кроз модели случајних променљивих. Трошкови изградње и трошкови одржавања и управљања су представљени као непрекидне случајне променљиве, док обим саобраћаја има додатну стохастичку компоненту представљајући промену саобраћаја кроз време као случајни процес.
Примена стохастичког модела је представљена кроз две студије случаја које су изабране како би се представило да утицај макро економске ситуације у земљи и саобраћајне прогнозе може да се обухвати моделом. Резултати показују да друштвено прихватљиве цене путарина покривају мањи удео ризика. У оквиру закључка истакнута је важност правовремених и разумљивих информација о потенцијалним сценаријима ризика које су кључне у подизању свести код доносиоца одлука и кључних интересних група о могућим скупим промашајима.
Кључне речи:
јавно-приватно партнерство, путеви са наплатом путарине, путарине, управљање ризицима, пројектни ризици, ризик саобраћаја, трошкови изградње, трошкови одржавања и управљања, финансијска оправданост, стохастички модели.

Научна област:
Грађевинарство

Ужа научна област:
Грађење и одржавање путева и аеродрома, Грађевински менаџмент

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CHAPTER I
INTRODUCTION

The total length of the road network and its level of service are among the most important indicators of the economic development of a country. Countries worldwide place significant efforts to keep their road infrastructure conditions at acceptable levels. Investments in road infrastructure improvements and upgrade can be quite substantial and require careful and robust analysis of a number of parameters. Managing the planning process and the delivery of road infrastructure projects represents a comprehensive set of activities including economic, financial, environmental and social impact assessments. Risk management per se is a long lasting, multidisciplinary, and complex process during the whole project’s life cycle and includes several project stakeholders.

Background

Large infrastructure investments represent significant burdens on public budgets. While public agencies have to comply with the limited budgets, they also have a social and economic responsibility to deliver projects of public interest. One possible model which enables the delivery of road projects under public budgetary constraints is a Public-Private Partnerships (PPP). Although there is no consensus on the definition of a PPP, in general a PPP can be described as a contractual agreement formed between a public agency and a private sector entity that allows for greater private sector participation in the delivery and financing of transportation projects (FHWA 2013a). Such partnerships serve as a model for helping overcome budgetary shortfalls, i.e., for filling the gap between assets and services required by the public and available funds for delivery of those assets and services. Transportation is one of the major sectors in which the implementation of PPPs has become a common approach in resolving infrastructure issues. For example, investment in transport infrastructure in 2014 accounted for 51% of the total investments in infrastructure (energy, transport, and water and sanitation) in emerging markets (Kasper 2015). The five-year average increase in transport investment is 40%.

Because of several technical, financial, legal, and economic issues which need to be addressed, PPP agreements are relatively complex. In addition, defining long-term
rights and obligations in these complex agreements between the public sector and the private concessionaire requires in-depth risk analysis because of the considerable number of risks usually encountered in PPP projects. The evaluation of such risks and the choice of adequate risk mitigation strategies are considered crucial parts of the preparation of a PPP contract. The assessment of a project’s financial soundness represents one of the basic underlying processes in the evaluation of PPP projects. The financial element of the project is a predominant factor for the private sector involvement in the first place. The project can be delivered as a PPP only if it reaches financial closure.

At the beginning of project development, some of the key input parameters of the project are assumed and not yet well defined. The problem of capturing uncertainties in PPP contracts in these early phases of project development is a challenging task. Estimates of long-term risks have been a topic of interest for many years. Use of probability theory has been recognized in academic literature, but has had limited use in practice to address and model such uncertainties.

In general, for PPP toll roads, the public sector is looking for a toll rate which is socially acceptable and which will encourage use of the facility. The issue of finding a socially acceptable toll rate involves, among other topics, the social perception of the value that a road generates as reflected in the users’ willingness to pay and users’ value of time. On the other side, the private sector is looking for a toll rate that will ensure project financial sustainability and adequate financial returns. Determining the financially required toll rate is relatively straightforward using appropriate financial models from the project's cash flows. Problems may arise if the socially acceptable toll rate is lower than the minimum financially required toll rate. This gap (usually called “affordability gap”) indicates that both the public and private partners have to consider other options for making the project’s financial structure acceptable for all involved stakeholders, including potential investors. These options may include, for example, government subsidies for the construction costs or a minimum revenue guarantee. Consequently, the term “acceptable,” as used in this dissertation, means acceptable to all stakeholders.

Factors such as the country’s economic profile, road network configurations, fuel prices, and fare prices of alternative public transportation can affect the public perception of what is an acceptable level of toll rates. These parameters can be classified as “external”
factors of the project, i.e. parameters which do not depend on the project’s technical characteristics and cannot be managed at the project level. They are either part of the network level decision making process or the result of the international market conditions. “Internal” factors of the project, such as construction and operation and maintenance costs, are part of the project’s characteristics and can be managed at the project level. A common practice is to assume deterministic values for these "internal" parameters early in the project assessment, but a more realistic consideration of the variability of “internal” parameters should be captured early in the financial analysis of the project. Taking these risks into consideration in the process of finding the minimum financially required and socially acceptable toll rates is an essential task.

**Problem Statement**

Traditional techniques of the financial analysis of projects are based on scenario analysis or sensitivity analysis. For scenario analysis, the foreseen possible outcomes are determined, and the financial assessment is conducted for each of them. For sensitivity analysis, input values of interest are increased or decreased by predefined steps to test the sensitivity of the outputs. Both of these techniques include a basic deterministic approach and provide a single value as an output. However, in the light of the recent financial crises and the historical cycles of high volatility in market conditions, advanced probabilistic models need to be introduced for long-term forecasts of PPP financial feasibility.

PPP projects congregate a number of stakeholders with different objectives. They all have a common goal to find a balance among different requirements and to reach an agreement which is mutually satisfactory and leads to the financial closure of the project. These differences in objectives have their roots in the fact that each side in the agreement is seeking the fulfillment of requirements set by the group they are representing. For example, the public sector, as mentioned earlier, is aiming at a project which will provide the required service for the society with a socially acceptable level of toll rates. On the other side, the private partner is seeking a project which will generate adequate return on capital with a relatively low-risk profile and for which it is possible to obtain funding at a low cost of debt. Lenders are looking for a project with a stable long term revenue forecast which is sustainable over debt maturity. Each group of
stakeholders, in their project assessments, relies on the level of toll rates as one of the key factors on which the project's financial strength can be evaluated.

The toll rate, with direct user charging, is one of the most important parameters in a PPP road project. However, the two opposite standpoints between the public and the private partners (to have socially acceptable toll rates and to have toll rates which will generate an adequate profit) define the issue of finding the acceptable toll rates as a key issue in the project’s feasibility analysis. The question which arises here can be stated as follows: What would be the toll rate which both fulfills financial constraints and, on the other side, captures a project’s uncertainties with certain probabilities?

**Research Objectives**

The objective of this research is to develop a methodology to assess the acceptable toll rate for all stakeholders, taking into account multiple uncertainties. In this research, the acceptable rate represents the toll rate which is socially acceptable and financially sufficient. The methodology adopted is based on the analysis of the following technical parameters: construction costs, operation and maintenance cost, and traffic volumes. Financial constraints such as debt service coverage ratios, and return on investment, are included in the analysis. The methodology uses both deterministic and probabilistic approaches. This research takes into consideration uncertainties regarding the estimates of construction, maintenance and operational costs estimates and the uncertainties of traffic forecasts.

**Expected Benefits**

The developed methodology establishes a framework for an early assessment of toll rates which addresses technical and financial aspects of the project. It is expected that this method will build a common ground for parties involved in the project to reach an agreement on an acceptable level of toll rates. Both the deterministic approach and the probabilistic approach help key stakeholders to evaluate the project's uncertainties more thoroughly and to understand the project's interdependencies more efficiently. The main contribution to the current state of risk assessment is the introduction of the stochastic component in the estimates of toll rates. This probabilistic approach represents a crucial step forward in the risk management practices in PPP road projects. It is expected that the research results will contribute to a more realistic technical analysis and financial modeling of PPP toll road projects. The potential users of the model presented in this
dissertation are road authorities, decision makers, development and commercial banks, concessionaires, infrastructure investment funds, rating agencies, and academic users.

**Organization of the Study**

The first chapter presents the background with the problem statement, research objectives, and expected benefits. Chapter II continues with an in-depth literature review with emphasis on existing tools for the feasibility assessment of PPP projects, characteristics of toll rates, uncertainties in traffic forecasts, construction costs, and operation and maintenance costs. The research framework and research methodology are presented in Chapter III. Chapter IV provides the results and discussion of the deterministic approach. Chapter V defines the basic principles of a stochastic approach and presents the results and associated discussion. Chapter VI provides two case studies which have been selected for the methodology validation. Chapter VII closes this dissertation with the main research findings, conclusions, and recommendations for the future research.
CHAPTER II
LITERATURE REVIEW

PPPs represent contractual relations between public and private partners. Overtime, some of these agreements may lead to larger issues, often resulting in expensive renegotiations for the public sector. Wide academic research efforts have sought to address these problems. This literature review focuses on a few aspects which are of interest for this research. First, background on PPPs in general, and PPPs for highway concessions in particular, is provided. Four toolkits, which are currently available for the assessment of PPP projects, are also reviewed. Then a literature review of the toll rate topic and PPP risks is carried out.

Background
There are many definitions of PPPs. While a PPP can be observed, in a broad sense, as an agreement between the public and the private partners for delivery of public projects and services, the legal definition is usually more complex. For example, the Green Paper of Commission of the European Communities (2004) defines the term PPP in general as the:

"cooperation between public authorities and the world of business which aim to ensure the funding, construction, renovation, management or maintenance of an infrastructure or the provision of a service."

The Law on Public-Private Partnership and Concessions (2011) in Serbia defines a PPP as:

"the long term cooperation between a public and a private partner for the purpose of providing financing, construction, reconstruction, management or maintenance of infrastructure and other facilities of public interest and provision of services, of public interest, which may be contractual or institutional."

Here, a contractual PPP represents the form in which the private and the public partner regulate their rights and obligations by a contract. An institutional PPP represents the form in which both partners are members of a joint venture and are responsible for the project’s delivery. Another example of a PPP categorization is in Brazil where the law defines a PPP as a concession contract in two possible forms: sponsored concessions or administrative concessions (Queiroz et al. 2014). Sponsored concessions represent
contracts where the private partner receives both revenues from user charges and financial subsidies from the public partner. Administrative concessions represent the type of contract in which the public partner provides (and guarantees) an income flow for the private partner. In this type of contract, the total amount of revenues to the private partner comes from the public partner.

The Serbian law recognizes and defines a concession as a type of PPP (The Law on Public-Private Partnership and Concessions 2011):

"with the elements of concession in which a public contract regulates the commercial use of natural resources or assets in general use which are publicly owned or the performance of an activity of public interest which the competent authority transfers to a national or foreign person, for a specified period of time, under specially prescribed conditions, against the payment of a concession fee by the private or the public partner, with the private partner bearing the risk associated with the commercial use of the subject of concession."

Another example of legal definition is from India (Government of India 2010a) where PPPs are defined as

"an arrangement between a government or statutory entity or government owned entity on one side and a private sector entity on the other, for the provision of public assets and/or related services for public benefit, through investments being made by and/or management undertaken by the private sector entity for a specified time period, where there is a substantial risk sharing with the private sector and the private sector receives performance linked payments that conform (or are benchmarked) to specified, pre-determined and measurable performance standards."

The World Bank, Asian Development Bank and Inter-American Development Bank's "Public-Private Partnerships Reference Guide" (2014) defines a PPP as a “long-term contract between a private party and a government entity, for providing a public asset or service, in which the private party bears significant risk and management responsibility, and remuneration is linked to performance.”

Looking at the PPPs for highways, there are also several different forms and models which have developed over time. For example, highways PPPs can be categorized based on their payment mechanism into toll concessions, shadow toll concessions, and
availability payment concessions (FHWA 2012). In the toll concession model, the private partner is entitled to charge tolls directly to users. In the shadow toll concession model, the private partner receives payments from the public partner for each vehicle that uses the road (no toll is collected from the users). In the availability payment model (also called availability fee or annuity model), payments from the public partner to the private partner are related to the road availability at pre-specified performance levels.

Another possible classification is based on the level of responsibility which is assigned to the private or the public partner (FHWA 2013a). In that context, for new facilities, i.e. greenfield projects, PPPs can be classified as private contract fee services, design build (DB), design build operate maintain (DBOM), design build finance (DBF), or design build finance operate maintain concession (DBFOM). For existing facilities, i.e. brownfield projects, there are two options: operation and maintenance concessions and long-term lease concessions. In this research, it is assumed that the project of interest is a greenfield toll concession based on the DBFOM model.

Existing toolkits

There are several toolkits available for the analysis and assessment of PPP highway projects. These toolkits provide a wide range of tools and manuals that may assist stakeholders involved in PPP projects from early phases of project development to financial closure and implementation. Some of the toolkits address various risks in PPP projects. These toolkits include probability and distributions of continuous random variables as input values in their models.

Federal Highway Administration (FHWA) Toolkit

In 2013, the Federal Highway Administration’s (FHWA) Office of Innovative Program Delivery launched a new toolkit, P3-Value (Public-Private Partnership Value-for-Money Analysis for Learning and Understanding Evaluation) (FHWA 2013b). Although the main purpose of the toolkit is to help decision makers in the “value-for-money” analysis, it covers other important aspects of PPPs such as risk evaluation and financial feasibility. This toolkit consists of four tools, namely a risk analysis tool, a public sector comparator (PSC) tool, a shadow bid tool, and a financial assessment tool, all of which are Microsoft Excel based and supported by associated manuals.

The risk assessment tool enables users to assess the impact that risks can have on a project’s cost and schedule starting with the basic project assumptions and identifying
risks of a project, both qualitatively and quantitatively. For identified risks, users can choose between the uniform and triangular distribution. The PSC tool enables users to determine the risk-adjusted costs of the project which are delivered under the public delivery (or traditional procurement) method (FHWA 2013c). Among other inputs, users have an option to use a simple toll scenario (toll rates can be assigned by vehicle type and road section) or a variable toll scenario (toll rates can be assigned by vehicle type, period of day/week and road section). Traffic is expressed by the number of vehicles per year, and users can provide an input for each year of the concession life and differentiate each section by vehicle type.

The shadow bid tool is used to calculate a project’s costs for the public sector if the project is delivered as a PPP concession (FHWA 2013d). Key components of this tool are: P3 contract payments which represent the cost the public sector has to pay to the private sector to make the project financially feasible; retained risks, i.e., risks which remain under the public sector responsibility; and other project costs that the private sector will have, such as procurement costs, etc. The financial assessment tool consists of two sections: a “value for money” analysis and a viability assessment (FHWA 2013e). The first section enables the comparison of the PSC and shadow bid costs and completes the value for money assessment. In the second section, users can review and analyze the project’s cash flows for better assessment of the project’s funding needs. Users have an option to conduct a sensitivity analysis of cash flows as well as a scenario analysis.

*World Bank Toolkit for Public-Private Partnerships in Roads and Highways*

The World Bank, with support of the Public-Private Infrastructure Advisory Facility (PPIAF), has developed a Toolkit for Public-Private Partnerships in Roads and Highways (World Bank and PPIAF 2009). The main objective of the Toolkit is to provide policy makers from countries with transitional and developing economies with some guidance and resources to design and implement PPP projects in the road subsector. The Toolkit consist of six modules: overview and diagnosis, key components, policy and planning, law and contract, implementation and monitoring, and tools. The first module provides general information about PPPs, their role in highway development and steps for defining a proper PPP policy. The second module presents information about the five main PPP components: scope, risks, revenues, finance, and
public accounting. The third module defines the role of the public sector in PPP arrangements with the goal to protect the public interest. The fourth module provides the legal and regulatory basis for the preparation and delivery of PPP contracts. The fifth module helps in the implementation of PPP highway projects through descriptions of a project’s key phases from project identification through preparation and tendering, contract monitoring and renegotiation. The sixth module provides a set of supporting tools such as case studies, financial models, bibliography, and key issues. Financial models enable the assessment of various financial parameters of a project over a concession period. Also, these models consider a large number of input parameters, allowing users to adjust the analysis to each project more closely. However, all of these input values are deterministic, values and to conduct sensitivity analysis, users have to run models for each set of input values manually.

Mladenovic and Queiroz (2014) have extended the Toolkit financial model to other infrastructure subsectors, in addition to roads. Based on the Toolkit toll road graphical financial model, their model can be used to assess the financial feasibility of Availability Payment (or Annuity) PPP Projects in any infrastructure subsector (e.g., roads, rail, airports, water, sanitation, schools, hospitals). As in the original model, the Availability Payment financial model comprises five worksheets (Data Sheet, Cash Flow Graph, Debt Graph, Dividend Graph, and Summary of Assumptions and Results). Default values are provided for each parameter defining a hypothetical PPP project. To define the project to be financially assessed, the user can change the parameter values using the arrow keys (scroll bars) provided in the Data Sheet and graph sheets.

**Texas Transportation Institute (TTI) Toll Revenue Estimation Model**

In 2012, the TTI published an Excel based spreadsheet tool for an early assessment of revenues generated on toll roads (Beaty and Lieu 2012). This tool allows users to specify several input parameters: general characteristics of the project, traffic data, and toll data. For traffic data, initial traffic is presented as a random variable and the user can choose between two types of distribution: normal and triangular. Over the concession period, traffic is adjusted by a growth rate every five years. This tool also considers two classes of vehicles: autos and trucks. One feature this tool provides is the maximum capacity constraint of the road which presents the upper limit in traffic simulations.
Toll rates, which need to be provided for autos and trucks separately, are adjusted over the concession period by the inflation rate. Users have an option to adjust toll rates manually every five years if the anticipated change is greater than inflation. This tool takes into account a ramp-up phase where users can specify the percentage of originally forecasted traffic, thus assuming a lower revenue during this phase. Transaction costs, which present costs occurring when vehicles use an electronic tolling system, are also covered in this tool. Yearly operational and maintenance costs are assumed to be fixed and are adjusted only for the inflation rate. Users can override these initial values by specifying values for each year separately. There are no options to include major maintenance costs like rehabilitation or reconstruction costs.

Government of India – PPP Toolkit for Improving PPP Decision-Making Processes

The Ministry of Finance in India released a web-based toolkit for the improvement of the decision-making process in PPP arrangements for the delivery of infrastructure projects (Government of India 2010b). This toolkit has five parts covering five sectors: state highways, water and sanitation, ports, solid waste management, and urban transport. For state highways, there are three modules: PPP background module, PPP process module, and tools and resources module. The first module provides background information and references on PPPs, while the second module provides guidance through the development of PPP projects from identification of potential projects, preparation and clearance, procurement, and contract management during the operational life. The third module provides a useful set of decision making tools for PPP practitioners, links to useful PPP resources and a set of case studies in India. This module contains six decision making tools: a Family Indicator, a Mode Validation Tool, a Suitability Filter, a Financial Viability Indicator Model, a Value-for-Money Indicator Tool, and a Readiness Filter. The PPP Family Indicator tool is set up to help with the preliminary selection of the mode that best fits the project for example, whether the project is a brownfield or a greenfield project or whether the primary resource of revenue is the user charge system, a shadow toll mechanism, or annuity payments by the public sector. The Mode Validation Tool further examines a risk allocation for the preliminary choice of the mode by allowing the user to choose the preferred risk allocation models. The tool then highlights the parts which are not in line with the chosen mode and the associated typical risk allocation. The PPP Suitability Filter
represents the preliminary qualitative value-for-money assessment which serves as a test of whether the project is suitable as a PPP or not. Users can answer a series of questions about the project and assess the level of difficulty to be delivered as a PPP on the scale from “NO GO” to “very attractive.”

The Financial Viability Indicator Model represents the quantitative assessment of a project’s financial feasibility delivered as a PPP. Users can test user-level charges, evaluate a project’s viability for the private partners, estimate the level of public support, and conduct sensitivity analysis of proposed financial schemes. Users have to provide an initial setup of the model to specify if the project involves capital expenditures, if it involves user charges, and if it is a brownfield or a greenfield project. Based on this initial setting, users further provide input data about the project such as the concession period, macroeconomic variables, traffic forecasts, capital expenditures, contingencies and insurance charges, financing structure, operating costs, and revenue options and outcomes. All of these input data are deterministic values. Results consist of a set of accounting ratios such as debt service coverage ratio, loan life cover ratio, return on assets, net profit margin, and return on equity. Also, results cover a set of output parameters related to the project, such as the project’s internal rate of return and net present value, and shareholder accounts, such as the equity internal rate of return and the equity net present value.

For the traffic forecasts, users can specify up to five different vehicle categories and road parameters (e.g., length, number of lanes, number of toll plazas). Traffic growth can be specified by the vehicle category and by phases, i.e. a project’s operational life can be divided into phases (up to three). Users can also specify the price elasticity of demand, i.e., the percent at which the traffic demand will change if the toll changes by 1%. Tolls can be specified by the vehicle category and by the phase. Discounts on user charges can be taken into account by specifying the percent of the discount (for example, for monthly passes) and the percent of users who use these discounts.

The Value for Money Indicator Tool presents a quantitative test which compares the costs of the traditional procurement option and the PPP option. Major risks which this tool considers are construction costs and time overrun, traffic risks (shortfall in traffic volume and revenue), operational expenditure risks, and the renegotiation risks. All of these risks are assumed to be random variables with bell shaped curves. Input for these
risks considers specifying the mean and the standard deviation. Financial output from
this tool serves as the input for the calculation of the value-for-money (VFM), which is
the difference between the net costs to the public if the project is publicly delivered and
managed and if the project is delivered as a PPP. The Readiness Filters include a series
of checklists and questionnaires for testing the level of a project’s readiness at major
steps in the process. The main purpose is to monitor the quality of the project
preparation process.

**Toll rates**

Models for establishing toll rates can be classified into supply-based and demand-based
categories (Gross and Garvin 2009). Average-cost pricing models belong to the supply-
based category. These models set tolls at the levels adequate to cover operations and
maintenance costs and, optionally, a project's capital expenditures. Marginal-social-cost
and revenue maximizing pricing models fall into the demand based category. Both
models take into account the demand elasticity. Marginal-social-cost models recognize
the relation between toll levels and traffic congestion and set levels of tolls based on the
impact of traffic on the surrounding environment. On the other hand, revenue-
maximizing models base the pricing scheme on the drivers value of time and vehicle
operation costs. Under various scenarios of traffic demand and toll rates, tolls are set to
the levels which provide the highest revenue for the operator. Although the revenue-
maximizing pricing approach is the most attractive for the concessionaire, the public
sector often sets restrictions on the pricing scheme during the concession duration.
Yescombe (2007), for example, recognizes three ways for setting the levels of toll rates
for a concession project: (i) competitive bidding which results in a fixed initial toll rate
that can be adjusted only for the inflation rate; (ii) bidders having the freedom to set
tolls; and (iii) the public authority setting the toll levels based on the national road
strategy.

However, the impact of incorrect estimates of toll rates can be significant. For example,
an analysis road concessions in Spain, Baeza and Vassallo (2010) found that 50% of
renegotiations of concession contracts ended up with an increase in toll rates. Another
example is an empirical study on the diversion of trucks from tolled highways to non-
tolled roads due to substantial rises in toll rates (Swan and Belzer 2008). The results
suggested that truck diversion is likely to take place if there is a substantial increase in
toll rates, which consequently may lead to safety issues and increased costs of freight transportation. Also, the increased maintenance costs of non-tolled roads due to an increased number of trucks place additional pressure on the budget available for the maintenance of the non-tolled road network.

**Toll elasticities**

Analyzing the price elasticity of travel demand, Hirschman et al. (1995) provided an empirical study on the sensitivity of traffic volumes with respect to the level of toll rates. They applied a multiple linear regression analysis to a time series of traffic volumes on bridges and tunnels in New York City. The authors included in the analysis, in addition to toll rates, four independent variables: employment, motor vehicle registration, subway fare, and gasoline prices. The analysis showed very low toll elasticities. For the data analyzed, an increase in toll rates did not have a substantial impact on traffic volumes, which seems to be typical for commuter traffic, as is mostly the case for New York tunnels and bridges.

The price elasticity of travel demand consists of several elasticities, i.e., cost components: operating costs, parking, tolls, travel time, accidents and insurance, and wear and ownerships (Burris 2003). The empirical estimates from various tolled facilities in the USA show toll elasticities in the range from -0.03 to -0.35 with, for example, less than -0.05 for the San Francisco Bay Bridge. Other studies show different values as, for example, Matas and Raymond (2003), which found a wide range of toll elasticities across motorway sections in Spain, varying from -0.21 to -0.83. They concluded that this range of variation can be explained by variables such as those related to the quality of the alternative roads, the length of the motorway section, and the location of the motorway (e.g., a tourist area). The more congested the alternative roads are, the higher the time benefits of using the tolled motorway will be, with demand consequently being more inelastic. They also concluded that the average aggregate toll elasticity cannot be used for the evaluation or forecasting purposes.

Similarly, Noland (2001) conducted an empirical study to test statistically whether an increase in highway capacity induces additional traffic growth. The analysis included data on lane mileage and vehicle-miles traveled (VMT) in the US. The multiple linear regression model also included data about the population, per capita income, and the cost of fuel as independent variables. The estimation of elasticities provided results
which clearly indicate that an increase in the highway capacity has an impact on VMT, thus increasing traffic on the highway.

*Estimating optimal toll levels*

Investigating the nature of PPPs, Yang and Meng (2000) developed a mathematical framework for the feasibility assessment of a new project as a function of optimal capacity and optimal toll rates, thus the maximizing profit and social benefit increment. This approach is useful for solving the issue of the different objectives of the public sector and the private firms, where the public sector is seeking a socially acceptable project while the private firms are looking for the maximization of revenues. Chen et al. (2001) extended this framework and included a simulation of traffic demand as a random variable. A standard normal distribution is used to model traffic forecasts. The optimal tolls and capacity probability distributions were obtained by solving the bi-level programming problem. Both probability distributions are bell shaped curves where the optimal capacity curve has a higher variation than the optimal toll capacity. The authors introduced the term "risk elasticity" as the measure of changes of profit standard deviation to changes in standard deviations of tolls and capacities. It is found that variations in capacity have a greater impact on profit variations than toll changes.

Subprasom et al. (2003) further extended the developed bi-level mathematical framework by including multiple uncertainties: forecasted travel demands, cost estimates, and value of time variations. Chen and Subprasom (2007) set the problem of finding optimal toll levels in a multi-objective optimization framework with the following objectives: the maximization of social welfare as the government objective, the maximization of profits as the private sector objective, and the minimization of inequality of benefits among road users. The project evaluation framework calculates three criteria for financial evaluation: the net present value (NPV), the internal rate of return (IRR), and the break-even year. If the project is not financially feasible, the calculation process is repeated with different government strategies which can modify the project's cash flows, e.g., construction costs subsidies, concession period extensions, etc.

*Risks in toll road concessions*

Following the concepts of traditional capital budgeting, the total risk of an asset can be divided between two risk components: systematic and unsystematic (Trigeorgis 1998).
Systematic (or market, or non-diversifiable) risks are related to the market movements which affect all securities and cannot be diversified away. Unsystematic (or diversifiable) risks are related to the variability of factors specific to the company or industry. These risks can be diversified and eliminated by various techniques such as portfolio hedging. For example, large construction companies may diversify unsystematic construction risks across a large portfolio of projects (Blanc-Brude and Makovsek 2013), while average-sized or local companies may not have the means to use this risk management strategy.

Based on the extensive literature review of risks in PPP projects in general and transport PPPs in particular, Pellegrino et al. (2013) grouped risks into four categories: technical, commercial, economic and financial, and political. Under each category there are several risks identified, with a total number of 19 risks. The risks in the technical category are site risks (land use and acquisition/resettlement and rehabilitation risk, site condition, site preparation), design risks (feasibility approvals, design approvals), construction risks (cost overrun, delay in completion, failure to meet performance criteria), and operating risks (operating cost overrun, delays or interruption in operation, shortfall in service quality). The risks in the commercial category are revenue risks (changes in taxes/tariffs, demand/usage); the risks in the economic and financial category are financial risks (interest rates, inflation, exchange rate, debt servicing risk); and the risks in the political category are regulator/political risks (changes in law, political issues).

A comparative analysis of PPP highway projects in Portugal, Spain, the United Kingdom, and Australia shows that these public agencies have 10 common categories of risks which are evaluated and allocated through PPP contracts: design, land acquisition, environmental compliance, construction, operations and maintenance, market/demand, latent defects, change in law, force majeure, and competing facilities (Brown et al. 2009).

The World Bank (2008) provides an example of the risk distribution matrix for PPPs in roads. This matrix defines 12 types of risk: design, site, construction, force majeure, revenue, operations and maintenance, performance, external, other market risks, political, default, and strategic risks.
Baeza and Vassallo (2010) highlighted several risks typical for toll motorway concessions: capital cost risks (land acquisition, construction and license approval), revenue risks, and maintenance and operation risks. This research will focus on traffic risks, construction risks, and operation and maintenance risks. Consequently, the following sections will provide some background on these selected risks.

Traffic forecasts

The history of PPPs in toll roads is full of examples of poor traffic forecasts. Analyzing traffic data from more than 100 international toll road projects, Bain (2009) reported that an average ratio of actual vs. forecasted traffic in the first year of road operation is 0.77, i.e., traffic forecasts, on average, had an error of 23 percent. Similarly, Flyvbjerg et al. (2005) reported that half of a sample of 210 road projects in general had an error in the traffic forecast greater than 20 percent. For example, in the case of the Dulles Greenway project, it was estimated that the traffic demand would be 34,000 vehicles per day and would increase at a 14 percent rate for the first six years (Garvin and Chea 2004). However, the original estimate was found to be too optimistic. The actual average traffic was only 11,500 vehicles per day in the first six months, less than half of the estimate (Fishbein and Babbar 1996).

There has been inconsistency even among traffic forecasts for the same road conducted by different consultants (Bain 2009). For example, four consultant companies provided traffic forecasts for one toll road. The difference between the highest and the lowest forecasted values for the first five years of the project's operation was 26 percent. In 2011, a survey was carried out among participants who are associated with transport modeling (Bain 2011). The goal was to learn their estimates of error range for traffic forecasts for existing (toll-free and toll) roads and new (also toll-free and toll) roads. Results of this survey revealed that there was a small, almost negligible, difference between results for toll-free and toll roads. For existing roads, even the next day forecast had an error of ±7.5 percent. As the forecast horizon increased, the forecast error also increased. For example, in the first year of the project opening, traffic forecast errors for existing roads can be expected to have a mean value of ±12.5 percent, while this value for new roads is ±17.5 percent. For a forecast horizon of 20 years, these values increase to ±42.5 percent for existing roads and ±47.5 percent for new roads. Baeza and Vassallo
(2010) found that traffic estimates for old concession contracts in Spain, in the period from 1967 to 1975, were very poor where real traffic was more than 60% lower than the forecasted level. Since traffic models have been improved, the new modern concession contracts in Spain, from 1996 to 2010, have an error of 25%.

The reasons for notable errors in traffic forecasts can be varied. For example, Vassallo (2007) pointed out that, in traffic forecasts for PPP road projects, there are two types of errors: natural and strategic errors. Natural errors represent inaccuracies of the forecasts which are the consequence of input and model uncertainties. Strategic errors represent manipulated traffic projections which are the consequence of strategic behavior of the stakeholders (for example, a bidder who intentionally increases traffic forecasts in order to win a tender). Bain (2009) listed some of the common reasons for modeling errors: a recession or economic downturn which is closely connected with the probable changes in the land use scenarios; an over-estimation of users' willingness to pay tolls, especially on roads which are charging higher than average toll rates; the complexity of the project and its tolling regime; an underestimation of the ramp-up period, etc. A synthesis report prepared by the US National Cooperative Highway Research Program (NCHRP 2006) provided an extensive list of factors influencing the traffic forecast performance: demographic and socioeconomic inputs, travel characteristics, value of time and willingness to pay, tolling “culture,” ramp-up period, truck forecasts, time choice modeling, risk, optimism bias, model validation, and peer reviews.

Revenue forecasts are dependent on demand forecasts and the associated assumptions (NCHRP 2006). This dependence results in a proportional level of uncertainties in both revenue and demand forecasts. The most widely used model for travel demand forecasts is the “four-step” model (Meyer and Miller 2001). This model is, with some modifications, also used for travel demand forecasts for toll roads (NCHRP 2006). However, there are other models developed and adopted for traffic forecasting purposes. For example, to model the uncertainty of future revenue, Irwin (2003), in the analysis of guarantees and other forms of public support for private investments in infrastructure projects, assumes that the revenue of a toll road project can be modeled as a stochastic process. Chow and Regan (2009) use, as the key concept in a real options analysis for managerial flexibility in network investments, a stochastic process such as the geometric Brownian motion (GBM) to model future demand. For the valuation of
flexibilities in the decision-making process for highway concessions, Galera and Soliño (2010) use traffic on the highway as the asset in the real options contract. The forecast of traffic volume, i.e. the Annual Average Daily Traffic (AADT), was modeled as a GBM. Moreover, the hypothesis that traffic volumes follow a GBM process was tested on Spanish toll highways (Soliño and Galera 2012). A sample of 11 highway concessions was used for the hypothesis testing with the data on AADT which were available, for most of the case studies, for a period of 30 years. The results showed that the null hypothesis could not be rejected.

Construction cost

Estimating the construction costs, especially in the early stages of project development, is a complex task. Many uncertainties are involved, and the estimates require careful consideration and due diligence in the evaluation process. Through the literature, two subjects related to construction cost uncertainties can be distinguished: first, the estimates of total construction costs of the projects, and second, the escalation of construction costs in transport projects. The first topic is part of the construction management practice where each project is evaluated for the existence of associated uncertainties in each cost component, leading to the overall estimate of total construction costs. The second topic draws on what is learned from statistical analysis of a large sample of transport projects and the relation between their actual and planned costs.

Elaborating on the first topic, Molenaar (2005) observed information aggregated in cost estimates as three cost categories: (i) known/knowns (known and quantifiable costs), (ii) known/unknowns (known and not quantified costs), and (iii) unknown/unknowns (unrecognized costs). It is important to consider and merge all three categories in the early stages of development of a highway project and to aggregate them in the complete cost estimate. Recognizing this issue, the Washington State Department of Transportation developed the Cost Estimation Validation Process (CEVP) for an early assessment of a highway project’s costs while considering various risks. The application of the CEVP methodology in nine case studies of highway megaprojects shows that some projects have a bell-shaped curve for the probability density function of total costs, while other projects have a long tail, or a long tail and a second hump. Although the probabilistic approach for the estimation of total construction costs by Monte Carlo
Simulations is commonly used, the fact that some of the construction cost components are correlated is often ignored (Touran 1993).

Flyvbjerg et al. (2003) conducted an extensive statistical analysis of construction costs covering 258 transport infrastructure projects over the period of nearly 70 years. Their goal was to test if the costs of transport infrastructure projects performed as planned at ex-post analysis. Results showed that the construction cost escalation, calculated as the difference between the actual and the planned cost as a percentage of the planned cost, is present in all modes of transport covered in the sample: rail, fixed links (tunnels and bridges) and roads. The distribution of cost escalation is a bell-shaped curve with a long right tail. For roads, in particular, the cost escalation has a mean of 20% with a standard deviation (SD) of 30%. The analysis also revealed that the construction cost forecast has not improved over time and that the magnitude of construction costs overrun has not decreased over time.

Upgrading the sample presented in Flyvbjerg et al. (2003), Cantarelli et al. (2012a) used a sample of 806 infrastructure projects worldwide (road, rail, tunnels, and bridges), out of which 537 were road projects. The mean cost overrun for roads is 19.8% with an SD of 31.4%, the lowest value of the four project types.

A similar study covered 78 Dutch infrastructure projects (Cantarelli et al. 2012b). Construction cost escalation is examined in relation to three independent explanatory variables: (i) project type (road, rail, and fixed links), (ii) project size, and (iii) the length of the project implementation phase. The sample size for road projects is 37 projects. First, road projects have the highest frequency of cost overruns. However, the mean value of cost overrun for road projects is not among the highest: the mean is 18.6% with an SD of 38.9%. Odeck (2004) analyzed construction cost overruns as the difference between estimated and actual costs in a sample of 620 completed road projects in Norway over the period 1992-1995. Results showed that 35.48% of the projects have cost underrun with a mean of -15% of cost underrun, 12.1% of projects do not have cost overrun, and 52.42% of projects have cost overrun with a mean value of 25%. Baeza and Vassallo (2010) found that capital cost underestimates are constantly present over the years in toll road concessions in Spain.
**Operation and maintenance costs**

Operation costs include costs associated with the business of running the project and collecting tolls. For example, operation costs can include, among others, transaction costs for each electronic toll collection, e.g. $0.085 per transaction (Beaty and Lieu 2012). They can also include marketing costs, courtesy patrols, and other costs tied to the organization which operates the facility. Operation costs can be assessed as the percentage of the annual revenue or construction costs. For example, Peng et al. (2014) adopted a rough estimate of operation costs as 7% of the annual traffic revenue. The FHWA PSC Tool assumes that annual operation costs are equal to a certain percentage of construction costs; the same assumption is made for routine and periodic or preventive maintenance costs (FHWA 2013b). Pantelias (2009) calculated the operation costs as a certain percentage of construction costs adjusted over time for inflation. For the Trans-Texas Corridor case study, the author used 3.5% of the construction cost as an approximation for the operation cost. Amdal et al. (2006) conducted an analysis of operating costs of the toll companies. Data were obtained from 26 Norwegian toll road companies. The main issue was to identify whether toll collection is affected by economics of scale and to identify economic, technological and institutional variables that affect operating costs. The results of the regression analysis indicated that the total number of paying vehicles per year, the share of vehicles using an on-board unit, and the competitive tendering for toll collection services as dummy variable were statistically significant. Also, the number of lanes, the debt level of the toll company, the use of a passenger-charging system, and whether the project uses a toll cordon or not will affect the average operating cost of the toll company.

Maintenance costs include the costs of activities to preserve the road conditions at a certain level of service. Road maintenance can be divided into several categories: routine works, periodic works, special works and development (World Bank 2014). Routine works are performed on a yearly basis and include works such as culvert cleaning and patching. Periodic works are carried out at intervals of several years with the goal of preserving the structural integrity of the road and include resurfacing, overlay, and pavement reconstruction. Special works include activities such as emergency works and winter maintenance. Development includes construction activities as part of the national development plan (e.g. the paving of unpaved roads in rural
areas). Maintenance costs can be estimated as a function of traffic volumes and construction costs. For example, Heggie (1995) uses a function of traffic volumes to estimate routine maintenance costs. Peng et al. (2014) use a function of both construction costs and traffic volumes for the estimate of annual maintenance costs. Pantelias (2009) estimates both routine maintenance and rehabilitation costs as percentages of construction costs adjusted over time for inflation.

**Summary**

The literature review reveals the complexity of PPP agreements in transport infrastructure. This complexity is present in all aspects, starting from the definition of PPP, which may vary from country to country, to the risks which need to be accounted for during the preparation, delivery, and operation of a single project.

The existing tools in use for the assessment of a project’s feasibility, i.e., a project's cash flow, lack the application of advanced probabilistic models. The more sophisticated multi-objective optimization models include multiple uncertainties in the optimization problem. However, these advanced models may seem complicated for wider use, as results are not easily transferred into easy to use information for decision making.

Information about optimal toll rates fulfilling different objectives for different stakeholders does not provide an answer to a simple question: what is a viable toll rate for a particular PPP project? This research addresses this issue and highlights the importance of the assessment of this parameter as a representation of a project's sustainability over time.

The literature review on risks in toll road concessions highlights presence of a number of risks. Models for estimation of selected risks are further reviewed. Information gathered in the presented literature review provides a background for the development of the methodology and an introduction to the uncertainties in the mathematical models used in this dissertation. The following chapter explains step-by-step the methodology for the assessment of an acceptable toll rate.
CHAPTER III
RESEARCH METHODOLOGY

This research is accomplished through several steps. After an extensive literature review which formed the basis for further research, the research framework can be defined. It introduces the model boundaries and examines these boundaries within the complex structure of PPP toll road projects. Further, the research methodology is developed with an explanation of the main steps for the assessment of an acceptable toll rate, i.e., a toll rate that is acceptable to all stakeholders. This chapter is dedicated to the detailed presentation of the research framework and the research methodology.

Research framework

PPP projects must define the interaction between several different aspects of the projects. In order to ensure project success, each segment of a project has to be properly addressed. The following aspects are considered in this research are: (i) project time frame, (ii) risk management context, (iii) stakeholder’s network, and (iv) implementation context. These aspects define the framework within which this research is conducted.

The time frame is an important component of PPP projects since the life span is very long, from the project development to the decommissioning event. In some cases, the life of the PPP is not even determined in advance but depends on the achievement of certain thresholds, for example, of demand or revenue. Risk management in PPPs, one of the main features of PPP contracts, defines the unique risk-sharing mechanism between the public and the private partners. PPP projects usually involve several stakeholders, each one with levels of participation changing over a project’s life (South et al. 2015). Implementation context represents a group of settings within which the project is developed and implemented. Regardless of whether they are project- or country-related, these settings can be categorized as technical, financial, social, economic, political, or legal.

Technical settings represent project parameters such as the project size, complexity, and costs. Financial settings represent the project’s financial soundness, the level of development of local financial markets, the availability of debt and equity providers, and all other factors which can affect the project’s financial outcome. Social settings are
rather twofold: one is the social acceptance of the project in general and the toll rates in particular, and the other is the impact that the project has on society. Economic, political, and legal settings are country-specific. The following headings present how and where the developed methodology fits within these aspects of a project.

**Time frame:** A PPP project’s life cycle can be divided into four phases as shown in Figure 1: phase I is the project’s development which includes an early assessment of the project’s economic, social, financial, and technical aspects; phase II represents the detailed preparation and procurement phase; phase III is the construction phase; and phase IV is the operation and maintenance phase. The developed methodology focuses on Phase I: the project development and the preliminary assessment.

![Figure 1. PPP project life cycle](image)

As the project progresses and some of the uncertainties are resolved, e.g. a better estimation of construction costs and maintenance costs during the detailed preparation (phase II) or obtaining the final value of construction costs at the end of the construction period (end of phase III or beginning of phase IV), the estimated values can be updated in the model. It is also recognized that this methodology can be used during the operation phase, especially during the project’s ramp-up phase when fluctuations in traffic volumes are still considerable. Also, if the concept of staged development is considered as one possible scenario, i.e., the option to expand the road capacity during the operation and maintenance phase, this methodology can be used for the assessment of anticipated uncertainties in traffic volumes after the expansion and associated assessment of toll rates.

**Risk management:** Regarding the most important aspect of PPP projects, several risks are addressed in this research: traffic risk, construction risk and operation and maintenance risk. Traffic risk is considered a major risk in road concessions. As traffic and revenue risks are correlated for toll roads, then both traffic and revenue risks represent the variability of cash flows needed for debt servicing during a project’s life. The traffic risk is modeled as the stochastic variable.
Second, construction risk represents the variability of estimates of upfront investments. The role of this risk is twofold: (i) it represents the project’s technical risk which is managed with traditional construction management practices; and (ii) it is the financial risk as construction costs are capital expenditures and thus significantly affect estimates of the project’s debt.

The third type of risk considered is operation and maintenance risk, which represents the risk of variability in ongoing expenditures, thus influencing the available cash flows needed to cover ongoing debt service and to provide adequate equity returns. All three risks represent risks related to the project’s financial structure.

**Stakeholders’ network:** Each infrastructure project involves several stakeholders with different levels of involvement. Stakeholders’ interrelations are dynamic and evolve over a project’s life (South et al. 2015). In the example of toll road SR91 in California, South et al. (2015) showed how the stakeholders’ dynamic looked over the project’s life. In the first phase of the project development, three groups of stakeholders were involved: sponsor (public), developer (private), and users (civic). Later, two more groups became involved: investors (private) and operator (private). The methodology developed in this research is of direct interest for the public and the private entities, users, and potential investors.

**Implementation context:** This research addresses the following aspects of the project implementation: (i) for the technical aspect, the model takes into account a project’s construction and operation and maintenance (O&M) costs; (ii) for the financial aspect, the methodology is based on the financial model for an early assessment of a project’s financial soundness; and (iii) for the social and economic aspect, the methodology provides an option to take into consideration the socially acceptable toll rate, even at the early stages of a project’s financial feasibility assessment. Economic aspect of the project implementation context also has its role in the model. In some cases, the public sector may consider financial support for the project to foster economic development of the region. Also, some input parameters are country-economic parameters, such as the inflation rate, the corporate tax rate, the state discount rate, and the VAT rate. The political and legal aspects are not part of this methodology.
All of these elements represent the model boundaries within which this research is developed and implemented. The following section introduces the methods used in this research.

**Research methodology**

For the development of the methodology for finding the acceptable toll rate for projects using a PPP scheme, the financial model of the Toolkit for PPPs in Roads and Highways (World Bank and PPIAF 2009) was used as a starting point. The Toolkit includes a financial simulation model presented in two forms, Graphical and Numerical. The Graphical Model is used as a diagnostic tool for preliminary assessments, while the Numerical Model is more detailed and can serve as a first project analysis at the pre-feasibility level.

The proposed methodology is presented in Figure 2. In the first step, toll rates are expressed as a function of other input parameters using the background mathematical settings from the Graphical Model. This is a new deterministic model that calculates the minimum toll rate for a given set of project parameters and for the minimum required values of financial constraints.

Further, an analysis is conducted to test the sensitivity of toll rates to changes in technical and financial parameters which are modeled as deterministic values. Sensitivity is expressed through the calculation of different elasticities which reveal the sensitivity of toll rates to changes in other parameters in the model. In the next step, some input parameters are modeled as random variables. The model takes into account the estimates of traffic volume, construction costs, operation and maintenance costs, and
their variability. Toll rates are determined from the probability distribution obtained from simulation. This approach enables the analysis of the possible range of toll rates, given the project’s risks and how the socially acceptable toll rate fits the financially required toll rate.

Trigeorgis (1998) explained the traditional approach for the Monte Carlo simulation technique in the analysis of managerial flexibility through several steps:

i) Modeling the project through the mathematical equations and identification of primary variables;

ii) Defining the probability distribution of primary variables. It is useful to conduct sensitivity analysis prior to simulation to confirm the importance of selected variables and to observe if other variables may be of importance. Also, the dependence between variables should be recognized;

iii) Drawing random samples from the probability distribution of selected variables to enable the calculation of interest;

iv) Repeating the process as many times as needed, each time storing the results, and obtaining the probability distribution of interest along with the expected value, standard deviation, and other statistics.

The adopted methodology in this research is in line with the described procedure, confirming the logic behind the development of the methodology and its intended application.

**Validation procedure**

Model validation represents the process which demonstrates how reasonably the developed model predicts the actual performance (NCHRP 2006). The first validation process is applied to the mathematical models for the calculation of toll rates. The models are checked for logical inconsistencies. As a second validation, the developed methodology is tested on two case studies of road projects which are currently under operation. The basic idea is to run developed methodology with project data from an early assessment, such as forecasted construction costs and traffic volumes, and to compare results with real expenses and traffic volumes, if this data is available. The distribution of toll rates obtained from simulations is compared with the current toll rates.
Summary
This chapter is dedicated to the research framework and the research methodology. It explains the model boundaries and steps which are followed in the model development based on the probabilistic approach. The mathematical background is presented in the next chapter as the first step of the methodology. It serves as the basis for both the deterministic and stochastic approaches.
CHAPTER IV
DETERMINISTIC MODEL

The 2nd edition of the World Bank (WB) Toolkit for PPPs in Roads and Highways, published in 2009, is used in this research.

**Mathematical background**

Step 1 in the research methodology is to use the WB Toolkit for setting the mathematical background and defining the problem. The model is derived from the Toolkit's Graphical Model which includes 18 input parameters, all of which are deterministic values. These parameters can be divided into four different groups:

i) technical parameters of the project: concession life, construction cost, construction period, distribution of works during the construction period, operation cost, initial daily traffic, traffic growth, and toll rate;

ii) the financial structure of the project with two input values: percent of equity and percent of government subsidies (the percent of debt is implied, as the total is equal to 100 percent);

iii) the debt structure with four input parameters: debt maturity, interest rate, type of repayment, and grace period; and

iv) country specific economic parameters: inflation rate, corporate tax rate, state discount rate, and VAT rate.

The Graphical Model calculates key financial parameters of the project for the given set of input values. These financial parameters are the project's financial internal rate of return \((FIRR)\), the equity internal rate of return (return on equity, \(ROE\)), the annual debt service cover ratio \((ADSCR)\), and the loan life cover ratio \((LLCR)\).

The debt service cover ratio \((DSCR)\) measures a company’s ability to meet the periodic interest and principal payments on its debt from available cash flows for a particular period (Delmon 2009). The most common approach is to examine the annual debt service cover ratio \((ADSCR)\). The loan life cover ratio \((LLCR)\), for a given year, is the present value of cash available for debt service \((CADS)\) up to the debt’s maturity divided by the outstanding debt at the date of calculation. The rate of return is the ratio of expected earnings or losses on an investment relative to the amount invested. The project’s internal rate of return \((IRR)\) represents the discounted rate for which the net
present value (NPV) over the project life is equal to zero. The IRR is the compounded rate of return which can be earned on both the debt and equity invested. The return on equity (RoE) is the rate of return which can be earned on the equity only.

On a trial and error basis, one can determine the minimum toll rate which will satisfy the predefined financial constraints. In other words, for the chosen set of input values, one can find the toll rate so that the FIRR, ROE, ADSCR and LLCR are above the minimum required values. This feature of the Toolkit serves as the starting point for the development of the deterministic model. The problem is set as calculating the minimum toll rate which will satisfy predetermined values of financial constraints instead of finding that toll rate on a trial and error basis. The mathematical relations between the input parameters and the financial parameters implemented in the Toolkit are used for this purpose. First, it is observed how financial constraints are calculated in the Toolkit.

Annual debt service cover ratio

If \( i \) is a year of a concession’s life \( i = 1,\ldots,T \) where the first year represents the start of the construction phase and \( T \) represents the end of a project’s operational life, then \( ADSCR_i \) is defined as the following:

\[
ADSCR_i = \frac{CF_{bds,i}}{DS_i}, \text{ where } i = CP + 1, \ldots, T
\]

where \( CP \) is the construction period, \( CF_{bds,i} \) is the cash flow before debt service in year \( i \) and \( DS_i \) is the debt service in year \( i \). Further, \( CF_{bds,i} \) is defined as

\[
CF_{bds,i} = R_i - OC_i - Tax_i, \text{ where } i = CP + 1, \ldots, T
\]

where \( R_i \) is revenue in year \( i \), \( OC_i \) is operating cost in year \( i \) and \( Tax_i \) are taxes in year \( i \).

These parameters are defined as

\[
R_i = 365 \times AADT_i \times t_{r,i} \times (1 + inf)^i
\]

\[
AADT_i = AADT_0 \times (1 + r)^i
\]

\[
OC_i = (P_i + a \times AADT_i + O_i) \times (1 + inf)^i
\]

\[
Tax_i = PR_{bt,i-1} \times C_{cr}
\]

where \( i = CP + 1, \ldots, T \), \( AADT_i \) is the annual average daily traffic in year \( i \), \( t_{r,i} \) is the toll rate (VAT excluded) in year \( i \), \( inf \) is the inflation rate, \( r \) is the traffic growth rate, \( P_i \) is the principal repayment in year \( i \), \( a \) is the variable part of a project’s maintenance cost, \( O_i \) is the operation cost in year \( i \), \( PR_{bt,i-1} \) is the profit before taxes in year \( i-1 \), and \( C_{cr} \) is the
The toll rate $t_{r,i}$ is the weighted toll rate, i.e., weighted across different categories of vehicles. The profit before tax $PR_{bt,i-1}$ is calculated as

$$PR_{bt,i-1} = R_{i-1} - OC_{i-1} - A_{i-1} - I_{i-1}, \text{where } i = CP + 1, \ldots, T$$

(7)

where $A_{i-1}$ is the amortization in year $i-1$ and $I_{i-1}$ is the financial cost in year $i-1$, or, in this case, interest in year $i-1$. The amortization is calculated over the amortization period $AP = T - CP$ and is equal to

$$A_i = \sum_{i=1}^{CP} CC_i (1 + inf)^t + \sum_{i=1}^{CP} CI_{D,i}$$

(8)

where $IS$ is a percentage of investment subsidies, $CC_i$ is the construction cost in year $i$, and $CI_{D,i}$ is the capitalized interests on debt in year $i$. If a debt service in year $i$ is defined as $DS_i = I_i + P_i$, where $P_i$ is a principal repayment in year $i$, then Equation 1 can be rewritten as

$$ADSCR_i = \frac{t_{r,i} 365 (AADT_i (1 + inf) t - AADT_{i-1} C_{tr} (1 + inf) t^{-1} - OC_i (1 - 0C_{i-1} - A_{i-1} - I_{i-1}) C_{tr}}{I_i + P_i}$$

(9)

where $i = CP + 1, \ldots, T$. During the analysis of a project’s financial feasibility, $ADSCR$ is defined in advance and set at a minimum required value. Then, its value is checked for each year of the project’s operation to see if it is equal or greater than that minimum required value. In this research, the objective is to find the minimum toll rate which satisfies the minimum required value of $ADSCR$ in each year of the project’s operation.

To find this value, Equation 9 needs to be rearranged so the toll rate can be calculated for the given $ADSCR$ and it becomes

$$t_{r,i} = \frac{ADSCR_{i} (I_i + P_i) + OC_i (1 - 0C_{i-1} - A_{i-1} - I_{i-1}) C_{tr}}{365 (AADT_i (1 + inf) t - AADT_{i-1} C_{tr} (1 + inf) t^{-1})}, i = CP + 1, \ldots, T$$

(10)

The toll rate has to be equal to or greater than the expression at the right side of Equation 10 for each year of the project’s operational life in order to satisfy the given constraint. Let us denote the toll rate for the given $ADSCR$ as $t_{r,ADSCR}$. From Equation 10, toll rates are calculated for each year of the project’s operational life $t_{r,ADSCR}$ which gives a set of solutions. Here, the solution to the problem is the maximum value of the toll rate from the obtained set of solutions:

$$t_{r,ADSCR} = \max \left( t_{r,i} \right), i = CP + 1, \ldots, T$$

(11)
**Loan Life Cover Ratio**

Similarly, the mathematical relations for other financial constraints from the Graphical Model of the Toolkit are also analyzed. Let us define \(\text{LLCR}_i\) in year \(i\) as the following:

\[
\text{LLCR}_i = \frac{\text{NPV}(CF_{dsk,i})}{\sum p_i}, i = CP + 1, ..., T
\]  

(12)

Comparing Equations 1 and 12, it can be observed that the calculation of both ratios takes into account the available cash flows and outstanding debt or principal repayments. If the project’s financial profile fulfills the requirements for \(\text{ADSCR}\), and given that \(\text{LLCR}\) is typically set to about 10% higher than \(\text{ADSCR}\) (Yescombe 2007), it can be assumed that the requirements for \(\text{LLCR}\) will also be fulfilled. For this reason, the calculation of the minimum toll rate for a given \(\text{LLCR}\) is not further considered. However, this assumption can be relaxed by checking the value of \(\text{LLCR}\) once the minimum required toll rate is determined. This also holds for other financial indicators which are not considered in this research.

**Internal rate of return**

The next constraint which is taken into account is a project's \(\text{IRR}\). It can be argued that the \(\text{IRR}\) is a result of the given cash flows rather than a constraint for the financial analysis. Here, however, it is assumed that the stakeholders are interested only in projects which will generate adequate return. Investors are aware, even in the early stages of project development, which value of \(\text{IRR}\) is acceptable based on the planned mix of equity and debt funding. Thus, the \(\text{IRR}\) as a constraint is kept in the model.

The Toolkit calculates the \(\text{IRR}\) both in nominal and real terms. Here, the \(\text{IRR}\) expressed in real terms is considered for calculations. The \(\text{IRR}\) is the discounted rate for which the project’s \(\text{NPV}\) over the project life is equal to zero:

\[
\sum_{i=1}^{T+1} \frac{OCF_{\text{real},i}}{(1+\text{IRR}_{\text{real}})^i} = 0
\]  

(13)

where \(OCF_{\text{real},i}\) is the operating cash flow in year \(i\) expressed in real terms without interests on debts.

Taking into account inflation \(inf\) and assuming it is constant over a project’s life, the relation between the operating cash flows in real terms and nominal terms is

\[
OCF_{\text{real},i} = \frac{OCF_{\text{nom},i}}{(1+inf)^i}
\]  

(14)

where \(OCF_{\text{nom},i}\) is the operating cash flow in nominal terms without interests on debts;
\[ OCF_{\text{void},i}^{\text{nom}} = \begin{cases} -CC_i(1 + inf)^i, & i = 1, \ldots, CP \\ R_i - OC_i - Tax_{c,i-1}^{\text{void}}, & i = CP + 1, \ldots, T + 1 \end{cases} \quad (15) \]

where \( CC_i \) is the construction cost in year \( i \), and \( Tax_{c,i-1}^{\text{void}} \) is the corporate tax rate without interests of debts calculated for year \( i-1 \) and paid in year \( i \);

\[ Tax_{c,i-1}^{\text{void}} = PR_{bt,i-1}^{\text{void}} \times C_{tr} \quad (16) \]

where \( PR_{bt,i-1}^{\text{void}} \) is the profit before taxes without interests of debts in year \( i-1 \); and

\[ PR_{bt,i-1}^{\text{void}} = R_{i-1} - OC_{i-1} - A_{i-1}^{\text{void}} \quad (17) \]

where \( A_{i-1}^{\text{void}} \) is the amortization in year \( i-1 \) without financial interests. Amortization is calculated over the amortization period as

\[ A_{i-1}^{\text{void}} = \frac{\sum_{t=1}^{CP} CC_t (1 + inf)^t}{AP} \quad (18) \]

Then, Equation 13 can be rearranged and rewritten as follows:

\[ \sum_{t=1}^{T+1} -CC_t(1 + inf)^t + t_r \times 365 (AADT_t(1 + inf)^t - AADT_{t-1} C_{tr}(1 + inf)^{t-1}) - OC_t \times \left(-OC_{t-1} - A_{t-1}^{\text{void}}\right) C_{tr} \times (1 + inf)^{t-1} = 0 \quad (19) \]

Again, if the toll rate is assumed to be constant over the concession period, then Equation 19 has a closed form solution:

\[ t_r = \frac{\sum_{t=1}^{T+1} -CC_t(1 + inf)^t - OC_t \times \left(-OC_{t-1} - A_{t-1}^{\text{void}}\right) C_{tr} \times (1 + inf)^{t-1}}{\sum_{t=1}^{T+1} 365 (AADT_t(1 + inf)^t - AADT_{t-1} C_{tr}(1 + inf)^{t-1}) \times (1 + inf)^{t-1}} \quad (20) \]

The toll rate from Equation 20, the toll rate for the given \( IRR \) in real terms, is denoted as \( t_r^{IRR} \).

**Return on equity**

The last constraint from the Graphical Model of the Toolkit is the equity \( IRR \) or return on equity (\( RoE \)). The \( RoE \) can be calculated as the rate of return when the \( NPV \) of cash flow available for distribution among shareholders is equal to zero:

\[ \sum_{t=1}^{T+1} \frac{SCF_{t}^{\text{real}}}{(1 + RoE_{t}^{\text{real}})^t} = 0 \quad (21) \]

where \( SCF_{t}^{\text{real}} \) is the cash flow available for distribution among shareholders in real terms and \( RoE_{t}^{\text{real}} \) is also in real terms. The cash flow available for distribution among shareholders \( SCF_{t}^{\text{real}} \) in real terms is
where \( SCF_{i}^{\text{nom}} \) is the cash flow available for distribution among shareholders in nominal terms;

\[
SCF_{i}^{\text{nom}} = \begin{cases} 
-E_i, i = 1, \ldots, CP \\
D_i - E_{r,i}, i = CP + 1, \ldots, T + 1 
\end{cases}
\]  

where \( E_i \) is the equity in year \( i \), \( D_i \) are the dividends in year \( i \) and \( E_{r,i} \) is an equity redemption in year \( i \). Dividends \( D_i \) are defined as

\[
D_i = (R_i - O_i - A_i - I_i)(1 - Tax_c)
\]  

Equation 21 then becomes

\[
\sum_{i=1}^{T+1} \frac{-E_i - E_{r,i} + 365t_r AADT_i(1+inf)^i(1-Tax_c) - (O_i + A_i + I_i)(1-Tax_c)}{(1+inf)^i(1+RoE_{\text{real}})^i} = 0
\]  

Following the previously introduced assumption that the toll rate is constant over the concession period, Equation 25 has a closed form solution:

\[
t_r = \frac{\sum_{i=1}^{T+1} E_i + E_{r,i} + (O_i + A_i + I_i)(1-Tax_c)}{\sum_{i=1}^{T+1} 365 AADT_i(1-Tax_c)(1+inf)^i} 
\]  

Solution of Equation 26, the toll rate for the given \( RoE \) in real terms, is denoted as \( t_{r,RoE} \).

The solution to the problem of finding the minimum toll rate which fulfills given financial constraints is the maximum value of the three toll rates:

\[
t_r = \max(t_r^{ADSCR}, t_r^{IRR}, t_r^{RoE})
\]  

Equation 27 concludes Step 1 of the methodology. Knowing the value of the weighted toll rate which fulfills the predefined constraints for \( ADSCR \), \( IRR \) and \( RoE \), makes it possible to analyze the sensitivity of toll rates to changes in other parameters in the model. For example, one can observe the sensitivity of the minimum financially required toll rate to changes in construction costs. The results of this and similar types of sensitivity analysis are presented in the following sections.

**Model validation**

Before continuing with the analysis of the sensitivity of toll rates to changes in selected parameters, one needs to test the presented mathematical model. The validation method chosen was a direct comparison of results obtained from Equation 27 and the results from the Toolkit for the same input parameters. Table 1 presents a summary of these input parameters. The values of construction costs and initial traffic are changed within
ranges presented in Table 1. This analysis enabled the comparison of 32 different results for toll rates.

**Table 1. Input parameters for the deterministic model validation**

<table>
<thead>
<tr>
<th>A. Project Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Concession term = 20 years</td>
<td></td>
</tr>
<tr>
<td>Construction cost = $1 million per km to $8 million per km</td>
<td></td>
</tr>
<tr>
<td>Construction period = 2 years</td>
<td></td>
</tr>
<tr>
<td>Distribution of works during the construction period = 1st year 50%, 2nd year 50%</td>
<td></td>
</tr>
<tr>
<td>Operation costs= $100,000 per km per year (no variable costs)</td>
<td></td>
</tr>
<tr>
<td>Equity=30% of the construction cost</td>
<td></td>
</tr>
<tr>
<td>Government subsidies to the capital costs=10% of the construction cost</td>
<td></td>
</tr>
<tr>
<td>Initial traffic=5,000 vehicles per day to 20,000 vehicles per day</td>
<td></td>
</tr>
<tr>
<td>Traffic growth=3% per year</td>
<td></td>
</tr>
<tr>
<td>Inflation=6% per year</td>
<td></td>
</tr>
<tr>
<td>Value added tax (VAT)=18%</td>
<td></td>
</tr>
<tr>
<td>Corporate tax = 10%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Loan Terms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of repayment = level-annuity basis (principal + interest = constant)</td>
<td></td>
</tr>
<tr>
<td>Nominal Interest rate=10% per year</td>
<td></td>
</tr>
<tr>
<td>Grace period=4 years</td>
<td></td>
</tr>
<tr>
<td>Repayment period=14 years</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Financial Constraints</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial internal rate of return of the project (FIRR) ≥12%</td>
<td></td>
</tr>
<tr>
<td>Return on equity (ROE) ≥16%</td>
<td></td>
</tr>
<tr>
<td>Annual debt service coverage ratio (ADSCR) ≥1.2</td>
<td></td>
</tr>
</tbody>
</table>

Results from this analysis are presented in Table 2. The analysis revealed that differences between toll rates calculated from the Graphical Model of the Toolkit ("Toolkit" raw in the table) and the deterministic model ("Model” raw in the Table) are in range from $0.0001 to $0.0018 per vehicle per kilometer. These differences are not a consequence of a random component of the Toolkit or the model because there is no such component. It is rather the consequence of rounding the numbers in the Toolkit calculations. The Toolkit has predefined increments for each input value including the increment for the toll rate. If the constraint for the IRR is set to 12%, it is likely that the calculated IRR in the Toolkit is a little bit higher than 12%. The user has to find the toll rate which calculates the value of this constraint on a trial and error basis. On the other
hand, in the deterministic model the toll rate is calculated directly based on predefined equations. These different approaches of finding the toll rate, one method using trial and error basis and the other by calculating it directly, causes the difference between the values. For this reason, no statistical analysis of errors or test for goodness of fit have been undertaken.

Table 2. Toll rates obtained from the Graphical Model and from the deterministic model ($/veh/km)

<table>
<thead>
<tr>
<th>Construction cost (US$ million per km)</th>
<th>Initial annual average daily traffic $AADT_0$ (vehicles per day)</th>
<th>5,000</th>
<th>10,000</th>
<th>15,000</th>
<th>20,000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Toolkit</td>
<td>Model</td>
<td>Toolkit</td>
<td>Model</td>
<td>Toolkit</td>
</tr>
<tr>
<td>1.0</td>
<td>0.139</td>
<td>0.1386</td>
<td>0.07</td>
<td>0.0693</td>
<td>0.047</td>
</tr>
<tr>
<td>2.0</td>
<td>0.224</td>
<td>0.223</td>
<td>0.112</td>
<td>0.1115</td>
<td>0.075</td>
</tr>
<tr>
<td>3.0</td>
<td>0.309</td>
<td>0.3075</td>
<td>0.154</td>
<td>0.1537</td>
<td>0.103</td>
</tr>
<tr>
<td>4.0</td>
<td>0.393</td>
<td>0.3919</td>
<td>0.197</td>
<td>0.196</td>
<td>0.131</td>
</tr>
<tr>
<td>5.0</td>
<td>0.478</td>
<td>0.4763</td>
<td>0.239</td>
<td>0.2382</td>
<td>0.16</td>
</tr>
<tr>
<td>6.0</td>
<td>0.562</td>
<td>0.5608</td>
<td>0.281</td>
<td>0.2804</td>
<td>0.188</td>
</tr>
<tr>
<td>7.0</td>
<td>0.647</td>
<td>0.6452</td>
<td>0.324</td>
<td>0.3226</td>
<td>0.216</td>
</tr>
<tr>
<td>8.0</td>
<td>0.731</td>
<td>0.7297</td>
<td>0.366</td>
<td>0.3648</td>
<td>0.244</td>
</tr>
</tbody>
</table>

Although the presented results are just a comparison between two models, they show that for a sample of 32 results, there is no inconsistency between the models. This sample size can be considered satisfactory because of the broad range of values.

One more method for model validation is used: checking the model for extreme values. The model was tested using extremely low or high traffic volumes. One scenario is that if there is almost no traffic, it is expected that the toll rate would be extremely high. The other scenario is the extremely high traffic volumes, when it is expected that the toll rate would be very low. In the case when the $AADT_0$ is set to zero, the deterministic model gives an output for the toll rate as infinite, which corresponds to the expected output. The other case, when $AADT_0$ was set to 1,000,000 vpd, the deterministic model calculated the value for toll rate close to zero, which is, again, in line with expectations. The Graphical Model of the Toolkit does not have an option to provide an input for
traffic volume as zero or one million vehicles per day; it has a predefined range of values. For this reason, the Graphical Model could not be tested with extreme case scenarios.

**Model limitation**

The following factors can be considered as the limitations of the model in its representation of real cases:

- the toll rate is adjusted only for inflation over the concession period;
- all input parameters are deterministic and adjusted over the concession period only for inflation;
- the model considers only a single road project and the behavior of the project within its own scope, regardless of its position within the network;
- it does not have a feature to take into account price (toll rate) elasticity of demand; however, as found in the literature, including elasticities is not recommended for forecasting purposes;
- there are no options to include major maintenance costs (reconstruction or rehabilitation costs) during the concession life; nevertheless, if such costs are annualized, they can be added to the O&M costs.

**Sensitivity analysis**

Step 2 of the presented methodology is a sensitivity analysis. The sensitivity of toll rates to changes in some of a project’s parameters is assessed through the estimation of elasticities (Vajdic et al. 2012). The elasticity \( e_{x,y} \) of parameter \( x \) to a change in parameter \( y \) can be expressed as follows:

\[
e_{x,y} = \frac{\Delta x}{\Delta y} \frac{x_0}{y_0}
\]  

(28)

where \( \Delta x \) is a change in the parameter \( x \), and \( x_0 \) is its initial value; \( \Delta y \) is a change in the parameter \( y \), and \( y_0 \) is its initial value. Here, elasticities represent the percent change in the toll rate due to a percent change of some technical or financial parameter, e.g., initial daily traffic. It is expected that results from this analysis will help identify the relationships between parameters of interest and values of toll rates.

First, toll rates were reviewed as a function of construction cost and initial daily traffic. This analysis serves as the basis for a better understanding of the toll rate sensitivity with respect to variations of construction costs and initial daily traffic. The initial daily
traffic is expressed as $AADT_0$ at the toll road opening year, while toll rates are expressed as the weighted average toll rate ($water$) in US dollar per vehicle. Assuming that traffic on the observed road section can be represented as a mix of cars, trucks and buses, the $water$ per vehicle is defined as follows:

$$water_r = \frac{(%C \cdot t^C_r + %T \cdot t^T_r + %B \cdot t^B_r)}{100} \quad (29)$$

where $%C$ is the percentage of cars, $%T$ is the percentage of trucks, and $%B$ is the percentage of buses; $t^C_r$, $t^T_r$ and $t^B_r$ are the toll rates for cars, trucks, and buses, respectively. Table 3 summarizes the input parameters and constraints which are used in this analysis. Values are assumed for a sample project with a relatively high percentage of government subsidies and high interest rates, indicating the potential implementation scenario in countries with transitional and developing economies.

**Table 3. Project input parameters (Vajdic et al. 2012)**

<table>
<thead>
<tr>
<th>A. Project Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concession term = 20 years</td>
</tr>
<tr>
<td>Construction period = 2 years</td>
</tr>
<tr>
<td>Operation costs = $100,000 per km per year (no variable costs)</td>
</tr>
<tr>
<td>Distribution of works during the construction period = 1st year 50%, 2nd year 50%</td>
</tr>
<tr>
<td>Equity = 40% of the construction cost</td>
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<td>Government subsidies to the capital costs = 40% of the construction cost</td>
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<tr>
<td>Traffic growth = 4% per year</td>
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<td>Inflation = 4% per year</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Loan Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Interest rate=15% per year</td>
</tr>
<tr>
<td>Type of repayment = level-annuity basis (principal + interest = constant)</td>
</tr>
<tr>
<td>Grace period= 2 years</td>
</tr>
<tr>
<td>Repayment period=14 years</td>
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</tr>
</tbody>
</table>

Figure 3 represents the results of this analysis. The horizontal axis represents the estimates of constructions cost needed to build the road and to put the project into operation; the vertical axis represents the toll rates, in terms of $water$. The results are as
expected: as construction costs increase, the minimum toll rate required to attract private investors also increases. And for lower values of initial AADT, toll rates increase at a higher rate. Analysis of elasticities provides more information about the sensitivity of toll rates with respect to changes in input parameters.

Figure 3. Toll rate $w_{at}$ estimated as a function of construction costs and $AADT$

The elasticity $\varepsilon_{tr,CC}$ of the toll rate $t_r$ (short form of $w_{at}$) to changes in construction costs $CC$ is defined as

$$
\varepsilon_{tr,CC} = \frac{\Delta t_r}{t_{r,0}} \frac{t_{r,0}}{\Delta CC} \frac{\Delta CC}{CC_0}
$$

where $\Delta t_r$ is the change in toll rate and $t_{r,0}$ is the toll rate at the initial point; $\Delta CC$ is the change in construction cost and $CC_0$ is the initial construction cost. The calculated elasticity depends on the selection of the initial point, and this type of elasticity is called point elasticity. Table 4 represents the summary of toll rate elasticities for each value of the planned initial construction cost.
Table 4. Elasticities of toll rates to changes in construction costs

<table>
<thead>
<tr>
<th>Construction costs (US$ million/km)</th>
<th>Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.609</td>
</tr>
<tr>
<td>2.0</td>
<td>0.757</td>
</tr>
<tr>
<td>3.0</td>
<td>0.823</td>
</tr>
<tr>
<td>4.0</td>
<td>0.862</td>
</tr>
<tr>
<td>5.0</td>
<td>0.886</td>
</tr>
<tr>
<td>6.0</td>
<td>0.903</td>
</tr>
<tr>
<td>7.0</td>
<td>0.916</td>
</tr>
<tr>
<td>8.0</td>
<td>0.926</td>
</tr>
</tbody>
</table>

For example, if the initial construction cost is $5 million/km and the $AADT$ is 5,000 vpd, the weighted average toll rate is $0.45 per vehicle-km. The elasticity of the toll rate for this initial construction cost is 0.89, regardless of the $AADT$. In other words, a 1% change in the initial construction cost would change the toll rate by 0.89%. Similarly, the elasticity of the toll rate with respect to an initial construction cost of $4 million/km is 0.86%, and with respect to an initial construction cost of $7 million/km is 0.92%. It can be observed that, as the initial construction cost increases, the elasticity also increases. In other words, for the larger estimated investment costs, the toll rate becomes more sensitive to variations in the original estimates of construction costs.

In this first analysis, the operation cost is assumed to be constant for all scenarios, regardless of the magnitude of the initial investment. As a more realistic approach, in a second analysis, it is assumed that the operation cost ($OC$) is a function of the construction cost, expressed as $OC=0.1*CC$. Similar to the first analysis, the toll rate $w_{t}$ was reviewed as a function of the $AADT$, construction cost, and operational cost. Figure 4 summarizes the results of this analysis.
Figure 4. Toll rate estimated as a function of construction cost and AADT with variable operational costs

Similar to the first analysis, the results are as expected: the minimum toll rate to attract private investors increases with construction costs and decreases with AADT. A lower AADT leads to a higher toll rate needed to generate the sufficient amount of revenue to keep the project financially viable.

Further, a comparison of Figures 3 and 4 shows that toll rates are higher in the second analysis. For example, if the AADT is 20,000 vehicles per day and the construction cost is $5 million/km, the calculated toll rate in the first analysis is $0.1 per vehicle per km, while in the second analysis it is $0.15/veh-km. Since the only difference in these analyses is in the approach used for the forecast of operational costs (in the first analysis it was constant and in the second it was function of the construction cost), it can be concluded that the inclusion of variable operational costs in the model may have a notable impact on the level of toll rates. Elasticities are also calculated for this analysis and are all equal to 1. This case is called unit elasticity, or, in other words, if the initial construction cost is changed by 1%, the toll rate will also change by 1%.

A third analysis was conducted for the analysis of the construction cost as a dependent variable. The reasoning behind this approach was to seek an answer to the following scenario: if the initial traffic AADT is known as well as the affordable toll rate, what
would be the maximum value of the construction cost which would provide a financially viable project?

Figure 5 presents the results of this analysis. The horizontal axis represents the initial daily traffic expressed as AADT, and vertical axis represents the maximum construction cost per kilometer. All other input parameters are same as specified in Table 3.

Figure 5. Maximum construction cost, toll rates and initial traffic volumes

The results are as expected: when the toll rate is known in advance, as AADT increases, the acceptable maximum construction cost increases as well. The elasticities determined in this case are also equal to 1. In other words, if the AADT changes by 1%, the maximum construction cost will change by 1%. The application of this analysis is useful in the decision-making process, as it provides an insight into the maximum amount of the initial investment. Based on this information, the decision about potential phases of the project can be made. If the maximum construction cost is sufficient for a single carriageway instead of an initially planned dual carriageway, the project might be considered to be built in phases (i.e., staged construction).

Summary

The first part of this chapter covers the mathematical basis of the deterministic model derived from the Graphical Model of the WB Toolkit. The model is then validated by comparing the results obtained from the Toolkit and those from the deterministic model. Also, the model was tested for the extreme values of initial traffic volumes. Further, the model was used to test the sensitivity of toll rates to changes in parameters of interest.
is shown that traffic, construction costs, and operational costs have an impact on toll rates. The following chapter further develops the deterministic model by introducing the uncertainties and using probabilistic models as proxies for risk quantification.
CHAPTER V
STOCHASTIC APPROACH

The deterministic model for the calculation of financially required minimum toll rates is a useful tool for the preliminary assessment of a project’s financial feasibility. It provides an insight from the user’s perspective by looking at the project’s viability through the level of toll rates which users need to pay. However, one of the major limitations of the model is that all input values are deterministic. This can be relaxed by introducing the probabilistic models into the calculation process, which would help in better capturing some of the uncertainties over the long-term horizon. This chapter presents a development of a stochastic model which can be used for the identification, assessment, and management of risks in PPP toll road projects. The following section presents steps 3 and 4 of the methodology developed in this research.

Risk quantification

There are numerous risks identified through the literature which may affect a project’s feasibility. Some are ranked as significant, while some are ranked as having a minor impact on a project’s outcome. The main risk for toll roads with the user charging system is traffic or demand risk. Recognizing the level of magnitude of this risk, various management techniques have been developed over the years, such as minimum revenue guarantees or a revenue sharing mechanism. Another major risk is the construction cost risk. The level of upfront investment has an impact on the financial structure of the project. Operation and maintenance costs are also considered a major risk as they represent the risk of not being able to service the debt in a timely way. These three risks are addressed in this research, and their probabilistic models are introduced. It should be noted that the models presented in the following sections are the ones used in this research. However, following the developed methodology, these models can be improved or replaced.

Traffic risk

As presented in Chapter 2, traffic risk is present throughout the life of the project. Most often, the traffic projections are overestimated. Traffic is a parameter which evolves over the concession period; thus, it is reasonable to use the probabilistic model for traffic forecasts with the time component. A stochastic process \( \{X(t), t \in T\} \) is a
process where, for each \( t \in T \), \( X(t) \) is a random variable (Ross 2007). The rationale behind the choice of a stochastic model is that variations of traffic volumes over time depend on many factors. Although this topic is very important for transport infrastructure, especially for PPP projects where the primary focus is on project’s financial feasibility, the theory behind traffic uncertainties goes beyond the scope of this dissertation. Here, it is assumed that a stochastic process, such as a random walk, can capture, to the certain extent, both the unsystematic and systematic risk components present in traffic forecast models.

The symmetric random walk represents the process which takes a unit step either to the left or right with equal probabilities in each time unit. If the process is speeded up with smaller and smaller steps and smaller and smaller time units going to the limit in the right manner, the process will reach Brownian motion (Ross 2007). Then, the Brownian motion \( \{X(t), t \geq 0\} \) can be defined as a stochastic process with the following properties:

(i) \( X(0) = 0 \);

(ii) \( \{X(t), t \geq 0\} \) has stationary and independent increments;

(iii) For every \( t > 0 \), \( X(t) \) is normally distributed with mean 0 and variance \( \sigma^2 t \).

These properties basically mean that changes of the value of the random walk in no overlapping time intervals are independent and that the distribution of values does not depend on \( t \), only on the length of that interval. The third property states that, over time, the mean value will not change and remains zero. For this reason, the application of Brownian motion for traffic risk modeling may not be adequate as it is usually expected that traffic will increase at some rate over time. This may be relaxed by using the Brownian motion with a drift coefficient \( \mu \). Then, property (iii) becomes:

(iv) \( X(t) \) is normally distributed with mean \( \mu t \) and variance \( \sigma^2 t \).

Now, over time, the mean value increases by the drift coefficient given the time increment. The remaining issue is that if \( X(t) \) is normally distributed, it means that for every \( t > 0 \) there is a probability that the underlying variable has a negative value. As this
cannot hold for modeling traffic volumes, i.e., traffic volumes cannot be negative, this can be also relaxed by introducing a process \( \{S(t), t \geq 0\} \) defined by

\[
S(t) = e^{\xi(t)}
\]

which is called a geometric Brownian motion (GBM). GBM can also be presented in the following format (Trigeorgis 1998):

\[
dS = \mu_s S dt + \sigma_s S dW_t
\]

(31)

where \( \mu_s \) is a drift term, \( \sigma_s \) is a measure of volatility and \( dW_t = \sqrt{dt} \varepsilon_t \) is a Weiner process where \( dt \) is a time increment and \( \varepsilon_t \sim N(0,1) \).

In applying GBM for modeling variables like financial or nonfinancial assets, the risk-adjusted drift rate for the market risk should be taken into account to simulate the risk-neutral environment. The discussion of risk-neutral settings for toll road concessions goes beyond the scope of this research (for further discussion see, for example, Soliño and Galera 2012, and Brandao and Saraiva 2008). Here, the risk-adjusted growth rate for traffic volume is used in calculations. GBM can be modeled in discrete time intervals as a function of the value in the previous time interval (Brandao and Saraiva 2008):

\[
S_{t+1} = S_t e^{\left(\frac{\mu_s - \sigma_s^2}{2}\right)\Delta t + \sigma_s \sqrt{\Delta t} \varepsilon_t}
\]

(32)

where \( \left(\frac{\mu_s - \sigma_s^2}{2}\right) \) represents the risk adjusted drift rate. Equation 32 is particularly important as here it is used for the simulation of GBM over specified time intervals using yearly periods. GBM can be completely simulated using only its initial value, a yearly growth rate, and the volatility.

GBM has applications in different branches of science and in real life, especially in the modeling of stock prices. As seen in the literature, GBM has also been used for modeling traffic or revenue uncertainties (see, for example, Chow and Regan 2009, and Galera and Soliño 2010). Moreover, Soliño and Galera (2012) use a sample of 11 highway concessions in Spain, covering a period of 30 years, to test the hypothesis that traffic volumes follow a GBM process. Results showed that the null hypothesis could not be rejected. Also, they have shown that the average value of the volatility of traffic is 0.075. Thus, in this research, GBM is adopted for modeling traffic volumes over the
concession period. It is assumed that the growth rate and the volatility of the traffic flow can be derived from similar projects and that they are constant over the concession period.

However, besides the described uncertainty in traffic behavior over time, the uncertainty in estimating initial traffic is equally important in some projects. This is particularly true for greenfield projects when there is no historic data on traffic. Estimates of initial traffic and forecasts of traffic volumes over time are based on the analysis of various exogenous parameters. Thus, capturing traffic risk properly in some cases represents capturing uncertainties - in both the initial traffic as well as the traffic forecast over time.

Here, it is assumed that the initial traffic can be modeled as a continuous random variable $S_0$. This assumption can be relaxed for brownfield projects with the history of traffic counts. In this research it is assumed that it initial traffic follows a normal distribution $S_0 = N\left(\mu_{i0}, \sigma^2_{i0}\right)$ where $\mu_{i0}$ is the mean value, and $\sigma^2_{i0}$ is a variance. Other distributions may be also used.

**Construction risk**

As seen in the literature, estimates of construction costs have not improved over the years and costs underestimates are constantly present. There are different forecast techniques: regression models developed from data on similar projects, or analysis of data based on other parameters like the level of economic development in the country, level of corruption, etc. (see, for example, Cirilovic et al. (2014) for estimation models of road reconstruction and rehabilitation costs or Alexeeva et al. (2011) for contract analysis of road works). These forecasts can provide an estimate of a single value for construction costs or a range of possible values for the potential escalation construction costs. In this research, construction risks are taken into account ex-ante as variations of total construction costs, i.e., as variations in the capital expenditures. The construction risks may also include delay in the completion time due to unforeseen events, resulting in a delay in the revenue collection. Here it is assumed that, for forecasting purposes, construction delays may be managed at certain costs, thus increasing the total construction costs.

In order to establish the connection between the estimated value of total construction costs and the cost performance for road projects derived from the literature, the
distribution of the cost development is adopted as a mid-step for the assessment of the mean value and the variance of estimated total construction costs. It is assumed that cost development is a continuous random variable \( Y \), i.e., it follows normal distribution \( Y \sim N(\mu_Y, \sigma_Y^2) \). This assumption may be relaxed by using some other types of continuous random distributions. Defining the cost development \( Y \) as the difference between the actual \((A)\) and the estimated cost \((E)\) compared with the estimated cost \((E)\), the following can be derived \( A = E \cdot Y + E \). Then, \( A \) is normally distributed with a mean \( E\mu_Y + E \) and standard deviation \( E\sigma_Y \).

**Operation and maintenance risk**

In this research, an operation and maintenance risk is a risk of not being able to cover ongoing debt obligations and other yearly expenses of a project or, in other words, a risk of having higher expenses for operation and maintenance activities than expected. As seen in the literature, the forecast models for operation and maintenance costs mainly include either estimates of construction costs or estimates of traffic volumes as the basis for the forecast. They are commonly presented as a certain percentage of construction costs or traffic volumes. In this research, it is assumed that O&M costs are functions of both construction costs and traffic volumes. Having previously defined construction cost as continuous random variable \( A \) and traffic as stochastic random variable \( S(t) \), then O&M costs become a function of two variables \( g(A, S(t)) \). Other models for O&M costs can be used to reflect users' perception or knowledge of O&M risks.

**Monte Carlo simulations**

Simulation techniques represent a method of repeated random sampling from the probability distribution of interest. Let us define \( X \) as a random vector having a density function \( f \). Then, the problem is set to compute some function \( g(X) \). One way to solve this problem is to estimate the parameters of function \( g(X) \), i.e., to estimate \( E[g(X)] \) by simulation (Ross 2007). This process starts by generating independent random vectors \( X^{(i)}, i = 1, \ldots, r \) where \( r \) is a fixed number. The result of the process is a large number of independent and identically distributed random variables \( Y^{(i)} = g(X^{(i)}) \). By the strong law of large numbers, the estimate \( E[g(X)] \) can be calculated as
\[ E[g(X)] = E[Y^{(i)}] = \lim_{r \to \infty} \frac{Y^{(1)} + \ldots + Y^{(r)}}{r} \] (33)

This approach to estimate \( E[g(X)] \) is called Monte Carlo simulation. In Monte Carlo simulations, the stochastic process is generated by simulating the sequence of random variables.

In this research, Monte Carlo simulation is applied to selected random variables traffic \( S(t) \) and \( S_0 \) and actual construction costs (\( CC \)), i.e., development cost \( Y \). The O&M costs are a function of these variables. Looking at the problem of finding the minimum financially required toll rate and following Equation 27, the solution to the problem is a probability distribution of toll rates. This probability distribution is estimated from the sample of toll rates generated by the simulation process. Generating the sample of toll rates concludes Step 3 of the adopted methodology.

**Parameter estimation**

Once the random sample of size \( n \) of toll rates is obtained from the simulation process, the statistical analysis of the underlying distribution and the calculation of point estimators can be applied as Step 4.

The problem of finding the probability distribution of toll rates is a problem of testing the hypothesis that an underlying distribution is satisfactory in representing the population (Montgomery and Runger 2007). One useful method is a graphical method of probability plotting. Histograms can help in the visual identification of the form of the underlying distribution, but the error may be large if the sample size is small. Probability plots enable the visual examination of the data to determine if the data fits the distribution under consideration. A more formal method is the goodness-of-fit procedure to test if the distribution under consideration can be adopted as the distribution of the population.

Let us define the financially required toll rate as a random variable \( T^{\text{fin}}_r \). If the \( T^{\text{fin}}_{r,1}, T^{\text{fin}}_{r,2}, \ldots, T^{\text{fin}}_{r,n} \) is a random sample of size \( n \) from the population which represents \( T^{\text{fin}}_r \), then the following is true for unbiased estimators (Montgomery and Runger 2007):

\[
E\left( \bar{T}^{\text{fin}}_r \right) = \mu_r
\]

\[
E\left( S^2_r \right) = \sigma^2_r
\] (34)
where $T^\text{fin}_r$ is a sample mean, $S^2_n$ is a sample variance, $\mu_r$ is a population mean and $\sigma^2_n$ is a population variance. The standard error is commonly reported along with the point estimator. The estimated standard error of $T^\text{fin}_r$ when the population variance is unknown is:

$$\sigma_{T^\text{fin}} = \frac{S^\text{fin}_n}{\sqrt{n}}$$

(35)

In some cases, it is necessary to use a biased estimator. Then, the mean squared error of an estimator needs to be observed. Also, there are different methods for point estimation, such as the method of moments and the method of maximum likelihood. Users can use relevant literature on statistical and probabilistic analysis for their choice of methods and further analysis of point estimators. In this research, available computational resources for presenting statistical analysis have been adopted to illustrate the methodology.

**Social acceptance**

Solving the problem of finding the minimum financially required toll rate represents valuable information for decision makers about a project's financial feasibility. However, this information is only partially useful, as it also needs to be compared with the socially acceptable level of toll rates. The level of socially acceptable toll rates depends on the social and economic environment where the project is planned for implementation. Thus, the last analysis of the methodology is set as a problem of finding the probability that the socially acceptable toll rate is, at the same time, financially sufficient.

Let us define $T^\text{soc}_r$ as a socially acceptable toll rate. Then, the probability that the socially acceptable toll rate is financially sufficient can be defined as

$$P\left(T^\text{fin}_r \leq T^\text{soc}_r\right) = \int_{0}^{T^\text{soc}_r} f\left(t^\text{fin}_r\right) dt^\text{fin}_r, \quad T^\text{soc}_r \in T^\text{fin}_r$$

(36)

where $f\left(t^\text{fin}_r\right)$ is a probability density function of random variable $T^\text{fin}_r$. It is a probability that the minimum financially required toll rate is less than the socially acceptable toll rate. This information may indicate that the project will be accepted among the users as long as the toll rate is lower than the toll rate users are willing to pay. The probability that this will hold true represents the probability of a project's
success in terms of the financial perspective, or, in other words, the probability that the project will be financially self-sustained without additional financial support. This analysis concludes Step 4 and, at the same time, concludes the methodology for the assessment of the minimum financially required toll rate given multiple uncertainties.

**Building the model**

The stochastic model is developed from the deterministic model by substituting chosen deterministic parameters with probability distributions. This process includes four steps: (i) introducing $AADT$ as a GBM, (ii) modeling the initial traffic $AADT_0$ as a continuous random variable, (iii) defining the O&M costs as a function of $AADT$ and $CC$, and (iv) adding the continuous distribution as $CC$ input. Building the model step-by-step enables the analysis of how each of the introduced risks integrates its price with the price of tolls.

Table 5 presents the overview of the results obtained from each step of this process. The starting point is a set of input values used for the deterministic model validation with the combination of $AADT_0=20,000$ vpd and $CC=1$ million/km. The toll rate calculated with the deterministic model is $0.035$ per vehicle/km. The probabilistic models and the set of input values are provided for each risk to illustrate the process. The number of random samples is $n=100$. Results are reported as the sample mean $\bar{T}_{r}$ and the sample variance $S_{r}^2$ (in the parentheses).
Table 5. Overview of the toll price share of each assumed risk in the stochastic model

<table>
<thead>
<tr>
<th></th>
<th>$\text{AADT}$ as GBM</th>
<th>$\text{AADT}\sim N(\mu, \sigma^2)$</th>
<th>$\text{O&amp;M}=f(\text{AADT}, \text{CC})$</th>
<th>$\text{CC}\sim N(\mu, \sigma^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu, \sigma$</td>
<td>$\mu=0.03, \sigma=0.2$</td>
<td>$\mu=20,000, \sigma=4,000$</td>
<td>$OC=0.1*\text{CC}+0.085<em>365</em>\text{AADT}$</td>
<td>$\mu=$1\text{ million}$, $\sigma=$100,000$</td>
</tr>
<tr>
<td>$\alpha\beta$</td>
<td>$\alpha\beta$</td>
<td>$\alpha\beta$</td>
<td>$\alpha\beta$</td>
<td>$\alpha\beta$</td>
</tr>
<tr>
<td>$\text{AADT}$</td>
<td>0.047 (3.46e-004)</td>
<td>0.053 (5.74e-004)</td>
<td>0.1745* (7.92e-004)</td>
<td>0.049 (5.52e-004)</td>
</tr>
<tr>
<td>$\text{AADT}\sim N(\mu, \sigma^2)$</td>
<td>0.035 (4.86e-033)</td>
<td>0.1722 (6.72e-004)</td>
<td>0.035 (4.36e-006)</td>
<td></td>
</tr>
<tr>
<td>$\text{O&amp;M}=f(\text{AADT}, \text{CC})$</td>
<td></td>
<td></td>
<td>0.1745* (7.92e-004)</td>
<td>0.1745* (7.92e-004)</td>
</tr>
<tr>
<td>$\text{CC}\sim N(\mu, \sigma^2)$</td>
<td></td>
<td></td>
<td></td>
<td>0.035 (4.30e-006)</td>
</tr>
</tbody>
</table>

Since O&M risks are a function of both traffic and construction cost risks, the results with the asterisk indicate the results which include the following three risks: traffic over time, construction, and O&M. The result which is bolded includes the additional risk of initial traffic estimate, resulting in inclusion of all four risks.

The results show that each risk either adds value to the sample mean, i.e., to the expected value of the toll rate, or increases the variance of the toll rate estimate. Risks which are modeled as continuous random variables with normal distributions, e.g., initial traffic and construction costs risks, create the variability component of toll rate assessment which helps evaluate the toll rate risk. Adding the risks one by one increases the variance. Introducing a stochastic component, i.e., a random walk element, increases both the sample mean and the sample variance. The introduction of the O&M costs as a function of traffic volumes and construction costs also increases the sample mean and the sample variance. This risk component has the highest impact on the price of a toll rate. The magnitude of this risk and its impact on toll rates depends on many factors: the
selection of probabilistic models for random variables included in the forecast model, the estimate of the ongoing expenses as a certain percentage of construction costs, the selection of the price for the single vehicle electronic operation, etc. However, these results indicate that the O&M expenses may have noticeable impact on required level of toll rates.

**Model validation**

Building the stochastic model from the deterministic model provides an insight into the structure of the model. The stochastic model validation included the reverse process: cancelling all uncertainties yielded back to the deterministic model. The model with all risks omitted provided the same results as the deterministic model. Another test was conducted in order to validate the model: standard deviations of all variables were multiplied by two. It was expected that the sample variance would also increase. The results showed that both the sample mean and the sample variance increased: the sample mean increased from 0.17 to 0.49 and the sample variance increased from 8.37e-004 to 0.33 (the input values for model validation are taken from Table 1).

**Model limitations**

Some of the limitations of the deterministic model are discussed in Chapter IV. The stochastic model is an improved version of the deterministic model in terms of uncertainty recognition and inclusion. But some limitations still exist, including several new points:

- as with the deterministic model, the toll rate is adjusted only for inflation over the concession period. Here, this may be considered a relaxed assumption since the toll rate is a random variable thus covering a scale of potential scenarios and uncertainties;
- the model considers only single road projects and behaviors of the projects within its own scope, regardless of its dependency on the surrounding network. The model does not recognize the level of the monopoly of the project;
- the model does not explicitly include the price elasticity of demand. Here, it is assumed that the elasticities cannot be used for forecasting purposes. As shown in the literature, the price elasticity of demand has several components where toll rates are one parameter. Also, there is evidence that elasticities depend on the type of traffic, e.g. commuter traffic can be considered inelastic.
The stochastic model includes volatility of initial traffic volumes and the volatility of traffic over time. It is assumed that this approach reflects a number of factors that may have an impact on traffic on a particular road thus potentially covering some of the factors influencing demand elasticities;
- there are no options to include major maintenance costs (reconstruction or rehabilitation costs) during the concession life. As with the deterministic model, if these costs are annualized, they can be added to the O&M costs;
- operation costs are modeled as a function of traffic and construction costs, adjusted for inflation rate over the concession period, thus reflecting the average price change. This assumption may be further relaxed to include items such as the evolution of labour costs, price of oil, etc. if it is anticipated that these parameters may have a high impact on operation costs beyond costs adjustments which inflation is already covering;
- the model does not have an option to specify boundaries for construction costs and traffic volumes. As they are modeled as random variables, some random values may be sampled from extreme regions which is not realistic for these parameters;
- the model considers three random variables, but the variability of other parameters over time may also be present. Moreover, some of these risks may be correlated, thus increasing the variance of the resulting distribution;
- it is assumed that financial parameters \( ADSCR, ROE, \) and \( FIRR \) for particular PPP projects are known in advance. It may be argued that these parameters depend on the project's cash flow and associated risks and thus they are result of the project's financial assessment. The logic behind this assumption is that, for the selected PPP project, investors' financial requirements are set based on the current financial climate, capital markets activities, the political and legal environment, and other "external" factors. They are seeking an investment which will fulfill these requirements.
Numerical example

To illustrate the methodology, a simple road section with four lanes is considered. Toll rates are expressed as the weighted average toll rate $\text{wat}$, in US dollars per vehicle. Assuming that traffic on the observed road section can be represented as a mix of cars, trucks and buses, the $\text{wat}$ per vehicle is determined from Equation 29. Table 6 summarizes the input parameters for the concession and constraints which were used in this analysis.

Table 6. Concession parameters

<table>
<thead>
<tr>
<th>A. Project Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concession term = 20 years</td>
</tr>
<tr>
<td>Construction period = 2 years</td>
</tr>
<tr>
<td>No of lanes = 4</td>
</tr>
<tr>
<td>Section length = 10km</td>
</tr>
<tr>
<td>Distribution of works during the construction period = 1st year 50%, 2nd year 50%</td>
</tr>
<tr>
<td>Equity = 40% of the construction cost</td>
</tr>
<tr>
<td>Government subsidies to the capital costs = 0% of the construction cost</td>
</tr>
<tr>
<td>Inflation = 4% per year</td>
</tr>
<tr>
<td>Value added tax (VAT) = 18%</td>
</tr>
<tr>
<td>Corporate tax = 10%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Loan Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Interest rate = 12% per year</td>
</tr>
<tr>
<td>Type of repayment = level-annuity basis (principal + interest = constant)</td>
</tr>
<tr>
<td>Grace period = 2 years</td>
</tr>
<tr>
<td>Repayment period = 14 years</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Financial Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial internal rate of return of the project (FIRR) ≥ 12%</td>
</tr>
<tr>
<td>Return on equity (ROE) ≥ 16%</td>
</tr>
<tr>
<td>Annual debt service coverage ratio (ADSCR) ≥ 1.2</td>
</tr>
</tbody>
</table>

Following Equation 32, the future traffic volume can be defined by knowing its starting value $AADT_0$, the expected growth rate $\mu$, and the volatility $\sigma$. Initial traffic is set to 20,000 vpd and the drift rate for GBM is 0.05, and the volatility of 0.1. In this example, the initial traffic volume is not presented as a random variable.

It is assumed that the estimated total construction cost is US $10^7$. The normal distribution of cost escalation has a mean value of 20% and standard deviation of 30% (Flyvbjerg et al. 2003, Cantarelli et al. 2012a). Thus, in this case, the actual construction

---

1 This numerical example is partially presented in Vajdic et al. (2015)
cost follows a normal distribution with a mean of US $1.2 \times 10^7$ and standard deviation of $3 \times 10^6$.

As already mentioned, the estimation of operation and maintenance costs is commonly related to the estimates of traffic volumes or construction costs. In this numerical example, it is assumed that the operation cost is equal to the sum of two elements: the percentage of the construction cost and the part related to the cost of having a transaction per vehicle:

\[ OC = p \times CC + q \times 365 \times AADT \]

where \( OC \) is the operation cost, \( p \) is the percentage of construction costs, and \( q \) is the cost of transaction per vehicle per year. Further, the maintenance cost is adopted from Heggie (1995):

\[ MC = 1700 + 0.5 \times AADT \]

where \( MC \) is the financial cost of the routine maintenance on two-lane roads.

Parameter \( p \) is set to 10% of the construction costs and a fixed fee for servicing each vehicle \( q = $0.085 \). Since Heggie's formula is for two-lane roads, it is modified to represent the maintenance costs for roads with four-lanes:

\[ MC = 3400 + 0.5 \times AADT \]

The assessment of the appropriate distribution for generated data for toll rates is conducted using the MATLAB built-in functions. This software is also used for the estimate of the mean \( E(t_r) \) and variance \( V(t_r) \). The method of maximum likelihood is used for point estimates. The number of generated random values of the stochastic model and the sampling values from normal distribution is \( n = 1000 \). In order to generate \( T_{r_i}^{fin,j}, i=1,\ldots,r \), the process of generating random values and determining toll rates data is repeated \( r = 1000 \). Following Equation 33, the result of Monte Carlo simulation is an estimate of toll rates. Data analysis reveals that the distribution has a long right tail. Following this information, several distributions were visually inspected for goodness of fit. Three distributions were chosen based on their fit to generated data: lognormal, t location-scale, and log-logistic. Figure 6 represents a histogram of generated data and probability density functions (pdf) of fitted distribution.
Further analysis included the assessment of point estimators for the mean and variance of these distributions, the standard errors of these estimates, and the associated log likelihood values. These results are presented in Table 7.

**Table 7. Analysis of point estimates from Monte Carlo simulation for toll rates ($/veh-km)**

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Mean</th>
<th>Std. error</th>
<th>Variance</th>
<th>Std. error</th>
<th>Log likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>lognormal</td>
<td>0.056</td>
<td>0.0074</td>
<td>0.0002</td>
<td>0.0053</td>
<td>2943.21</td>
</tr>
<tr>
<td>t-location scale</td>
<td>0.056</td>
<td>0.0004</td>
<td>0.0001</td>
<td>0.0004</td>
<td>2938.32</td>
</tr>
<tr>
<td>log-logistic</td>
<td>0.056</td>
<td>0.0073</td>
<td>0.0002</td>
<td>0.0035</td>
<td>2943.77</td>
</tr>
</tbody>
</table>

The t location-scale has one more parameter, degrees of freedom, which is equal to 14 in this case. As the analysis shows, all three distributions provide the same results for the estimates of the mean. Log likelihood values are also in a close range, where the log-logistic distribution value is slightly higher. Chosen distributions are also tested for the goodness of fit. Two tests are used: chi-square test and one-sample Kolmogorov-Smirnov test.
The null hypothesis is that the data in vector $T_{r}^{fin}$ comes from a chosen distribution. The test to determine if the null hypothesis is rejected or not is performed at the 5% significance level. The summary of results is presented in Table 8.

**Table 8. Summary of results for goodness-of-fit tests**

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Chi-square test</th>
<th>p-value</th>
<th>Kolmogorov-Smirnov test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>lognormal</td>
<td>do not reject</td>
<td>0.3408</td>
<td>do not reject</td>
<td>0.7581</td>
</tr>
<tr>
<td>t location-scale</td>
<td>reject</td>
<td>5.299e-004</td>
<td>do not reject</td>
<td>0.7344</td>
</tr>
<tr>
<td>log-logistic</td>
<td>do not reject</td>
<td>0.1510</td>
<td>reject</td>
<td>1.153e-168</td>
</tr>
</tbody>
</table>

Neither tests do not reject the null hypothesis only for lognormal distribution. Following these results, the cumulative distribution functions (cdf) of the original data and estimated distributions were also visually checked, as shown in Figure 7. As can be observed from the figure, both log-logistic and lognormal seems to have good fit. Thus, lognormal distribution is chosen as the fitting distribution, and the estimates of mean and variance for toll rates are further analyzed. The estimated standard error of the mean is 0.0001.
The mean toll rate of the lognormal distribution is $0.056 per vehicle per km, representing the expected value of the toll rate for the assumed risks of construction costs, O&M costs, and traffic volume estimates. In other words, there is a probability of 0.5 that the weighted toll rate of $0.056 per vehicle per km is financially sufficient for the project to be self-sustained over the concession life without additional financial support. It can be expected that negotiations will likely not take place due to the partial realization of the above mentioned risks.

By knowing the distribution parameters, it is then possible to examine how the socially acceptable toll rate corresponds to the distribution of the weighted toll rate assuming multiple project risks. For example, if the socially acceptable toll rate is $0.08 per vehicle per km, then, in this numerical example, the probability that this toll rate is financially sufficient is 0.95. In other words, the price of $0.08 per vehicle per km financially covers 95% of the assumed traffic, construction costs, and O&M risks. There is a probability of 0.95 that the project will be accepted by users since the toll rate is

![Cumulative distribution functions of toll rates from Monte Carlo simulation and fitted distributions](image)
lower than the toll rate already publicly accepted as the level users are willing to pay to use the facility. This may also imply that there is probability of 0.05 that the project will need some additional financial mechanism over its concession life. If the socially acceptable toll rate is $0.04 per vehicle, then there is probability of 0.09 that it will be financially sufficient. In other words, there is a probability of 0.91 that the project will need some additional financial support over its concession life. If the project commences with a financial structure that neglects the multiple uncertainties, then it can be expected that the project terms will need to be renegotiated at some point.

For projects with no previous traffic records, it is anticipated that the variability in the estimate of initial traffic should be included and thus added to the stochastic model. In order to include this risk in the assessment of financially optimal toll rates, the initial traffic is modeled as a random variable following a normal distribution. The input parameters are the same as for the previous example with one additional parameter - the standard deviation of initial traffic which is set to 4,000 vpd. Toll rates are presented as values for the whole section, i.e., for the section of ten kilometers.

In order to reduce computational efforts, the number of generated random values of the stochastic model and sampling values from normal distribution is set to $n=100$ and number of Monte Carlo simulations is $r=100$. Figure 8 represents a histogram of the generated data and the probability density functions (pdf) of the fitted distribution. Similar to the example above, the distribution has a long right tail, so the same three distributions were chosen based on their fit to generated data: lognormal, t location-scale, and log-logistic.
Following the methodology, the assessment of point estimators for the mean and variance of selected distributions is performed as a next step. These results are presented in Table 9 along with standard errors of these estimates and associated log likelihood values.

**Table 9.** Analysis of point estimates for toll rates from Monte Carlo simulation with the additional initial traffic risk ($/veh)

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Mean</th>
<th>Std. error</th>
<th>Variance</th>
<th>Std. error</th>
<th>Log likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>lognormal</td>
<td>0.63</td>
<td>0.0305</td>
<td>0.039</td>
<td>0.0217</td>
<td>27.688</td>
</tr>
<tr>
<td>t location-scale</td>
<td>0.58</td>
<td>0.0172</td>
<td>0.091</td>
<td>0.0170</td>
<td>22.057</td>
</tr>
<tr>
<td>log-logistic</td>
<td>0.62</td>
<td>0.0288</td>
<td>0.039</td>
<td>0.0139</td>
<td>29.686</td>
</tr>
</tbody>
</table>

The t location-scale has one more parameter, degrees of freedom, which is equal to two in this case. As the analysis shows, the mean values have increased while the variances showed a slight increase compared with the previous example. The log likelihood value for the log-logistic distribution is the highest indicating the log-logistic distribution is
the best fit. Chosen distributions are also tested for the goodness of fit. Two tests are used: chi-square test and one-sample Kolmogorov-Smirnov test.

The null hypothesis is that the data in vector $T_{r, \text{fin}}$ comes from a chosen distribution. The tests to determine if the null hypothesis is rejected or not are performed at the 5% significance level. The summary of results is presented in Table 10.

Table 10. Summary of results for goodness-of-fit tests

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Chi-square test</th>
<th>p-value</th>
<th>Kolmogorov-Smirnov test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>lognormal</td>
<td>do not reject</td>
<td>0.1898</td>
<td>do not reject</td>
<td>0.5868</td>
</tr>
<tr>
<td>t-location scale</td>
<td>do not reject</td>
<td>N/A</td>
<td>do not reject</td>
<td>0.2796</td>
</tr>
<tr>
<td>log-logistic</td>
<td>do not reject</td>
<td>0.2976</td>
<td>do not reject</td>
<td>0.9686</td>
</tr>
</tbody>
</table>

Neither tests do not reject the null hypothesis that data comes from all three distributions. The log-logistic distribution has the highest p-value, so this distribution is chosen as the best fit. The cumulative distribution functions (cdf) of the original data and estimated distributions are also visually checked, as shown in Figure 9. As can be observed from the figure, the log-logistic fit seems better than the other two. Thus, the log-logistic distribution is chosen as the fitting distribution, and the estimates of the mean and variance for toll rates are further analyzed. The estimated standard error of the mean is 0.004.
The mean of the log-logistic distribution represents the estimate of the toll rate $0.62 per vehicle per 10 km section which is financially sufficient to cover 50% of the assumed risks. Comparing the results to the previous example, the difference of $0.06 per vehicle per 10 km section may be interpreted as the price of the additional risk, the variability of the initial traffic estimate.

Following the previous example, the socially acceptable toll rate of $0.8 per vehicle for the whole section has the probability of 0.86 to be financially sufficient compared with the previous example when the probability was 0.91. These differences may not seem significant at first glance. However, they indicate that considering more risks in the model increases the level of the toll rate needed to keep the project financially sustainable. The probability distribution of the toll rate changes by increasing the expected value of the toll rate. In other words, more money is needed to cover more risks. Also, the socially acceptable toll rate loses its financial strength and, in this case, covers a lower percentage of the realizations of potential risks.

**Figure 9.** Cumulative distribution functions of toll rates from Monte Carlo simulation and fitted distributions
Summary
A methodology for the assessment of toll rates is developed based on probabilistic principles with the goal of enabling a more realistic assessment of project risks. Uncertainties captured in the model include traffic risks, construction risks, and operation and maintenance risks. The toll rates that the stochastic model calculates are derived from the financial model, given specific requirements, i.e. constraints, for the values of financial indicators. Thus, the toll rates calculated from the model represent financially required toll rates. They are sufficient to ensure the financial success of a project. In order to obtain the toll rate that is acceptable for all stakeholders, the distribution of toll rates serves to determine the probability that the socially acceptable toll rate is financially sufficient or, in other words, to quantify the toll rate risk, i.e., the risk that the agreed toll rate will not be financially sufficient.
CHAPTER VI
CASE STUDIES

A methodology for the assessment of toll rates was developed (see Chapter V) based on probabilistic principles with the goal of enabling more realistic assessment of project risks. An application of the stochastic model is illustrated in two case studies: the Olympia Odos Motorway Concession in Greece and the Belgrade Bypass in Serbia. The Olympia Odos is a tolled motorway under construction/operation which has experienced financial difficulties due to the high impact of the financial crisis in Greece. The Belgrade Bypass is a highway project delivered as a publicly procured project financed exclusively from loans. It is a non-tolled highway project which was, during the project development phase, considered as a candidate for the tolling system.

Olympia Odos Motorway Concession, Greece

Greece has a long history of PPP projects. One of the first examples is the Corinth Canal which was commissioned in 1881 as a concession for 99 years (Roumboutsos 2013). Two major concessions were completed by the early 1970s: the water supply system and the generation and distribution of electrical power. During the 1990s, the Greek Authorities and the European Commission agreed on the framework with the goal of maximizing the private sector involvement in the delivery of the transport related infrastructure. Similar provisions were made under the next framework in the early 2000s and, more recently, under the National Strategic Reference Framework implemented in the period 2007 – 2013.

The PPP implementation in the transport sector is characterized by the small number of large projects awarded in two waves (Roumboutsos 2013). The first group of projects was awarded in the late 1990s and includes the Athens International Airport, the Athens Ring Road Attica Tollway, and the Rio-Antirio Bridge. The second group of projects was awarded between 2007-2008 and included the so-called “axis of development” motorways: the Maliakos-Kleidi Motorway (Aegean Motorways), the Elefsina-Corinth-Patra-Tsakona (Olympia Odos), the Antirio-Ioannina Motorway (Ionia Odos), the Central Greece Motorway (E65 motorway), and the Corinth-Tripoli-Kalamata Motorway (Moreas). The Port of Piraeus Transhipment Terminal Concession was also awarded in the second wave. The Egnatia Odos motorway, which services the east-west
axis of northern Greece, is the only motorway in recent years delivered as a publicly procured project. Figure 10 represents the map of major highways in Greece.

![Map of major highways in Greece](http://en.wikipedia.org/wiki/Highways_in_Greece), accessed 24.03.2015.

The impact of the financial crisis on PPP projects in the transport sector was significant (Roumboutsos 2013). The project revenues significantly dropped due to the reduced demand, and users demonstrated elasticity to the level of toll tariffs. As concessions mostly rely on charging user fees, the projects under the operation experienced difficulties in their debt servicing. The projects under the construction had to stop construction works since a significant part of the construction was budgeted from the brownfield toll revenues. Concessions were under negotiation with the Greek government. In April 2013, the terms of the new agreement were announced which included the reduction of scope, increased public financial contribution, and the payment of claims.

The Olympia Odos is a toll motorway concession located in Northern Peloponnese, Greece (Roumboutsos and Nikolaidis 2013). The concession is approximately 365 km in length and consists of four sections that are a mix of brownfield and greenfield sections (Koklas et al. 2011, Academic Dictionaries and Encyclopedias 2015):
- The Elefsina – Korinthos section is an existing motorway section with a length of 63.6 km. The AADT is 30,000 vehicles per direction. The existing cross section includes 3 lanes and one emergency lane per direction, a complex of tunnels 4.5 km in total length, and two toll plazas, Elefsina and Isthmos, to be repaired to fulfill current motorway standards;
- The Korinthos-Patra section is an existing road with poor geometrical design and a high accident rate. The length of the section is 120 km. The AADT varies from 7,500 to 11,000 vehicles per direction with high seasonal peaks (summer holiday traffic etc.) up to 30,000 vehicles per direction. A new motorway section is planned to be built along the old motorway;
- The Patra Bypass section is an existing motorway section with two lanes and an emergency lane per direction. The length is 18.3 km and it includes a complex of tunnels 4.7 km total length. The AADT is approximately 8,000 vehicles per direction with seasonal peaks up to 15,000 vehicles per direction. No tolls are to be received on this section throughout the concession period. It is planned to be repaired to fulfill current motorway standards;
- The Patra-Pyrgos-Tsakona section is a completely new section that is 163.3 km long. New alignment of this new motorway section is planned to be built along the old road.
The Olympia Odos project was initiated prior to 1998 (Roumboutsos and Nikolaidis 2013) and the call for prequalification was launched in 2001 followed by invitations to selected bidders to submit respective bids in 2006. The total budget for the project was €2.2 billion which included design and construction, financing costs, and operation costs during the construction. Financing was structured in the following way: Shareholders Equity €160 million; Debt Capital €1,140 million, out of which €990 million was a senior debt with 13 years maturity (Ferron-Hugonnet 2008); the Greek State/EU funds €500 million; and tolls received from brownfield sections were expected to bring in €400 million. The concession term is 30 years, and the financial close was reached in 2008, and the construction works were planned to be completed in 2014. The construction works were divided into two phases: the first phase for the construction of the Korinthos-Patras section was planned to take 3.5 years, and the second period was planned for another 2.5 years for the construction of the Patra-Pyrgos-Tsakona section.
(Academic Dictionaries and Encyclopedias 2015). At the time of the procurement phase, the country’s sovereign debt was rated AA (Roumboutsos and Nikolaidis 2013). Given the information retrieved from the literature, some model input values are missing and have to be assumed. Since the invitation for project bids was published in 2006, it is assumed that the financially pre-feasibility base year for the methodology application is 2005. It is assumed that the operation costs consists of two segments: the percentage of the construction cost $p= 10\%$ and the cost of having transaction per vehicle $q= €0.05$. Maintenance costs are estimated using the modified formula adapted from Heggie (1995). Traffic growth is assumed based on the forecasted average annual growth rate for GDP for the period between 2001 and 2020 which was 3.8% (TEN-STAC 2004). Here, the traffic growth rate is assumed to be 4%. Based on the inflation rate for the period 2004-2005, it is assumed that the inflation rate was set to 3% (Worldwide Inflation Data 2015). The VAT rate in 2005 was 19% (Living in Greece 2010) and the corporate tax was 32% (Trading Economics 2015). Since the country rating was AA during the procurement phase, it is assumed that the nominal interest rate is 6%. Also, it is assumed that the grace period is 2 years.

The initial $AADT$ is determined as a weighted average of the $AADTs$ of each section over the length of the entire project:

$$AADT_{0}^{av} = \frac{\sum_{i=1}^{4} AADT_i * L_i}{L}$$

where $AADT_{0}^{av}$ is the averaged initial traffic, $AADT_i$ is the annual average daily traffic of each section, $L_i$ is the length in kilometers of each section, and $L$ is the total length in kilometers. The reported $AADT_i$ are traffic volumes counted at toll plazas when the project was already in operation (Musso et al. 2013). However, the Concessionaire reported traffic decreases of 30% compared to the traffic forecasts (Lambropoulos et al. 2012). Thus, the reported $AADT_i$ are increased to reflect this information. The $AADT_{0}^{av}$ is set at 39,300 vpd, and the traffic volatility over time is assumed to be 0.2. The initial traffic is assumed as a single value since three sections out of four have a history of traffic counts, i.e., three sections are brownfield.

Variations in construction cost estimate, based on the evidence from the literature, is applied in this case study. Following Flyvbjerg et al. (2003) and Cantarelli et al.
(2012a), the average cost escalation applied here is 20% with a standard deviation of 30. 
Thus, the construction cost is the random variable with a normal distribution with mean 
value of €2.64 billion and standard deviation of €0.66 billion. Based on the discussion 
with experts, financial constraints are assumed as following: $ADSCR=1.2; IRR=0.12$ and 
$RoE=0.16$. Table 11 represents the summary of input values with the note if the value is 
assumed or obtained from the literature.

**Table 11.** Overview of input values for the Olympia Odos case study (base year 2005)

<table>
<thead>
<tr>
<th>A. Project Parameters</th>
<th>Note*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concession term = 30 years</td>
<td>I</td>
</tr>
<tr>
<td>Total investment = €2,640 billion</td>
<td>I</td>
</tr>
<tr>
<td>Standard deviation of total investment = €0.66 billion</td>
<td>A</td>
</tr>
<tr>
<td>Construction period = 6 years</td>
<td>I</td>
</tr>
<tr>
<td>Distribution of works during the construction period: 1st year 15%, 2nd year 20%, 3rd year 20%, 4th year 15%, 5th year 20%, 6th year 10%</td>
<td>A</td>
</tr>
<tr>
<td>$AADT_0^{av}$ = 39,300 vehicles per day</td>
<td>I</td>
</tr>
<tr>
<td>Standard deviation of $AADT = 0.2$</td>
<td>A</td>
</tr>
<tr>
<td>Traffic growth = 4%</td>
<td>I</td>
</tr>
<tr>
<td>No of lanes = 4</td>
<td>I</td>
</tr>
<tr>
<td>Project length = 365km</td>
<td>I</td>
</tr>
<tr>
<td>Equity = 7% of the total investment</td>
<td>I</td>
</tr>
<tr>
<td>Government subsidies to the capital costs = 41% of the total investment</td>
<td>I</td>
</tr>
<tr>
<td>Inflation = 3% per year</td>
<td>A</td>
</tr>
<tr>
<td>Value added tax (VAT) =19%</td>
<td>I</td>
</tr>
<tr>
<td>Corporate tax = 32%</td>
<td>I</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Loan Terms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Interest rate=6% per year</td>
<td>A</td>
</tr>
<tr>
<td>Type of repayment = level-anuity basis (principal + interest = constant)</td>
<td>A</td>
</tr>
<tr>
<td>Grace period= 2 years</td>
<td>A</td>
</tr>
<tr>
<td>Repayment period=13 years</td>
<td>I</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Financial Constraints</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial internal rate of return of the project ($FIRR) \geq 12%$</td>
<td>A</td>
</tr>
<tr>
<td>Return on equity ($ROE) \geq 16%$</td>
<td>A</td>
</tr>
<tr>
<td>Annual debt service coverage ratio ($ADSCR) \geq 1.2$</td>
<td>A</td>
</tr>
</tbody>
</table>

*I-values retrieved from the literature, A-assumed values

The number of generated random values of traffic volume over time and construction costs is $n=1,000$ and the number of simulations for toll rate calculations is $r =1,000$. The generated data of random vectors for toll rates $T^{fin}_r$ are divided by the project length or, in this case, by the project length which is tolled. For Olympia Odos it is 346.7 km, so
the data represent the weighted average toll rates in € per vehicle per kilometer. A histogram of toll rates $T_i^{fin}$ is presented in Figure 12. As with the numerical examples in Chapter V, the distribution has a long right tail. Again, three distributions were chosen based on their fit to generated data: lognormal, log-logistic and t-location scale. Their probability density functions (pdf) are also presented in Figure 12.

![Histogram of results and pdf of fitted distribution for Olympia Odos](image)

**Figure 12.** Histogram of results and pdf of fitted distribution for Olympia Odos

Table 12 represents an overview of the point estimates for the tested distributions. Log likelihood value is the highest for lognormal distribution, indicating the best fit.

**Table 12.** Point estimates for toll rates fitted distributions for Olympia Odos ($/veh-km$)

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Mean</th>
<th>Std. error</th>
<th>Variance</th>
<th>Std. error</th>
<th>Log likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>log-logistic</td>
<td>0.192</td>
<td>0.016</td>
<td>0.015</td>
<td>0.008</td>
<td>1054.43</td>
</tr>
<tr>
<td>lognormal</td>
<td>0.188</td>
<td>0.016</td>
<td>0.010</td>
<td>0.011</td>
<td>1065.01</td>
</tr>
<tr>
<td>t-location scale</td>
<td>0.174</td>
<td>0.003</td>
<td>0.009</td>
<td>0.009</td>
<td>969.49</td>
</tr>
</tbody>
</table>
The chosen distributions are also tested for the goodness of fit using the chi-square test and the one-sample Kolmogorov-Smirnov test. The null hypothesis is that the data in vector $T_r^{fin}$ comes from a chosen distribution. Tests to determine if the null hypothesis is rejected or not are performed at the 5% significance level. The summary of results is presented in Table 13.

Table 13. Summary of results for goodness-of-fit tests for Olympia Odos

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Chi-square test</th>
<th>p-value</th>
<th>Kolmogorov-Smirnov test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>log-logistic</td>
<td>Reject</td>
<td>0.0128</td>
<td>do not reject</td>
<td>0.8512</td>
</tr>
<tr>
<td>lognormal</td>
<td>do not reject</td>
<td>0.7136</td>
<td>do not reject</td>
<td>0.9917</td>
</tr>
<tr>
<td>t location-scale</td>
<td>Reject</td>
<td>1.117e-13</td>
<td>reject</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

Following the presented analyses, the lognormal distribution is chosen for further analysis. The mean value of the lognormal distribution represents the estimate of the mean for the weighted average toll rate of €0.188 per vehicle per kilometer. The obtained mean value of the $\text{wat}_r$ is compared with the level of toll tariffs on Olympia Odos. The toll rates are defined by a flat rate per kilometer indexed every year according to the Concession Agreement (Olympia Odos 2015c). The price per kilometer was set to €0.04 per kilometer for cars, excluding VAT, for a base year 2003. It is assumed that the major part of traffic is cars, while it can be expected that trucks are also a significant share of the traffic composition since Patra is a major port connecting Greece and Italy. The relationship between the toll rates for all four vehicle categories is calculated based on the toll tariffs charged at toll plazas (Olympia Odos 2015b). Table 14 presents the summary of the analysis and comparison of the toll rates which were declared in 2003 and the calculated mean value of the toll rate from the stochastic model.
Table 14. Comparison of $w_{at}$ for Olympia Odos case study

<table>
<thead>
<tr>
<th></th>
<th>I Motorcycles, tricycles</th>
<th>II Vehicles with or without trailers and height up to 2.20 m</th>
<th>III Vehicles with or without trailers with two or three axles and height up to 2.20 m</th>
<th>IV Vehicles with or without trailers with four or more axles and height in excess of 2.20 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumed traffic</td>
<td>2%</td>
<td>70%</td>
<td>2%</td>
<td>26%</td>
</tr>
<tr>
<td>composition</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relationships</td>
<td>0.7</td>
<td>1</td>
<td>2.5</td>
<td>3.5</td>
</tr>
<tr>
<td>between toll rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weighted average</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>toll rate €/veh/km</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(forecasted, VAT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>included, base</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>year 2003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weighted average</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>toll rate €/veh/km</td>
<td>0.082</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(mean value, VAT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>included, base</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>year 2005)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

As can be observed, the initial toll rate set for the Olympia Odos concession is lower than the mean value obtained from the simulation using the stochastic model. For the given set of input values, the toll rate distribution reveals that there is a probability of 0.5 that the weighted average toll rate of €0.188 per vehicle is financially sufficient, assuming traffic and construction costs risks. The toll rate for cars, recalculated from the model, is €0.112 per vehicle, which exceeds by 2.24 times the predefined toll rate of €0.050 per car (VAT included) that was set before the concession started. Figure 13 represents the cumulative distribution function of vector $T_{r}^{fin}$ and the lognormal distribution with estimated parameters.
As specified in the methodology, the final step is to compare the socially acceptable toll rate with the data. More specifically, to find the probability that the socially acceptable toll rate will be financially sufficient. Here, the toll rate set in the contract is compared with the values of toll rates following selected lognormal distribution. There is a probability of 0.1 that the predefined weighted average toll rate is financially sufficient given the assumed risks (gray line in Figure 13). This result may not be surprising today since the project was already a subject of re-negotiation with the government due to the realization of some of risks (low traffic, opposition of users to pay tolls, etc.).

At the time the project was tendered, the economic situation in Greece was much different, yielding different economic and financial environments. Although the government provided significant subsidies for the project, the financial crisis caused overall instability in the country which is reflected in the drop of traffic demand and lack of users willingness to pay for using roads. The results showed that even when the project is supported with a loan-free investment covering significant capital

Figure 13. Cumulative distribution functions of toll rates for Olympia Odos with the predefined toll rate
expenditures, the project cannot continue its operation as financially self-sustained if traffic decreases, i.e., if the traffic risk materializes.

The applied stochastic model, as shown, is able to capture uncertainties and provide valuable insights into the project's behavior under different circumstances. It is difficult to capture macroeconomic uncertainties at the project level, but the stochastic approach for modeling traffic behavior over time may be a good context for capturing systematic risks. Extreme events which are outside the scope of project management may be captured even in the early phases of project development. This early information is valuable to help in the timely identification of potential risks and to foster adequate risk management strategies.

Now it is known that long-lasting contractual obligations are subject to a number of risks which are dynamic in nature. Some of them may have a high impact on the project such as the world financial crisis which had a significant influence on Greece's economic and financial stability. A lesson which can be learned from this case study is that the risk evaluation needs to be upgraded to more sophisticated models, such as stochastic models, which are able to capture various levels of risks over time. This enables a better understanding of a project's dynamic behavior over time for all stakeholders and allows the timely development of appropriate risk management strategies.

**Belgrade Bypass, Serbia**

Serbia is located in South East Europe (Figure 14). Its state road network is 16,700 km in total length, out of which 667 km are state highways (PE Roads of Serbia 2015a). Due to a lack of proper maintenance and a deficiency of new investments for a long period of time, the road network is in relatively poor condition with a low level of service. In recent years, most of the infrastructure project investments have been financed through loans from major development banks with a couple of projects financed from the budget. So far, in Serbia there was one unsuccessful attempt to deliver a PPP highway project – the Horgos–Pozega project in 2007.
The Belgrade Bypass project dates back to 1972 when it was included in the city’s Master Plan. During the 1980s, the European Investment Bank (EIB) provided a loan for the project development which led to the beginning of the construction phase in 1991. Soon after, the loan was canceled due to political problems related to the disintegration of former Yugoslavia. During the 1990s, one section (from Dobanovci to Ostruznica) was partially constructed from public funds (only one carriageway) along with the bridge over the Sava river. It was ready to be opened for traffic in 1999, but the bridge was heavily damaged in the NATO bombing campaign and the whole section was closed to traffic until 2005. The repair and reconstruction of the bridge was financed from the public funds. In 2005, a new feasibility study for the Belgrade Bypass project was prepared. Following this study, the EIB and the European Bank for Reconstruction and Development (EBRD) provided funds for the construction works.
As of today, some sections of the project are completed, while some sections are still under construction.

The Belgrade Bypass project is divided into three sections:
- Section A, L=9.7km from Batajnica to Dobanovci;
- Section B, L=37.3km from Dobanovci to BubanjPotok;
- Section C, L=22km from BubanjPotok to Starcevo.

It represents the connections among the parts of the road network from west, north, southwest, south, and east (Figure 15). Traffic coming from the north (from the Hungarian border and the city of Novi Sad) on highway E-75 is redirected on the northern city border to by-pass the city (section A). It has a connection with highway E-70 on the western city border (that comes from Croatia) and continues to the south over the Sava river. It is again connected with E-75 at the southern city border at Bubanj Potok (section B). The last section, section C, goes east to Pancevo and Starcevo.

Figure 15. Belgrade Bypass map
Sections A and B (partially) are currently under operation and/or upgraded. Section C is still in the early phases of project development, and the delivery of this section is not anticipated in the near future. Section A and parts of section B have been delivered through the public procurement process (design-build or design-bid-build, depending on the section procured). Operation and maintenance are the responsibility of the Public Enterprise “Roads of Serbia.” All sections are non-tolled, although there have been some discussions of the introduction of toll tariffs. For this reason, the project is selected as a case study to test how the currently acceptable level of toll rates in Serbia corresponds with the financially required toll rates, assuming that the project would be delivered as a PPP.

Most of the input data are adopted from the feasibility study prepared in 2005 (Scott Wilson Kirkpatrick 2005). Assuming that the delivery of the project occurred in the next few years after the feasibility study was prepared, the first year of project operation phase would have been set to be 2010. The base year for the application of the stochastic model, i.e., for the financial feasibility assessment, is 2005. In the feasibility study, several scenarios were analyzed. Staged development included the analysis of scenarios with building one carriageway as the first phase and adding the second carriageway later as the second phase. Some scenarios included the construction of the carriageway in full width. Here, it is assumed that the project is delivered in the full carriageway width.

It is considered that sections A and B are of interest as one PPP project. Section A is a greenfield section while section B is considered as brownfield. Section B represents the new alignment along with the old route and thus it is assumed that the estimates of initial traffic volumes are not subject to large errors. The total investment for section A is € 74.6 million and for section B is € 280.2 million. These investments include the construction costs, land acquisition, project design, and supervision of works. The total investment is considered a random variable with a normal distribution representing the construction risk. The mean value of the total investment for the greenfield section A is increased by 20%, and the standard deviation is assumed to be 30%, reflecting the evidence from the literature about the typical cost escalation of road projects. The mean of the total investment for the brownfield section B is adopted from the feasibility study,
and the standard deviation is assumed to be 10%. The full width of the carriageway consists of 4 lanes with two emergency lanes. It is assumed that the construction phase would last 2 years. It is also assumed that the operation cost has two parts: the percentage of the construction cost $p = 10\%$ and the cost of having a transaction per vehicle $q = £0.05$. Maintenance costs are estimated using the modified formula adapted from Heggie (1995).

The economic analysis in the feasibility study included a sensitivity analysis of different scenarios: realistic and pessimistic scenarios (Scott Wilson Kirkpatrick 2005). These two scenarios are applied for forecasts of GDP growth, which was then related to forecasts of traffic growth and changes in traffic composition, i.e., an increased number of trucks with two or three axles. The elasticity of traffic demand to changes in per capita income was estimated as 1.2 which is considered suitable for central and east Europe. According to the results of traffic simulations, traffic volume on the Belgrade Bypass would drop with the introduction of toll tariffs. It is expected that 5% of traffic, for the realistic scenario, would diverge to other available routes, while the pessimistic scenario yields to a diversion of 16%.

In the case of having toll charges on sections A and B of the Belgrade Bypass, the advanced model for traffic forecasts is adopted in order to capture related uncertainties. Beside capturing the risks of traffic shifts to other alternative routes, it is also important to capture the uncertainties in the estimates of the initial traffic, especially for the greenfield sections. The deterministic model is adopted for the initial traffic estimate for brownfield sections.

It is assumed that the initial traffic for the greenfield Section A is a continuous random variable which follows a normal distribution. The mean of the normal distribution of initial traffic is calculated as the averaged initial traffic $AADT_{0,av}$ from data given in the feasibility study for the year 2010, reduced by 5% to reflect the expected shift of traffic to other routes due to the introduction of tolls. For Section A, the mean of the initial traffic $AADT_{0,A}$ is set at 71,500 vpd, and the standard deviation is assumed to be 10,000 vpd. For the brownfield Section B, the initial traffic averaged across all sub-sections $AADT_{0,B}^{av}$ is set at 40,800 vpd. No variability in initial traffic is assumed.
In the feasibility study, the traffic growth rate was estimated at 6% for period 2006-2011 and 3.5% after 2011. Here, it is assumed to be constant throughout the concession period at 5%. In 2005, the inflation in Serbia was 16.1%, but it was lower in the following years (World Bank 2015a). Here, it is assumed that the inflation rate is 10%. The corporate tax rate is 10% (Tesche 2005), and it is assumed that the nominal interest rate is 12%. Table 15 summarizes the input parameters for the Belgrade Bypass case study.

Table 15. Overview of input values for the Belgrade Bypass case study (base year 2005)

<table>
<thead>
<tr>
<th>A. Project Parameters</th>
<th>Note*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concession term = 25 years</td>
<td>A</td>
</tr>
<tr>
<td>Investment costs</td>
<td>I</td>
</tr>
<tr>
<td>Section A = €89.5 million; Section B = €280.2 million</td>
<td></td>
</tr>
<tr>
<td>Standard deviation of total investment</td>
<td>A</td>
</tr>
<tr>
<td>Section A = €27 million; Section B = €28 million</td>
<td></td>
</tr>
<tr>
<td>Construction period = 2 years</td>
<td>A</td>
</tr>
<tr>
<td>Distribution of works during the construction period:</td>
<td></td>
</tr>
<tr>
<td>1st year 50%, 2nd year 50%</td>
<td>A</td>
</tr>
<tr>
<td>$E[AADT_{0,t}] = 71,500$ vehicles per day</td>
<td>I</td>
</tr>
<tr>
<td>$std[AADT_{0,t}] = 10,000$ vehicles per day</td>
<td>A</td>
</tr>
<tr>
<td>$AADT_{0,t}^{av} = 40,800$ vehicles per day</td>
<td>I</td>
</tr>
<tr>
<td>Standard deviation of $AADT = 0.2$</td>
<td></td>
</tr>
<tr>
<td>Traffic growth = 5%</td>
<td>A</td>
</tr>
<tr>
<td>No of lanes = 4</td>
<td>I</td>
</tr>
<tr>
<td>Project length = 47km</td>
<td>I</td>
</tr>
<tr>
<td>Section A = 9.7km; Section B = 37.3km</td>
<td>I</td>
</tr>
<tr>
<td>Equity = 20% of the investment costs</td>
<td>A</td>
</tr>
<tr>
<td>Government subsidies to the capital costs = 20% of the investment costs</td>
<td>A</td>
</tr>
<tr>
<td>Inflation = 10% per year</td>
<td>A</td>
</tr>
<tr>
<td>Value added tax (VAT) =18%</td>
<td>I</td>
</tr>
<tr>
<td>Corporate tax = 10%</td>
<td>I</td>
</tr>
</tbody>
</table>

| B. Loan Terms                                       |       |
| Nominal Interest rate=12% per year                  | A     |
| Type of repayment = level-annuity basis (principal + interest = constant) | A |
| Grace period= 2 years                               | A     |
| Repayment period=15 years                           | A     |

| C. Financial Constraints                            |       |
| Financial internal rate of return of the project ($FIRR$) ≥12% | A |
| Return on equity ($ROE$) ≥16%                        | A     |
| Annual debt service coverage ratio ($ADSCR$) ≥1.2     | A     |

*I-values retrieved from the literature, A-assumed values
Simulations are run separately for section A and for section B. In order to reduce computational efforts for section A, the number of generated random values of the initial traffic volume, traffic volatility over time, and construction costs is \( n = 200 \). The number of simulations for the toll rate calculations is \( r = 200 \). For section B, the number of generated random values is \( n = 1,000 \), and the number of simulations for toll rate calculations is \( r = 1,000 \). The generated data of random vectors for toll rates are divided by the project length to obtain weighted average toll rates in € per vehicle per kilometer.

A histogram of toll rates \( T^{fin}_r \) for section A is presented in Figure 16. The distribution has a long right tail and, again, the same three distributions were chosen based on their fit to generated data: lognormal, log-logistic, and t-location scale. Their probability density functions (pdf) are also presented in Figure 16.

![Figure 16. Histogram of results and pdf of fitted distribution for the Belgrade Bypass Section A](image)

Figure 16. Histogram of results and pdf of fitted distribution for the Belgrade Bypass Section A
Table 16 represents the overview of point estimates of the toll rate for the tested distributions. Log likelihood value is the highest for log-logistic distribution, indicating the best fit.

**Table 16.** Point estimates for toll rates fitted distributions for the Belgrade Bypass Section A (€/veh-km)

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Mean</th>
<th>Std. error</th>
<th>Variance</th>
<th>Std. error</th>
<th>Log likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>log-logistic</td>
<td>0.118</td>
<td>0.029</td>
<td>0.003</td>
<td>0.014</td>
<td>333.20</td>
</tr>
<tr>
<td>lognormal</td>
<td>0.118</td>
<td>0.030</td>
<td>0.003</td>
<td>0.022</td>
<td>331.56</td>
</tr>
<tr>
<td>t-location scale</td>
<td>0.110</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>319.48</td>
</tr>
</tbody>
</table>

Chosen distributions are also tested for the goodness of fit using the same two tests as earlier: chi-square test and one-sample Kolmogorov-Smirnov test. The null hypothesis is that the data in vector $T_{i}^{fin}$ comes from a chosen distribution. Tests to determine if the null hypothesis is rejected or not are performed at the 5% significance level. The summary of results is presented in Table 17.

**Table 17.** Summary of results for goodness-of-fit tests for the Belgrade Bypass Section A

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Chi-square test</th>
<th>Kolmogorov-Smirnov test</th>
</tr>
</thead>
<tbody>
<tr>
<td>log-logistic</td>
<td>do not reject</td>
<td>do not reject</td>
</tr>
<tr>
<td>lognormal</td>
<td>do not reject</td>
<td>do not reject</td>
</tr>
<tr>
<td>t-location scale</td>
<td>reject</td>
<td>do not reject</td>
</tr>
</tbody>
</table>

Following the presented analyses, the log-logistic distribution is chosen as the distribution with the best fit. The mean value of the log-logistic distribution represents the estimate of the mean for a weighted average toll rate of €0.118 per vehicle per kilometer for Section A.

The same steps are repeated for section B. A histogram of toll rates $T_{i}^{fin}$ is presented in Figure 17. Distribution has a long right tail and, again, the same three distributions were chosen based on their fit to generated data: lognormal, log-logistic, and t-location scale. Their probability density functions (pdf) are also presented in Figure 17.
Figure 17. Histogram of results and pdf of fitted distribution for the Belgrade Bypass Section B

Table 18 represents an overview of point estimates for the tested distributions. Log likelihood value is the highest for log-logistic distribution, indicating the best fit.

Table 18. Point estimates for fitted distributions of toll rates for the Belgrade Bypass Section B (€/veh-km)

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Mean</th>
<th>Std. error</th>
<th>Variance</th>
<th>Std. error</th>
<th>Log likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>log-logistic</td>
<td>0.154</td>
<td>0.010</td>
<td>0.003</td>
<td>0.004</td>
<td>1625.73</td>
</tr>
<tr>
<td>lognormal</td>
<td>0.156</td>
<td>0.010</td>
<td>0.003</td>
<td>0.008</td>
<td>1613.16</td>
</tr>
<tr>
<td>t-location scale</td>
<td>0.144</td>
<td>0.001</td>
<td>0.004</td>
<td>0.001</td>
<td>1547.84</td>
</tr>
</tbody>
</table>

The chosen distributions are also tested for the goodness of fit using the same two tests as earlier: chi-square test and one-sample Kolmogorov-Smirnov test. The null hypothesis is that the data in vector $T_{n}^{a}$ comes from a chosen distribution. Tests to determine if the null hypothesis is rejected or not are performed at the 5% significance level. The summary of results is presented in Table 19.
Table 19. Summary of results for goodness-of-fit tests for the
Belgrade Bypass Section B

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Chi-square test</th>
<th>p-value</th>
<th>Kolmogorov-Smirnov test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>log-logistic</td>
<td>reject</td>
<td>8.345e-004</td>
<td>do not reject</td>
<td>0.4040</td>
</tr>
<tr>
<td>lognormal</td>
<td>reject</td>
<td>1.666e-007</td>
<td>reject</td>
<td>0.0147</td>
</tr>
<tr>
<td>t-location scale</td>
<td>reject</td>
<td>9.277e-013</td>
<td>reject</td>
<td>8.908e-006</td>
</tr>
</tbody>
</table>

Following the presented analyses, the log-logistic distribution is chosen as the
distribution with the best fit. The mean value of the log-logistic distribution represents
the estimate of the mean for a weighted average toll rate for Section B of €0.154 per
vehicle per kilometer.

Although having different values of toll rates for section A and section B can be
expected, it might be surprising that the mean value of toll rates is lower for the
greenfield section A than for the brownfield section B. With the assumption that
greenfield sections bring higher risks, the presented results confirm the well-known fact
that traffic risk is the main risk in PPP contracts. Section A has higher initial traffic
which reduces the impact of construction and maintenance risks on financial outcomes.
On the other side, section B has lower initial traffic, and the project remains vulnerable
to other risks which, in this case, have higher impact on financial strength of the project.
The mean value of the $\text{wat}$, for both sections is compared with the level of toll tariffs in
Serbia. The defined toll rates in the feasibility study are based on a flat rate per
kilometer according to the PE Roads of Serbia (Scott Wilson Kirkpatrick 2005). The
price per kilometer was set to €0.02 per kilometer for cars, €0.06 per kilometer for
vehicles with two or three axles with or without trailers, and €0.12 per kilometer for
vehicles with four or more axles with or without trailer. Traffic composition was also
introduced in the feasibility study where it is estimated that 85% of traffic are cars, 6%
of traffic are the second category and 9% of traffic are the third category vehicles.
However, the toll rates given in the feasibility study are lower than the actual toll rates
currently charged on motorways in Serbia. Thus, the toll rates which are considered as
socially acceptable are €0.03 per kilometer for cars, €0.08 per kilometer for the third
category, and €0.15 per kilometer for the fourth category (Automobile and Motorcycle
Association of Serbia 2014). Table 20 presents the summary of the analysis and the comparison of weighted average toll rates which are socially acceptable in Serbia with the calculated mean value of the weighted average toll rate from the stochastic model.

**Table 20. Comparison of \( w_{at} \), for the Belgrade Bypass case study**

<table>
<thead>
<tr>
<th></th>
<th>I Cars</th>
<th>II Vehicles with or without trailers with two axles</th>
<th>III Vehicles with or without trailers with two or three axles</th>
<th>IV Vehicles with or without trailers with four or more axles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current toll rates in Serbia €/km (socially acceptable, VAT included)</td>
<td>0.03</td>
<td>N/A</td>
<td>0.08</td>
<td>0.15</td>
</tr>
<tr>
<td>Assumed traffic composition</td>
<td>85%</td>
<td>N/A</td>
<td>6%</td>
<td>9%</td>
</tr>
<tr>
<td>Weighted average toll rate €/veh/km (socially acceptable, VAT included)</td>
<td></td>
<td>0.044</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weighted average toll rate Section A €/veh/km (mean value, VAT included)</td>
<td></td>
<td>0.118</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weighted average toll rate Section B €/veh/km (mean value, VAT included)</td>
<td></td>
<td>0.154</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The acceptable toll rate is lower than the mean value obtained from simulations for both sections. For the given set of input variables, the weighted average toll rates of €0.118 per vehicle per km and €0.154 per vehicle per km have a probability of 0.5 to be financially sufficient. The weighted average toll rate which is socially acceptable €0.044 per vehicle per km corresponds, for example for section B, to less than 0.01 of the cumulative distribution function. In other words, there is a probability of 0.01 that the socially acceptable toll rate is at the same time financially sufficient. The toll rate for cars, which is calculated from the weighted average toll rate, is €0.11 per vehicle, which is almost three times more than the current level of toll rates in Serbia.

The analysis of the Belgrade Bypass case study as a PPP toll road shows that the weighted average toll rate for both sections is higher than the level of toll rates which users are currently paying in Serbia. Assuming the variability of input parameters reveals that the socially acceptable toll rate covers less than 1% of assumed risks, even for the brownfield section. The main drawbacks of the project delivered as a PPP would be: the relatively low percentage of vehicles which pay high toll tariffs and the project’s position in the surrounding network which allows relatively easy access to the competing non-tolled motorway.

Since some sections of the project are currently under operation on a non-tolled basis, it is possible to observe how some of the risks have materialized. Both sections A and B have experienced lower traffic than forecasted. The AADT is in the range from 9,000 vpd to 15,000 vpd for the period from 2009-2013 (PE Roads of Serbia 2015b). On the other hand, the number of heavy trucks (vehicles in the 4th category) is higher than estimated on some sections: from 9% to 17% for the same time period. It can be assumed that posing tolls would change the composition of traffic and additionally reduce it. The financial structure of the project as assumed in this case study can be considered as not sustainable over time and the project would probably be subject to renegotiation. The contribution from the government would probably need to be more substantial, possibly both as a subsidy to the construction costs and as some kind of revenue guarantee active throughout the life of the project.
Summary
Case studies are used to illustrate the developed methodology and to assess the level of the impact that the stochastic model has in the price of tolls. The case study of the Olympia Odos in Greece has shown that the project’s external uncertainties, such as the country's sovereign debt crisis, can be captured with the random component of traffic forecasts. The case study of the Belgrade Bypass represents an example when traffic forecasts for greenfield sections can be captured by using the additional risk component – initial traffic as a random variable. This case study also confirms that the magnitude of an error in the traffic forecast leads to a financially weak project. Both case studies reveal that the socially acceptable toll rate, i.e., the toll rate currently charged to the users, financially covers only a minor share of the assumed risks.
CHAPTER VII
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Following the presented methodology, the developed models and the lessons learned from the case studies, this chapter concludes this dissertation with the main findings, conclusions, and recommendations for future research.

Summary
Public-private partnerships are currently being implemented in infrastructure projects through a range of different contracts from maintenance and repair projects to expansion and development of new public infrastructure. The available tools and models for project evaluation are largely focused on the generalized cases. In reality, only a small portion of the projects are full-scale PPPs where the private partner has the autonomy to select the toll rate and make independent engineering and construction decisions. A large portion of projects delivered as PPPs are agreements between the public and the private partners with predefined roles, obligations, constraints, and risk-sharing mechanisms.

Observing a single road project, the associated risks can be grouped as systematic and unsystematic risks. Systematic risks, in the case of PPP toll roads, relate to the macroeconomic conditions and other market risks which cannot be controlled and managed at the project level. In this research, it is assumed that the stochastic traffic model can capture the systematic component of project risks, thus integrating the price of those risks in the toll tariffs. Unsystematic risks are risks related to the project itself, like construction and O&M elements of the project, and thus can be controlled and managed at the project level. It is assumed that these risks can be captured with the introduction of these elements as continuous random variables.

This dissertation introduces a new model that can capture the effect of a predefined socio-economically acceptable toll rate set by the public sector on the feasibility of the project and the risk allocation policy. Therefore, it can be a useful tool for the decision-makers and stakeholders in PPP toll road agreements for an early assessment of a project’s financial soundness.

The purpose of the presented methodology is to assess a single value, the probability that the socially acceptable toll rate is financially sustainable. This interpretation can be
used to identify potential pitfalls at the beginning of the financial analysis and risk assessment of a project, indicating the possibility of the project’s financial success or failure. This interpretation also gives an estimate of the “toll rate risk,” the risk that the agreed or assumed toll rate is not financially sufficient.

The methodology has several potential applications and may assist several stakeholders. One of the main purposes may be in the procurement processes where the bidder is bidding on the toll rate. The public sector may estimate the level of risk that the concessionaire is willing to accept through the toll rate which is offered. Also, the risk behavior of the concessionaire may be estimated, i.e., which bidder is more aggressive and willing to take risk in the bidding process. The concessionaire may use this tool to upgrade their current evaluation methods to an advanced level of volatility assessment. This tool will help in better understanding the interrelations among project parameters and their uncertainties and evolution over time. Another potential application is in the evaluation of the project based on other types of repaying mechanisms such as shadow tolls and availability fee.

The rating agencies, development, and commercial banks may also benefit from this tool. Although the stochastic models are already in use by rating agencies, the model which calculates the distribution of toll rates enables new insights into the project’s financial strength. It assembles several aspects of the project into single information that is easy for understanding the probability that the socially and economically acceptable toll rate is sufficient and sustainable to keep the project feasible.

The developed methodology and applied stochastic model represent a considerable step forward in the risk management of PPP road projects. The timely risk identification would allow better allocation of time for identification and development of the appropriate risk-management strategy.

**Research findings**

Developing the methodology for the assessment of the financially and socio-economically acceptable toll rates reveals several research findings. Drawing on the lessons learned from the deterministic and stochastic model development and the application of the methodology on case studies, the following findings can be highlighted:
looking at the changes of toll rates to any change in the \( AADT \) or construction costs within the project scope exclusively, i.e., “inner” elasticities, it can be observed that changes in the \( AADT \) or construction costs require adjustments of toll rates in order to keep the predefined financial constraints fulfilled as expected. It is anticipated that the stochastic model can capture this more realistically through the distribution of toll rates covering possible uncertainties of traffic and the materialization of construction risks.

- sensitivity analysis of the deterministic model revealed that operation and maintenance costs do have a certain impact on the financial dynamic of the project. The increase in O&M costs increases “inner” elasticities indicating the project’s internal vulnerability to any change in other “inner” parameters. Following these findings, the analysis and testing of the stochastic model showed that each risk has its share in the toll price. The analysis revealed that changes in O&M costs may have a considerable impact on toll tariffs.

- uncertainty in the initial traffic estimates, when introduced in the stochastic model, increases the toll rate needed to maintain the financial structure of the project at the required level. In other words, more risks utilize more money to cover those risks.

- both case studies revealed that socially acceptable toll rates financially cover only a minor share of the assumed risks. These findings highlight the importance of timely risk assessment and proper risk allocation since the risk materialization may significantly impact the operational life of the project and fulfillment of debt obligations.

Conclusions

Forecasting a system behavior in a dynamic environment is a challenging task, especially if the system under consideration is part of a wider system which also evolves over time. In this research, a toll road project and its defining parameters is the system under consideration. Such a system is delivered in the dynamic environment comprised of the larger road and transport network. Interactions among all elements of the system are numerous and quite complex. It is difficult to capture all these interactions under one PPP agreement which is designed to keep the project feasible over time for all stakeholders.
Decision-making in such a dynamic environment is a challenging task. Prioritizing transportation projects which need to be delivered is a socio-economic, multi-objective optimization problem. However, once the project gets a green light, the next step is to decide on the model for project delivery and finance, e.g., from the budget, from government-backed loans, or as a PPP project.

If a PPP project gets approved but the risks are not assessed and analyzed in the early phases of project development, the picture of project success can be misleading and can lead decision makers to controversial decisions. Even if the project's delivery can be fostered by government subsidies, its operational and financial life can be jeopardized due to the realization of risks. Timely and understandable information about potential risk scenarios is crucial in raising awareness among decision makers and stakeholders about possible costly downturns.

**Recommendations for future research**

Further research will expand the analysis of the maintenance costs to include periodic activities such as rehabilitation or reconstruction of the carriageway. Since these activities usually involve large investments, the question which arises here is how the selection of different maintenance strategies impacts the financially required toll rates. Also, options to expand the capacity, e.g., staged development, are subjects of further research as their application in PPP projects are getting attention. These options enable the better use of needed funds over time and the better management of associated uncertainties.

Operation costs are modeled as a function of construction costs and traffic volumes. A subject of further research is to investigate which part of the function has a higher impact on toll rates.

Options to specify the limit for traffic volumes is also a subject of interest. The number of lanes determines the maximum capacity of the highway, so introducing the model of the maximum cap for $AADT$ will help in better representing the real cases.

The presented model considers three random variables, but the variability of other parameters over time may also be present and considered. The model can be expanded to include other risks such as performance risk, etc. Moreover, some of these risks may be correlated, thus increasing the variance of the resulting distribution. The price
elasticity of demand and its correspondence with the model is also subject of further research.

In some cases, the $ADSCR$ is the strictest constraint, while in some cases $IRR$ or $RoE$ are prevailing. This depends on the mix of equity, debt, and subsidies, as well as the level required for these constraints. A subject of further research is to investigate which constraint is prevailing in the composition of financial structure.
VIII REFERENCES

Academic Dictionaries and Encyclopedias, Olympia Odos,


Beaty, C., and Lieu, H. (2012). Development of an Early-Stage Toll Revenue Estimation Model. UTCM project #09-22-02, Texas Transportation Institute, College Station, TX, USA.


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IX APPENDICIES

Appendix I
User’s guide

This appendix summarizes the information needed to run the set of simulations for the stochastic model. Its intent is to provide prospective users with sorted parameters as an input to the model and to clarify its output. Input parameters are divided into five different groups: technical parameters, financial structure, debt structure, country specific economic parameters and financial constraints. Type of repayment is assumed to be on level-annuity basis – principal plus interest is constant over the repayment period. This assumption cannot be changed in this version of the model. The output is the vector of random values of financially required toll rates. Notation is the same as in the code for better following.

Input

- **MC**: number of Monte Carlo simulations
- **No_sim**: Number of sampling

Technical parameters:

- **CL**: Concession life
- **CP**: Construction period
- **PW**: Distribution of works during the construction period
- **L**: project length in km, both tolled and non-tolled sections, if any
- **Lt**: length of tolled sections
- **Lanes**: number of lanes for maintenance
- **CCmean**: Estimated construction cost
- **CCstd**: standard deviation of construction costs
- **OC**: Operation cost
- **MC**: Maintenance costs
- **AADT0**: Estimated initial daily traffic
- **stdAADT0**: Standard deviation of initial daily traffic
- **TG**: Traffic growth
- **Std**: Volatility of traffic over time

Financial structure:
• IS: Percent of government subsidies
• E: Percent of equity

Debt structure:
• RP: Debt maturity, repayment period
• IR: Interest rate
• GP: Grace period

Country economic parameters:
• Inf: Inflation rate
• CTR: Corporate tax rate
• VAT: value added tax rate

Financial constraints:
• ADSCR: Annual debt service ratio
• IRR: internal rate of return
• ROE: return on equity

Output
• toll_rate_MC: vector with random sample from the population which represents
  the financially required toll rate (weighted average toll rate)

This output ends the code for the simulation of the stochastic model for calculation of
financially required toll rates. Users need to continue with the available statistical
program packages to test the sample, to find the distribution with the best fit and to
determine the probability that the socially acceptable toll rate is, in the same time,
financially sufficient.
Appendix II

Code for application

This appendix is a copy of the code developed in MATLAB for the simulation of the stochastic model. The basic code (marked *) calls two functions for the simulation of the geometric Brownian motion (marked ** and ***).

*Stochastic model

clear all
CL=25; %concession life, in years
CP=2; %construction period, in years
L=47; %section length in km, both greenfield and brownfield, needed to calculate maintenance costs
Lt=47; %length of the project with toll tariffs
lanes=4; %number of lanes
PW=zeros(1,CP);
PW(1,1)=0.5; %percent of construction work in 1st year
PW(1,2)=0.5; %percent of construction work in 2nd year
A=CL-CP; %amortization, in years
IS=0.2; %investment subsidies
E=0.2; %equity
RP=15; %debt maturity, in years
IR=0.12; %interest rate
INFR=0.10; %inflation rate
GP=2; %grace period, in years
CTR=0.1; %corporate tax rate
VAT=0.18;
q=0.05; %cost of transaction per vehicle as part of operation costs
pc=0.1; %percent of construction costs as operation costs
ADSCR=1.2; %annual debt service cover ratio
IRR=0.12; %internal rate of return
ROE=0.16; %return on equity
MC=200; %number of Monte Carlo simulations

No_sim=200; %No of sampling
toll_rate_sim=zeros(No_sim,MC); %create matrix with toll rates data
toll_rate_temp=zeros(No_sim,No_sim); %create temporary matrix with toll rates
for ii=1:1:MC

% initial traffic as random variable
AADT0=49100;%initial traffic
stdAADT0=7000;%standard deviation of initial traffic
AADT0 = randn(No_sim,1)*stdAADT0+AADT0; %initial traffic as normal distribution

for iii=1:1:No_sim

% generate construction costs CC from normal distribution
CCmean = 337800000;
CCstd = 30000000;
CC = randn(No_sim,1)*CCstd+CCmean; %construction cost, in US$

% generate traffic as Geometric Brownian motion
D=AADT0(iii,1); %start traffic, assumed value at the beginning of operation life at T=0, in veh/day
TG=0.05; %traffic growth
std=0.2; %GBM volatility of traffic
test_simplest_montecarlo0 %function as a separate code, marked with **
St=transpose(S); %returns matrix with traffic volumes over concession life

% set up matrix with AADT
temp=zeros(No_sim,CP);
St(:,1)=[];
AADT=[temp St];

% Inflation factor for concession period and number of simulations
I=zeros(1,CL+1);
for i=1:1:(CL+1)
    I(1,i)=(1+INFR)^i;
end

% generate operation and maintenance costs
OC=zeros(No_sim,CL);
M=zeros(No_sim,CL);
%Operation cost and Traffic volume for concession life
Operation=zeros(No_sim,CL);
Traffic=zeros(No_sim,CL);
for j=1:1:No_sim
  for i=1:1:CP
    M(j,i)=0;
    OC(j,i)=0;
    Operation(j,i)=0;
    Traffic(j,i)=0;
  end
  for i=(CP+1):1:CL
    M(j,i)=(1700*2+(0.5*AADT(j,i)))*L; %maintenance costs
    according to Heggie's formula
    OC(j,i)=pc*CC(j,1)+q*AADT(j,i)*365+M(j,i); %first
terms is 10% of CC as operation cost,
    %second term is cost per transaction per vehicle per
    year
    Operation(j,i)=OC(j,i)*I(1,i);
    Traffic(j,i)=AADT(j,i)*365;
  end
end

%Construction expenditures
Con_costs=zeros(No_sim,CP);
Con_cost_total=zeros(No_sim,1);
Equity=zeros(No_sim,CP);
Equity_total=zeros(No_sim,1);
Subsidies=zeros(No_sim,CP);
Subsidies_total=zeros(No_sim,1);
Debt_wo=zeros(No_sim,CP); %debt without capitalised interest
Cap_interest=zeros(No_sim,CP); %Capitalised interest
Cap_interest_total=zeros(No_sim,1);
Debt=zeros(No_sim,CP); %drawdowns
Debt_total=zeros(No_sim,1);

for j=1:1:No_sim
  for i=1:1:CP
    Con_costs(j,i)=PW(1,i)*CC(j,1)*I(1,i);
  end
end
Con_cost_total(j,1)=Con_cost_total(j,1)+Con_costs(j,i);
Equity(j,i)=Con_costs(j,i)*E;
Subsidies(j,i)=Con_costs(j,i)*IS;
Subsidies_total(j,1)=Subsidies_total(j,1)+Subsidies(j,i);
Debt_wo(j,i)=Con_costs(j,i)*(1-E-IS);
end
Cap_interest(j,1)=(Debt_wo(j,1)/2)*IR;
Debt(j,1)=Cap_interest(j,1)+Debt_wo(j,1);
Cap_interest(j,2)=(Debt(j,1)+(Debt_wo(j,2)/2))*IR;
Debt(j,2)=Cap_interest(j,2)+Debt_wo(j,2);
for i=1:1:CP
    Cap_interest_total(j,1)=Cap_interest_total(j,1)+Cap_interest(j,i);
    Debt_total(j,1)=Debt_total(j,1)+Debt(j,i);
    Equity_total(j,1)=Equity_total(j,1)+Equity(j,i);
end
end

%Amortization
Amort=zeros(No_sim,CL);
Amort_no_int=zeros(No_sim,CL);
for j=1:1:No_sim
    for i=(CP+1):1:(CP+A)
        Amort(j,i)=(Con_cost_total(j,1)+Cap_interest_total(j,1)-
                    Subsidies_total(j,1))/A;
        Amort_no_int(j,i)=Con_cost_total(j,1)/A;
    end
end

%Debt Service
P=zeros(No_sim,CL);
Int=zeros(No_sim,CL);
for j=1:1:No_sim
    [Principal,Interest,Balance,Payment]=amortize(IR,(RP-
                      GP),Debt_total(j,1));
    for i=1:1:(RP-GP)
        P(j,i+GP)=Principal(1,i);
    end
    for i=1:1:(RP-GP)
\begin{verbatim}

Int(j,i+GP)=Interest(1,i);
end
for i=(CP+1):1:GP
    Int(j,i)=Debt_total(j,1)*IR;
end
end

%toll rate calculation for minimum ADSCR
TR_ADSCR=zeros(No_sim,CL);
for j=1:1:No_sim
    for i=(CP+1):1:RP
        TR_ADSCR(j,i)=(ADSCR*(Int(j,i)+P(j,i))+Operation(j,i)-
        (Operation(j,i-1)+Amort(j,i-1)+Int(j,i-1))*CTR)/(Traffic(j,i)*I(1,i)-
        Traffic(j,i-1)*I(1,i-1)*CTR);
    end
end

%toll rate calculation for minimum project IRR, real terms
Temp_11=zeros(No_sim,CL);
Temp_21=zeros(No_sim,CL);
for j=1:1:No_sim
    for i=(CP+1):1:CL
        Temp_11(j,i)=(Operation(j,i)-(Operation(j,i-1)+Amort_no_int(j,i-1))*CTR)/I(1,i);
        Temp_21(j,i)=(Traffic(j,i)*I(1,i)-Traffic(j,i-1)*I(1,i-1)*CTR)/I(1,i);
    end
    for i=1:1:CP
        Temp_11(j,i)=Con_costs(j,i)/I(1,i);
    end
end
TR_IRR=zeros(No_sim,CL);
for j=1:1:No_sim
    for i=(CP+1):1:CL
        X11=Temp_11(j,1:i);
        X21=Temp_21(j,1:i);
        TR_IRR(j,i)=pvvar(X11,IRR)/pvvar(X21,IRR);
    end
end

\end{verbatim}
Temp_12=zeros(No_sim,CL+1);
Temp_22=zeros(No_sim,CL+1);
for j=1:1:No_sim
  for i=(CP+1):1:CL
    Temp_12(j,i)=(Operation(j,i)+Amort(j,i)+Int(j,i))*(1-CTR)/I(1,i);
    Temp_22(j,i)=(Traffic(j,i)*(1-CTR));
  end
  for i=1:1:CP
    Temp_12(j,i)=Equity(j,i)/I(1,i);
  end
  Temp_12(j,CL+1)=Equity_total(j,1)/I(1,CL+1);
end

TR_ROE=zeros(No_sim,CL+1);
for j=1:1:No_sim
  for i=(CP+1):1:(CL+1)
    X12=Temp_12(j,1:i);
    X22=Temp_22(j,1:i);
    TR_ROE(j,i)=pvvar(X12,ROE)/pvvar(X22,ROE);
  end
end

toll_rate=zeros(No_sim,1);
TR=zeros(No_sim,3);
for j=1:1:No_sim
  TR_ROE(~TR_ROE)=inf;%remove zeros from vector
  TR_IRR(~TR_IRR)=inf;
  TR(j,:)=[max(TR_ADSCR(j,:)) min(TR_IRR(j,:)) min(TR_ROE(j,:))];
  toll_rate(j,1)=max(TR(j,:))*(1+VAT);
end

for j=1:1:No_sim
  toll_rate_temp(j,iii)=toll_rate(j,1);
end
toll_rate_temp_average=mean(sort(toll_rate_temp),2);

for j=1:1:No_sim
    toll_rate_sim(j,ii)=toll_rate_temp_average(j,1);
end

h = waitbar(0, 'Please wait...');
waitbar(ii/MC)
close(h)
end

toll_rate_MC=mean(sort(toll_rate_sim),2)/Lt;

**test_simplest_montecarlo0
S0      = D;  %Traffic in year 0
mu      = TG; %drift, traffic growth
sigma   = std; %volatility, traffic dispersion over time
T       = CL-CP; %number of years of operation life
nb_traj = No_sim; %number of simulations
step    = 1;  %step at which simulation is calculated, if is equal
to 1, it is simulated each year

[S, t]  = simplest_montecarlo0( sigma, T, nb_traj, S0, mu, step);
%function as a separate code, marked with ***

***simplest_montecarlo0

function [S, t] = simplest_montecarlo0( sigma, T, nb_traj, S0, mu, step)
nT = ceil(T/step);
W  = sigma * sqrt(step) * cumsum(randn(nT, nb_traj));
c  = repmat((mu - sigma^2/2) * step * (1:nT)',1,nb_traj); %GBM with
S  = [repmat(S0,1,nb_traj); S0 * exp( c + W)];
if nargout > 1
    t = [0;step * (1:nT)'];
end
VITA

Nevena Vajdić obtained her Diploma in Civil Engineering from the University of Belgrade (UoB) in 2002. Following her graduation from UoB, she worked as a project design engineer in Belgrade. In 2007, she was admitted to the graduate program at the Zachry Department of Civil Engineering, Texas A&M University, College Station, TX, from which she obtained a Master of Science in Civil Engineering. Her studies at Texas A&M mainly focused on risk assessment of toll roads delivered as public-private partnerships. After her graduation from Texas A&M in 2009, she pursued her education towards a Doctor of Philosophy degree in Civil Engineering at the Faculty of Civil Engineering in Belgrade. Her research focus is in risk management, real options theory, toll roads and public-private partnerships.

She lives and works in Belgrade. She is married and has two daughters.
Прилог 1.

Изjava о ауторству

Потписани-а: Невена Вајдић
број индекса: ________________________________

Изјављујем

да је докторска дисертација под насловом

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СТОХАСТИЧКИ МОДЕЛ ЗА УТВРЕЂИВАЊЕ
ОПТИМАЛНЕ ПУТАРИНЕ
---

- резултат сопственог истраживачког рада,
- да предложена дисертација у целини ни у деловима није била предложена за добијање било које дипломе према студијским програмима других високошколских установа,
- да су резултати коректно наведени и
- да нисам кршио/ла ауторска права и користио интелектуалну својину других лица.

Потпис докторанда

У Београду, [16.03.2016]
Прилог 2.

Изјава о истоветности штампане и електронске верзије докторског рада

Име и презиме аутора ________________________________

Број индекса ________________________________

Студијски програм ________________________________

Наслов рада ________________________________

Ментор ________________________________

Потписани/а ________________________________

Изјављујем да је штампана верзија мој докторског рада истоветна електронској верзији коју сам предао/ла за објављивање на порталу Дигиталног репозиторијума Универзитета у Београду.

Дозвољавам да се објаве моји лични подаци везани за добијање академског звања доктора наука, као што су име и презиме, година и место рођења и датум одбране рада.

Ови лични подаци могу се објавити на мрежним страницама дигиталне библиотеке, у електронском каталогу и у публикацијама Универзитета у Београду.

Потпис докторанда ________________________________

У Београду, ________________________________
Прилог 3.

Изјава о коришћењу

Овлашћујем Универзитетску библиотеку „Светозар Марковић“ да у Дигитални репозиторијум Универзитета у Београду унесе моју докторску дисертацију под насловом:

СТОХАСТИЧКИ МОДЕЛ ЗА УТРБИВАЊЕ
ОПТИМАЛНЕ ПУТАРИНЕ

која је моје ауторско дело.

Дисертацију са свим приликома предао/ла сам у електронском формату погодном за трајно архивирање.

 Моју докторску дисертацију похрањену у Дигитални репозиторијум Универзитета у Београду могу да користе сви који поштују одредбе садржане у одабраном типу лиценце Креативне заједнице (Creative Commons) за коју сам се одлучио/ла.

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(Молимо да заокружите само једну од шест понуђених лиценци, кратак опис лиценци дат је на полеђини листа).

У Београду, 16.03.2016.


Потпис докторанда

N. Vojdic